Operational Research II

June 01 2006

Test

Familiy Name:_____

Name:_____

Matricola:_____

Write your official answers in the dedicated space below each question. Justify your answers utilizing either the dedicated space or rough copy for the calculations. Not justified answers will not be considered.

1. A decision maker has the following utility function:

$$u(x) = e^{-\frac{1}{x^2}} \qquad \#$$

a. What is His/Her risk aversion function?

b. At x=1/2, is She/He risk prone or risk averse?

c. At x=30 is She/He risk prone or risk averse?

_____

$$u'(x) = \frac{2}{x^3} e^{-\frac{1}{x^2}} \qquad \#$$

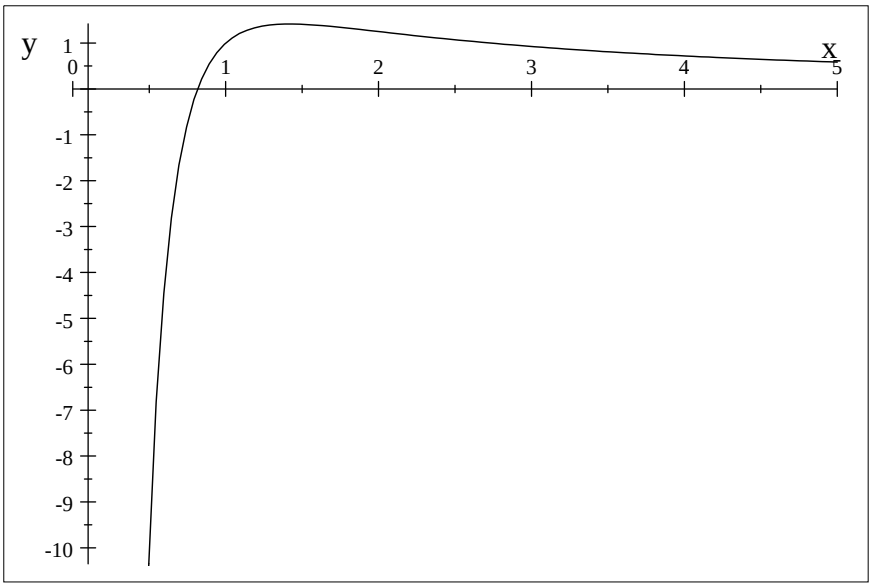
$$u''(x) = \frac{4}{x^6} e^{-\frac{1}{x^2}} - \frac{6}{x^4} e^{-\frac{1}{x^2}} \qquad \#$$

$$r(x) = -\frac{u''(x)}{u'(x)} = \frac{3x^2 - 2}{x^3} \qquad \#$$

d. $r(0.5) = -10.0 = -10.0$

e. $r(30) = \frac{1349}{13500} = \frac{1349}{13500} = 9.9926 \times 10^{-2}$

f. $\frac{3x^2-2}{x^3}$


y
x
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
1
2
3
4
5

2. Two identical components are characterized by a constant failure rate equal to $9 \cdot 10^{-2}[1/year]$. Compute the reliability of the system obtained setting:

a the components in series

b the components in parallel

c the components in a cold standby configuration

$\lambda := 9 \cdot 10^{-2}$

$n := 2$

$R(t) := e^{-\lambda \cdot t}$

$R(1) = 0.9139311852712$

Parallel $RP(t) := 1 - (1 - R(t))^{n}$

$RP(1) = 0.992592159131184$

$RS(t) := (R(t))^{n}$

$RS(1) = 0.835$

$Rss(t) := e^{-\lambda \cdot t} \cdot \left[1 + \lambda \cdot t + \frac{(\lambda \cdot t)^2}{2} \right]$

$Rssn(t) := e^{-\lambda \cdot t} \cdot \sum_{i=0}^{n-1} \frac{(\lambda \cdot t)^i}{i!}$

$Rss(1) = 0.999886413245987$

$q := 0.01$

$Rssn(1) = 0.996$

$Rsq(t) := e^{-\lambda \cdot t} \cdot [1 + (1 - q) \cdot (\lambda \cdot t)]$

$Rsq(8760) = 0$

3. A decision maker must choose between two systems, made of components 1 and 2, with failure rates $7 \cdot 10^{-2}[1/year]$ and $2 \cdot 10^{-2}[1/year]$ respectively. System A is composed of the series of the two components. If it fails within one year, the decision maker looses 95000EUR. If it does not fail, the decision maker does not loose anything. System B is composed of the parallel of the two components. If it fails within one year, the decision maker looses 180000EUR, otherwise the decision maker looses nothing.

a Draw the influence diagram for this problem

b Derive the corresponding decision tree

c Compute the failure probabilities of systems A and B

d Evaluate the best alternative

---

a

$\lambda 1 := 7 \cdot 10^{-2}$ $\quad \lambda 2 := 2 \cdot 10^{-2}$

$R1(t) := e^{-\lambda 1 \cdot t}$ $\quad R2(t) := e^{-\lambda 2 \cdot t}$

$R1(1) = 0.9323938199059$ $\quad R2(1) = 0.98$

Parallel $\quad RP(t) := 1 - (1 - R1(t)) \cdot (1 - R2(t))$ $\quad FP(t) := 1 - RP(t)$

$RP(1) = 0.998661307941475$ $\quad FP(1) = 1.339 \times 10^{-3}$

$RS(t) := R1(t) \cdot R2(t)$

$RS(1) = 0.914$

$(1 - RP(1)) \cdot 180000 = 240.965$

$(1 - RS(1)) \cdot 95000 = 8.177 \times 10^{3}$

5 A system is composed of two components in parallel, with failure rates equal to λ_1 and λ_2. There is only one repairman, who repairs the components with repair rate equal to μ.

a Draw the state diagram for this system

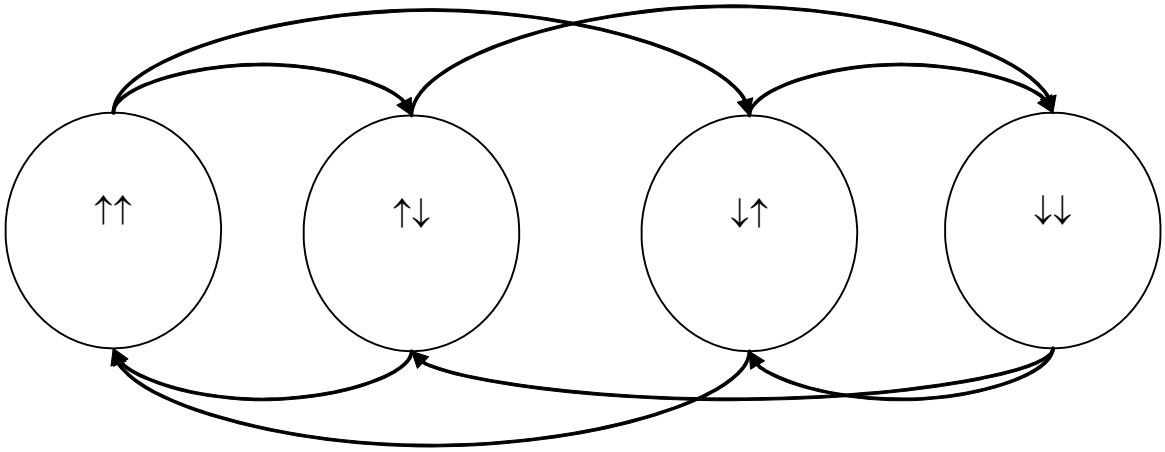
b Write the Markov Matrix

c Write the Markov-Kolmogorov Equation (Hint: $\frac{d\mathbf{P}}{dt} = A\mathbf{P}$)

d Write the equation from which one would obtain the steady state probabilities of the system and explain how one would obtain a solution (You do not need to perform the calculations.)

---

a



b

$$\begin{bmatrix} -\lambda_1 - \lambda_2 & \mu & \mu & 0 \\ \lambda_2 & -\lambda_1 - \mu & 0 & \mu \\ \lambda_1 & 0 & -\lambda_2 - \mu & \mu \\ 0 & \lambda_1 & \lambda_2 & -\mu - \mu \end{bmatrix} \qquad \#$$

c

$$\begin{bmatrix} \frac{dP_1}{dt} \\ \frac{dP_2}{dt} \\ \frac{dP_3}{dt} \\ \frac{dP_4}{dt} \end{bmatrix} = \begin{bmatrix} -\lambda_1 - \lambda_2 & \mu & \mu & 0 \\ \lambda_2 & -\lambda_1 - \mu & 0 & \mu \\ \lambda_1 & 0 & -\lambda_2 - \mu & \mu \\ 0 & \lambda_1 & \lambda_2 & -\mu - \mu \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \qquad \#$$

$$
\begin{aligned}
\frac{dP_1}{dt} &= \mu P_2 - P_1(\lambda_1 + \lambda_2) + \mu P_3 \\
\frac{dP2}{dt} &= \mu P_4 - P_2(\mu + \lambda_1) + \lambda_2 P_1 \\
\frac{dP_3}{dt} &= \mu P_4 - P_3(\mu + \lambda_2) + \lambda_1 P_1 \\
\frac{dP_4}{dt} &= \lambda_1 P_2 - 2\mu P_4 + \lambda_2 P_3
\end{aligned}
\qquad \#
$$

d

$$
\begin{aligned}
0 &= \mu P_2 - P_1(\lambda_1 + \lambda_2) + \mu P_3 \\
0 &= \mu P_4 - P_2(\mu + \lambda_1) + \lambda_2 P_1 \\
0 &= \mu P_4 - P_3(\mu + \lambda_2) + \lambda_1 P_1 \\
0 &= \lambda_1 P_2 - 2\mu P_4 + \lambda_2 P_3
\end{aligned}
\qquad \#
$$

$$
-P_1(0) + s\widetilde{P}_1 = \mu\widetilde{P}_2 - \widetilde{P}_1(\lambda_1 + \lambda_2) + \mu\widetilde{P}_3
$$

$$
-P_2(0) + s\widetilde{P}_2 = \mu\widetilde{P}_4 - \widetilde{P}_2(\mu + \lambda_1) + \lambda_2\widetilde{P}_1 \qquad \#
$$

$$
-P_1(0) + s\widetilde{P}_1 = \mu\widetilde{P}_2 - \widetilde{P}_1(\lambda_1 + \lambda_2) + \mu\widetilde{P}_3
$$