Operational Research II

June 29th 2007

Second Midterm

Familiy Name:_____

Name:_____

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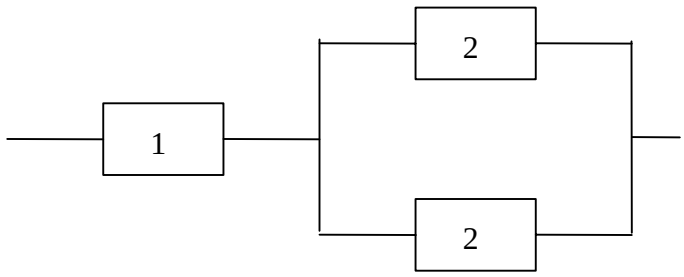
Write your official answers in the dedicated space below each question. Justify your answers utilizing either the dedicated space or rough copy for the calculations. Not justified answers will not be considered.

1 Given the following system, composed by two types of components arranged as follows:



1.a Determine the structure function of the system

1.b Determine the minimal cut sets

1.c Determine the expression of the system failure probability within time T, using the rare event approximation

Suppose that the component failure rates are constant and given by:

$$\lambda = \begin{bmatrix} 1 \cdot 10^{-2} \\ 2 \cdot 10^{-2} \end{bmatrix} \qquad \#$$

in units 1/year.

1.d Determine the system average unavailability over 1 year using Taylor approximation and the rare event approximation.

1.e Suppose that the system is subject to periodic maintenance of lenght $\tau_r = 1day$. Derive the equation that determines the optimal maintenance interval

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1.a

$$X_T = 1 - (1 - X_1)(1 - X_2X_3) = X_1 + X_2X_3 - X_1X_2X_3 \qquad \#$$

1.b

$$1, \text{ and } (2,3) \qquad \#$$

1.c

$$1 - e^{-\int_0^t \lambda_1(s)ds} + (1 - e^{-\int_0^t \lambda_2(s)ds})(1 - e^{-\int_0^t \lambda_3(s)ds}) \qquad \#$$

1.d

$$F(t) = \lambda_1 t + \lambda_2 t^2 \qquad \#$$

$$\frac{\int_0^\tau (\lambda_1 t + \lambda_2 t^2)dt}{\tau} = \frac{1}{6}\tau(3\lambda_1 + 2\tau\lambda_2)$$

$$\overline{q}(\tau) = \frac{\int_0^\tau (\lambda_1 t + \lambda_2 t^2)dt}{\tau} = \lambda_1 \frac{\tau}{2} + \lambda_2 \frac{\tau^2}{3} \qquad \#$$

1.e

$$\overline{q}(\tau) = \lambda_1 \frac{\tau}{2} + \lambda_2 \frac{\tau^2}{3} + \frac{\tau_r}{\tau} \qquad \#$$

$$\frac{d}{d\tau}(\lambda_1 \frac{\tau}{2} + \lambda_2 \frac{\tau^2}{3} + \frac{\tau_r}{\tau}) = \frac{1}{6\tau^2}(4\lambda_2\tau^3 + 3\lambda_1\tau^2 - 6\tau_r) \qquad \#$$

$$4\lambda_2\tau^3 + 3\lambda_1\tau^2 = 6\tau_r \qquad \#$$

2 A component is characterized by the following failure rate, which is a function of time:

$$\lambda(t) = \begin{cases} \frac{3}{t+1} & if \quad 0 < t < 2 \\ t & if \quad t \geq 2 \end{cases} \qquad \#$$

2.a What is the reliability of the component at $t = 1$?

2.b What is the reliability of the component at $t = 3$?

Hint: $\int \frac{3}{t+1} dt = 3\ln(t+1)$

2.a $e^{-\int_0^1 \lambda(t)dt} = \frac{1}{8}$

2.b $e^{-\int_0^3 \lambda(t)dt} = e^{-3\ln 3 - \frac{5}{2}} = 3.0402 \times 10^{-3}$

$-3\ln 3 - \frac{5}{2} = -5.7958$

3 A system is repairable. It has 3 states: fully working, partially working and damaged. When the system is working, it can become partially working with failure rate λ. If it is partially working, the repair has a rate equal to μ. If it is partially working, then it can become damaged with a rate 2λ. The repair then brings the component to the fully working state with a rate $\frac{\mu}{2}$.

3.1 Draw the state diagram for this problem

3.2 Write the corresponding transition matrix (A)

3.3 Write the markov-Kolmogorov equations for the system

3.4 If the system is considered "available" in states 1 and 2, find the system steady state unavailability

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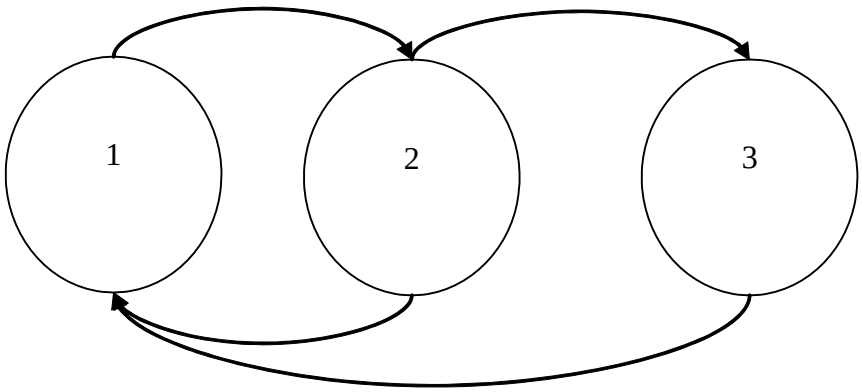
3.1



3.2

$$\begin{bmatrix} -\lambda & \mu & \frac{\mu}{2} \\ \lambda & -2\lambda+\mu & 0 \\ 0 & 2\lambda & \frac{\mu}{2} \end{bmatrix} \qquad \#$$

$$A = \begin{matrix} -\lambda & \mu & \frac{\mu}{2} \\ \lambda & -2\lambda+\mu & 0 \\ 0 & 2\lambda & \frac{\mu}{2} \end{matrix} \qquad \#$$

3.3

$$\begin{matrix} \frac{dP_1}{dt} \\ \frac{dP_2}{dt} \\ \frac{dP_3}{dt} \end{matrix} = \begin{matrix} -\lambda & \mu & \frac{\mu}{2} \\ \lambda & -2\lambda+\mu & 0 \\ 0 & 2\lambda & \frac{\mu}{2} \end{matrix} \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} = \begin{matrix} \mu P_2 - \lambda P_1 + \frac{1}{2}\mu P_3 \\ \lambda P_1 + P_2(\mu - 2\lambda) \\ 2\lambda P_2 + \frac{1}{2}\mu P_3 \end{matrix} \qquad \#$$

3.4 If the system is considered "available" in states 1 and 2, find the system steady state unavailability

$$\begin{matrix} 1 & 1 & 1 \\ \lambda & -2\lambda+\mu & 0 \\ 0 & 2\lambda & \frac{\mu}{2} \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \qquad \#$$

, Solution is:

$$\frac{1}{2\lambda}\frac{2\lambda\left(\frac{1}{2}\mu^2-\frac{3}{2}\lambda\mu\right)+\lambda^2\mu}{2\lambda^2-\frac{3}{2}\lambda\mu+\frac{1}{2}\mu^2}$$

$$-\frac{1}{2}\lambda\frac{\mu}{2\lambda^2-\frac{3}{2}\lambda\mu+\frac{1}{2}\mu^2}$$

$$2\frac{\lambda^2}{2\lambda^2-\frac{3}{2}\lambda\mu+\frac{1}{2}\mu^2}$$