Operational Research II

MidTerm

1. A firm is selecting a new energy generator. It can buy the generator now (generator A) sustaining a cost of 500 and receiving a sving benefit of 120 per year over the next five years, for sure. Or it can decide to wait for the new type of energy generator (generator B). Generator B is a clean one, whose cost is -700. However, there is the possibility of a government incentive of 30% of the cost. The probability that the government will introduce the incentive in one year (PI) is $PI = 0.5$. The new type of generator, however will have an incertain performance. It can perform well, guaranteeing a recovery of 180 per year over the next five years, or it can show a poor performance, guaranteeing savings of up to 120 per year. The probability of good performance is $PG \sim u(0,1)$. Knowing that the discunt rate of the firm is 5%,

- **a.** Draw the influence diagram for this decision
- **b.** Draw the corresponding decision tree
- **c.** What is the preferred alternative?
- **d.** What is the value of PG for which you are indifferent among the two alternatives?
- **e.** Compute the expected value of perfect information on the government incentive
- **f.** Suppose that you gather further evidence on the performance of new generators looking at 11 new products. In 6 out eleven cases the performance is good. What do you decide now? (Hint: $\frac{\int_0^1 p^7(1-p)^5 dp}{\int_0^1 s^6(1-s)^5 ds} = \frac{7}{13} = .54$)

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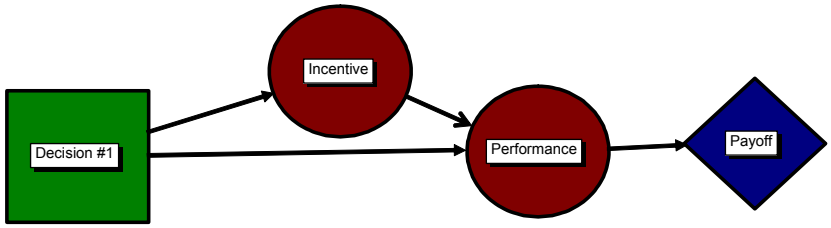
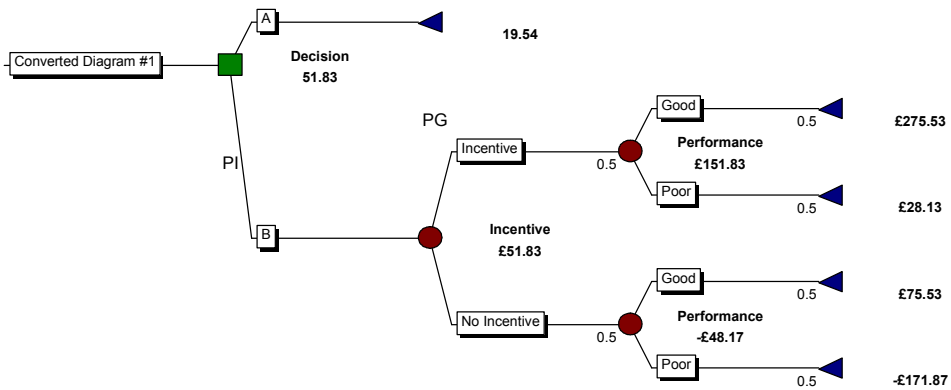
a.

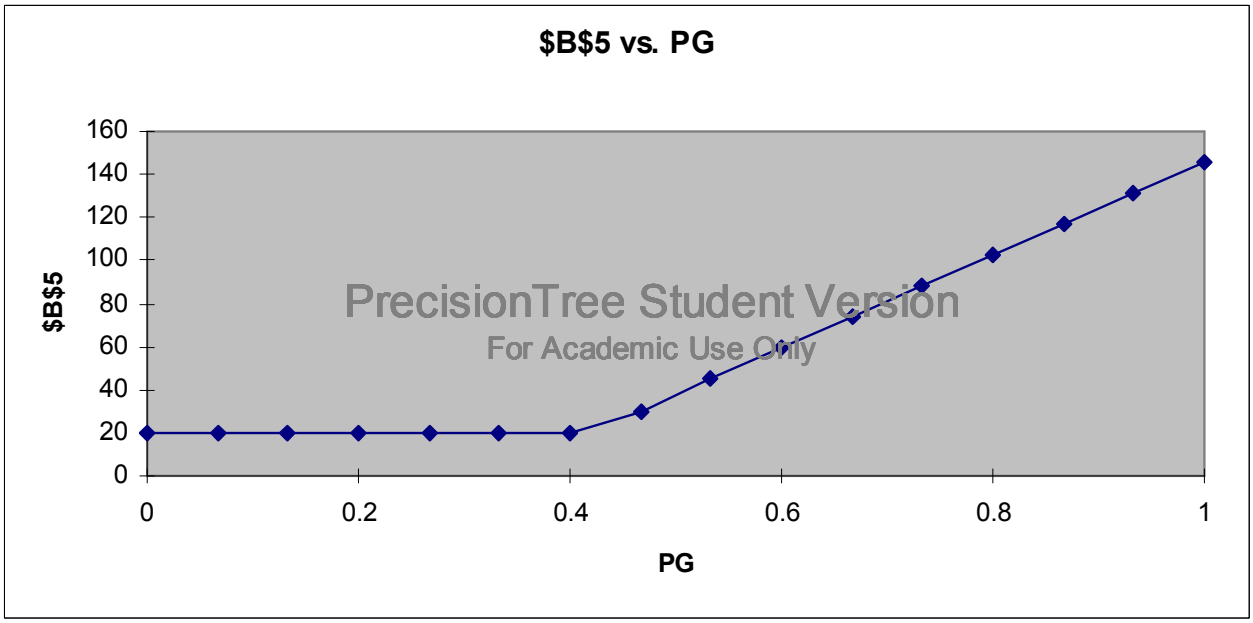


b.

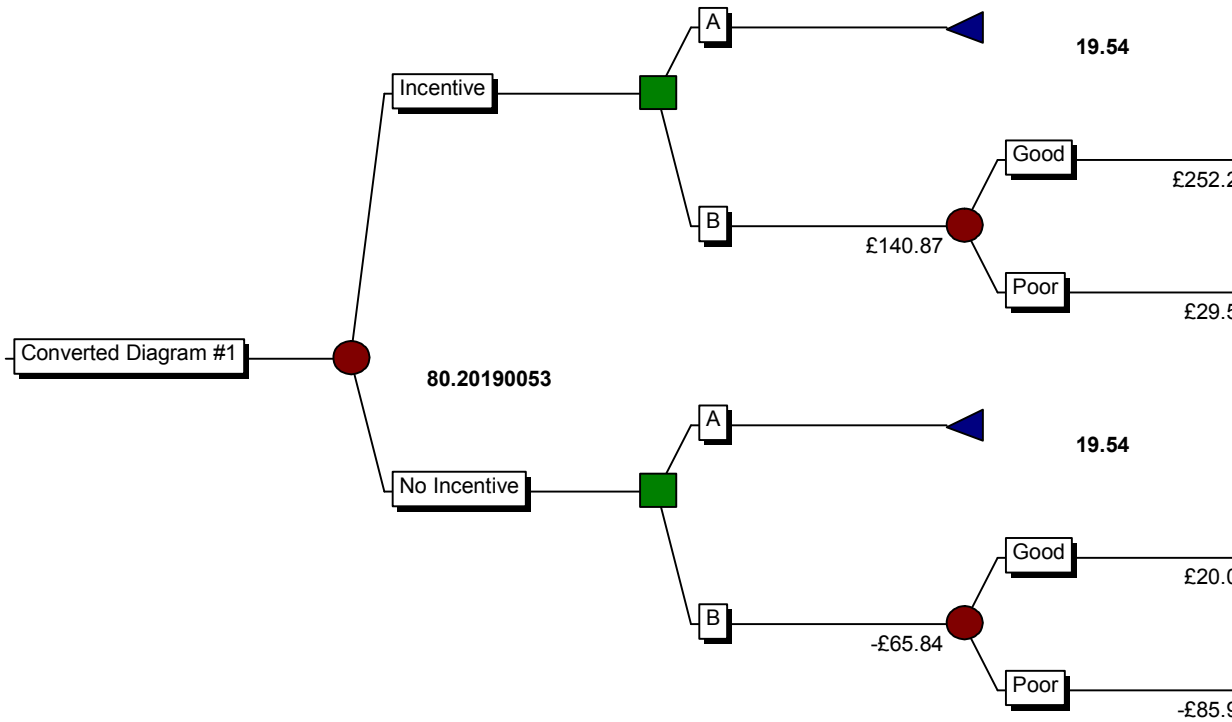


c. See above

d. 0.41



e.



EVPI **42.68926663**

f. From d, still B.

2. A random variable is characterized by the following:

$$f(x) = \begin{cases} k\cos x & if \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & otherwise \end{cases} \qquad \#$$

Using a perfect random number generators that produces uniform numbers bewteen 0 and 1, you want to generate random variables distributed according to $f(x)$.

a. What is the value of k that makes $f(x)$ a density function

b. What is the inversion equation?

c. Suppose the first random number generated is 0.5. What are the corresponding values of $F(x)$ and x?

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a.

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)dx = 2k \qquad \#$$

Hence:

$$k = \frac{1}{2} \qquad \#$$

b.

$$\int_{-\frac{\pi}{2}}^{x} k\cos s ds = \frac{1}{2}\sin x + \frac{1}{2} \qquad \#$$

Hence:

$$u = \frac{1}{2}\sin x + \frac{1}{2} \qquad \#$$

Solution is :

$$x = \arcsin(2u - 1) \qquad \#$$

c.

$$F(x) = 0.5 \qquad \#$$

$$u = 0.5 \qquad \#$$

implies

$$x = 0 \qquad \#$$

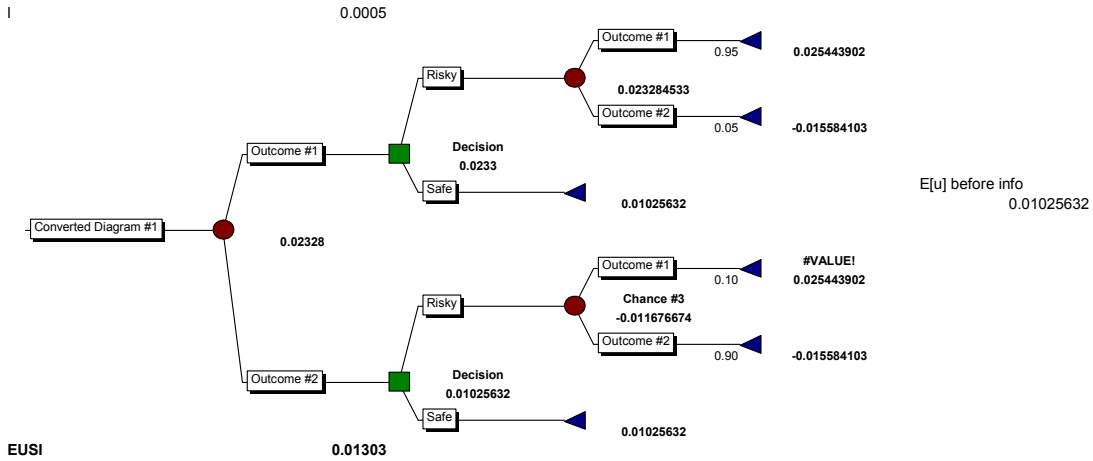
3. You have to decide bewteen two investments, A and B. A gives for sure $x_1 = 20$. B gives $x_2 = 50$, if the market goes up, with probability $P = .5$ and of $x_3 = -30$ with probability $1 - P = 0.5$. You have the following utility function:

$$u(x) = 1 - e^{-\alpha x} \qquad \#$$

A source of information says that the market goes up when eventually the market goes up with probability 0.9 ($P\text{sup}|up = 0.9$). And it also tells you that it goes down when eventually the market goes down, with probability 0.95(Psdown$|down = 0.95$). Set $\alpha = 1/40$.

What is the expected utiliy of sample information for this source?

a. _____



says	up	down
sup	0.90	0.05
sdown	0.10	0.95
pup	0.5	
pdown	0.5	
psup	0.48	0.48
psdown	0.53	0.53

	sup	sdown
up	0.94737	0.10
down	0.05263	0.90

pup_sup	psup_up*pup/psup	pup_sdown	psdown_up*pup/psdown
pdown_sup	psup_down*pdown/psup	pdown_sdown	psdown_down*pdown/psdown