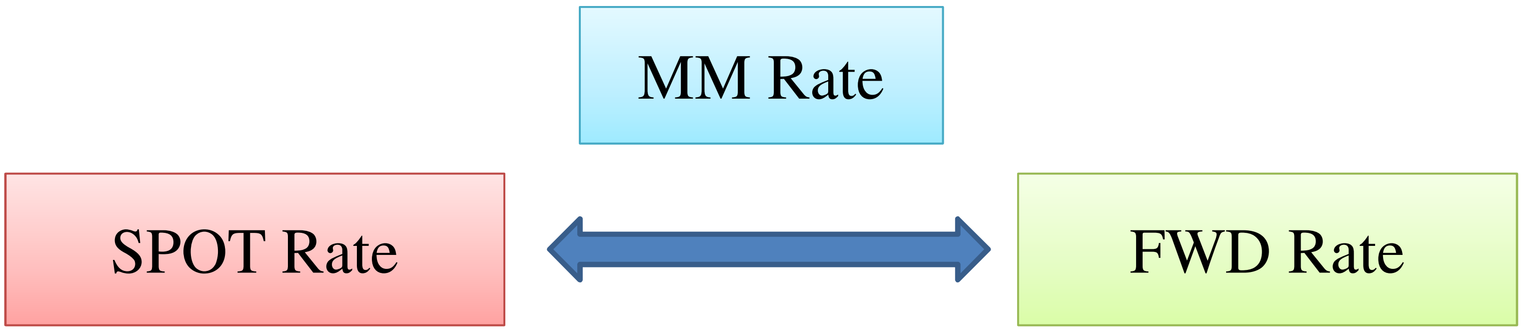
Lesson III: Overview

1. The relationship between spot, forward and money market rates
2. How to construct synthetic securities

The relationship between spot, forward and money rates

Spot, Fwd & MM Rates



Investment/borrowing decisions and currencies of denomination I

Assume you have some funds to place in the money market for 3 months: how to choose between domestic and foreign currency-denominated securities?

↓

Relying exclusively on interest rate differentials might be seriously misleading: both **interest and exchange rates** should be taken into due account.

Investment/borrowing decisions and currencies of denomination II

1st option: invest in a USD-denominated security (assuming $ is the domestic currency)

↓

At the end of a 3 months period, this would yield

$$\left(1+\frac{r_{\$}}{4}\right)$$

Investment/borrowing decisions and currencies of denomination III

2nd option: buy £ to invest in a GBP-denominated security and finally sell GBP foward after 3 months

1. Buy £: $\frac{1}{S_{\$/£}}$
2. Invest $\frac{1}{S_{\$/£}}$ in a GBP - denominated security

   and get $\frac{1}{S_{\$/£}}\left(1+\frac{r_{£}}{4}\right)$
3. Sell GBP forward to receive $\frac{F_{0.25\$/£}}{S_{\$/£}}\left(1+\frac{r_{£}}{4}\right)$ USD

Investment decisions and currencies of denomination IV

You will be indifferent between the two options only if

$$\left(1+\frac{r_{\$}}{4}\right) = \frac{F_{0.25\$/£}}{S_{\$/£}}\left(1+\frac{r_{£}}{4}\right)$$

Investment/borrowing decisions and currencies of denomination V

Rearranging the terms we would get:

$$r_{\$} = r_{£} + 4\left[\frac{F_{0.25\$/£} - S_{\$/£}}{S_{\$/£}}\right]$$

Annualised fwd premium/discount on £

Annualised GBP interest rate

Covered Interest Rate Parity I

More generally, if we allow for compound interest, an investor/ borrower would be indifferent between domestic and foreign currency denominations of investment or debt if

$$(1+r_D)^n = \frac{F(_nD/F)}{S(D/F)}(1+r_F)^n$$

Covered Interest Rate Parity II

When steps have been taken to **avoid foreign exchange ris**k by use of **forward contracts** (hence the term "covered"), **rates of return on investments and costs of borrowing will be equal, irrespective of the currency of denomination** (*ceteris paribus*)

Covered Interest Rate Parity III

Delving with the *ceteris paribus* condition

↓

There must be no frictions for the CIRP to hold, meaning no legal restrictions on the movement of K, no tax advantages among different countries…

Covered Interest Rate Parity IV

The CIRP links tightly together Spot and Forward rates → persistent deviations are **unlikely** to occur, because this would give rise to arbitrage opportunities (**No Free Lunch Principle**)

Deviations from CIRP and arbitrage opportunities I

Suppose that

$$(1+r_D)^n < \frac{F_{(nD/F)}}{S_{(D/F)}}(1+r_F)^n$$

$$\Downarrow$$

The best thing to do would be to borrow in your domestic currency and to invest simultaneously in a foreign currency-denominated security. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt (i.e. you will receive more domestic currency)

Deviations from CIRP and arbitrage opportunities II

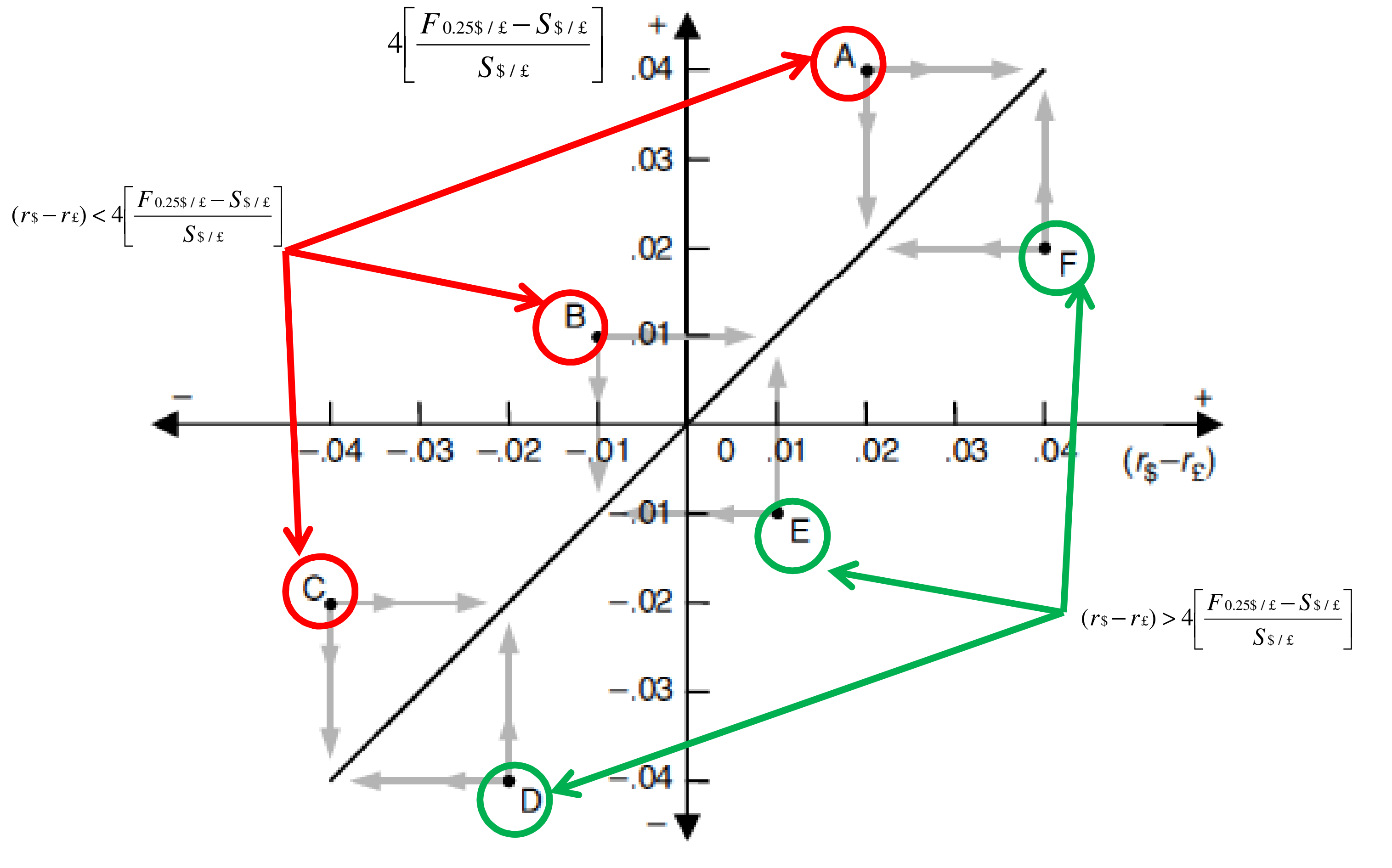
What if

$$(1+r_D)^n > \frac{F(_nD/F)}{S(D/F)}(1+r_F)^n$$

↓

The best thing to do would be to borrow foreign currency and to invest simultaneously in a domestic currency-denominated security. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt

CIRP: a graphical approach I



CIRP: a graphical approach II

For all the points lying above the 45°line (A,B and C), it must be that

$$(r_{\$} - r_{£}) < 4\left[\frac{F_{0.25\$/£} - S_{\$/£}}{S_{\$/£}}\right]$$

⇓

This further implies:

- Covered investment in £ yields more than in $;
- Borrowing in $ is cheaper than covered borrowing in £

CIRP: a graphical approach III

The adjustment procedure driving A, B, and C down towards the 45° line works as follows:

1. Borrow \$, thus tending to increase $r_{\$}$;
2. Buy spot £ with the borrowed \$, thus tending to increase $S_{(\$/£)}$;
3. Buy a £ security, thus tending to reduce $r_{£}$;
4. Sell the £ investment proceeds forward for \$, thus tending to reduce $F_{0.25\ (\$/£)}$.

⇓

Points 1 to 4 will all push A, B and C back down to the CIRP line

CIRP: a graphical approach IV

For all the points lying below the 45°line (D, E and F), it must be that

$$(r_{\$} - r_{£}) > 4\left[\frac{F_{0.25\,\$/£} - S_{\$/£}}{S_{\$/£}}\right]$$

This further implies:

- Covered investment in $ yields more than in £;
- Borrowing in £ is cheaper than covered borrowing in $

CIRP: a graphical approach V

The adjustment procedure driving D, E, and F up towards the 45° line works as follows:

1. Borrow £, thus tending to increase $r_{£}$;
2. Buy spot \$ with the borrowed £, thus tending to decrease $S_{(\$/£)}$;
3. Buy a \$ security, thus tending to reduce $r_{\$}$;
4. Sell the \$ investment proceeds forward for £, thus tending to increase $F_{0.25\ (\$/£)}$.

Points 1 to 5 will all push D, E and F back up to the CIRP line

When transaction costs are brought into the picture…I

Covered investment/borrowing involve two FX transactions (one on the spot market and the other on the forward market)

↓

Transaction costs have to be bore twice

↓

There could be deviations from interest rate parity due to the extra transaction costs of investing/borrowing in foreign currency…

Is it always so?

When transaction costs are brought into the picture…II

It can be shown that transaction costs do not always contribute to deviations from CIRP.

Some preliminary notation:

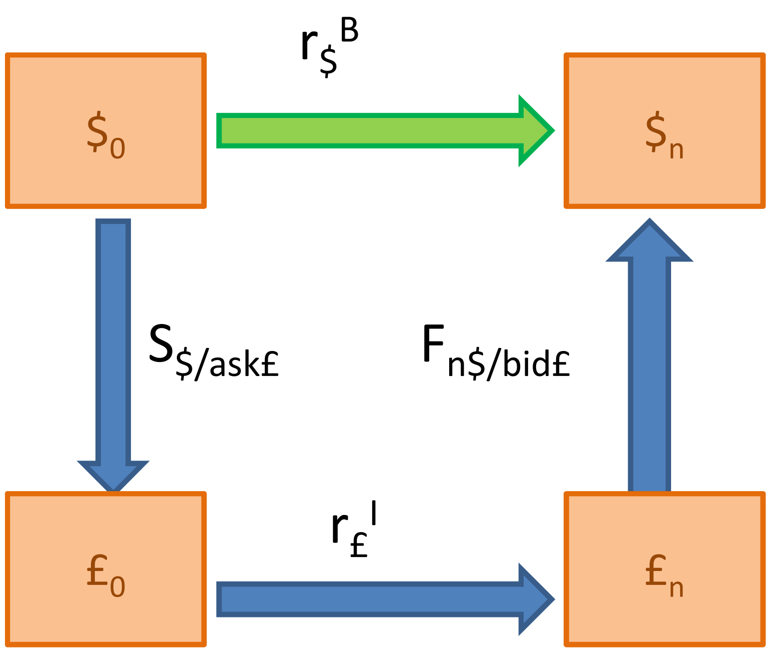
- S($/ask£), S($/bid£)= spot exchange rate when buying/selling £ with $ respectively
- F_n($/ask£), F_n($/bid£) = n-year forward exchange rate when buying/selling £ with $
- $r_{\$}^{I}$ and $r_{£}^{I}$ = interest rates earned on USD/GBP-denominated investments
- $r_{\$}^{B}$ and $r_{£}^{B}$ = interest rates due on USD/GBP-denominated borrowings

When transaction costs are brought into the picture…III

Case a) round-trip covered interest rate transaction

Suppose that, at time 0, you borrow $\$_{0}$. Assume further that the interest rate on such borrowing is $r_{\$}^{B}$…

…the borrowed \$ are exchanged into £ at S(\$/ask£)…



…$£_n$ are sold at the forward rate F(\$/bid£) stipulated at time 0, thus obtaining $\$_n$ (USD required to repay the debt).

…and placed in a £-denominated deposit yielding $£_n$ after n-months…

When transaction costs are brought into the picture…IV

Based on the CIRP, we could write

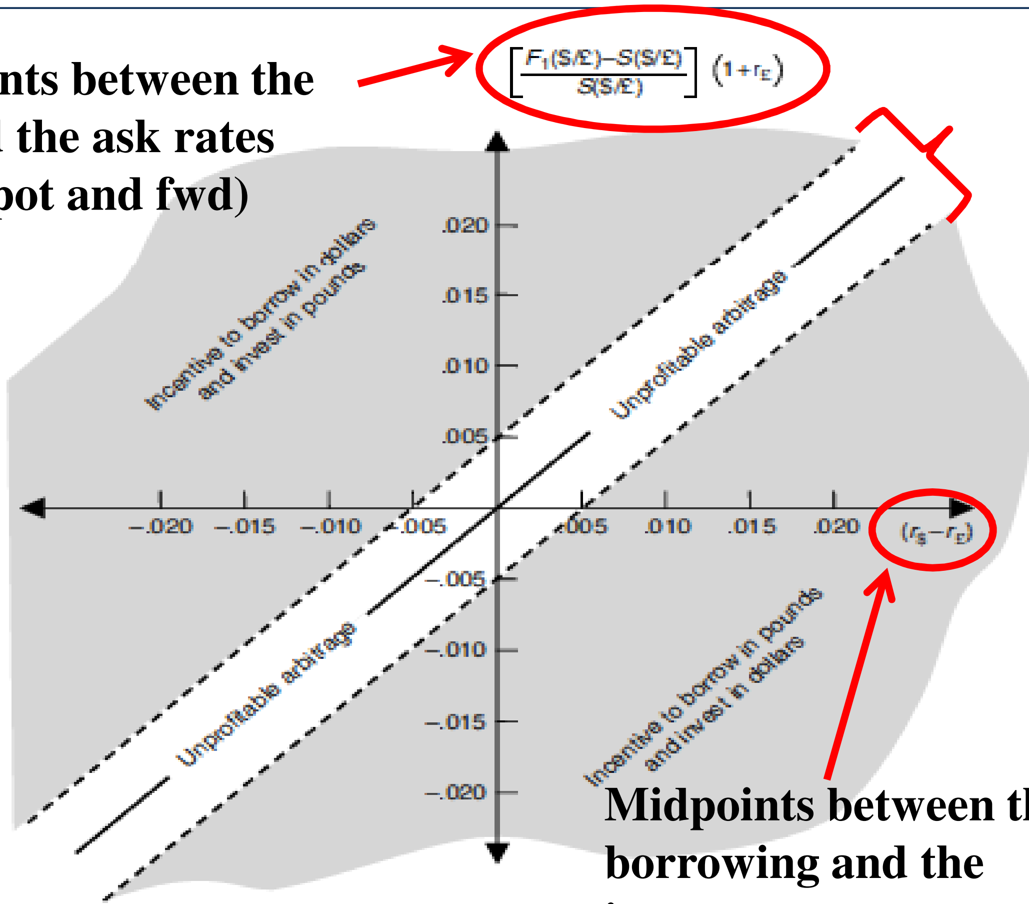
$$(1+r_{\$}^{B})^{n} = \frac{F_{n}(\$/bid\,£)}{S(\$/ask\,£)}(1+r_{£}^{I})^{n}$$

This is **NOT** a perfect 45°-line on the CIRP diagram, but more a "band" drawn around mid-rates. This is because of the transactions costs to be bore:

- Bid-ask spread = (S($/**ask**£) - F_n($/**bid**£))
- Borrowing-investment transaction costs = $(r_{\$}^{B} - r_{£}^{I})$

When transaction costs are brought into the picture…V

Midpoints between the bid and the ask rates (both spot and fwd)



Round-trip arbitrages (and the related transaction costs) determine a "band" around the CIRP line, within which arbitrages would be unprofitable

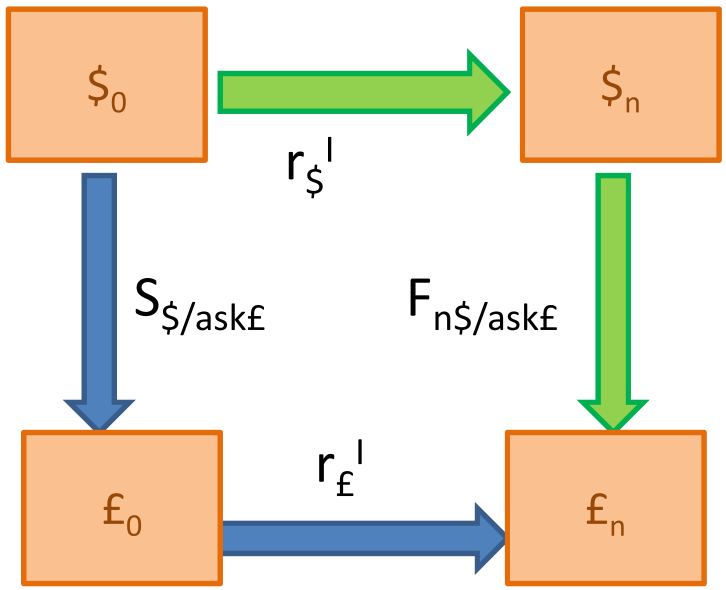
Midpoints between the borrowing and the investment rates

When transaction costs are brought into the picture…VI

Case b) one-way covered interest rate transaction

If you need £$_n$ sometime in the future and you have \$$_0$ today, you could either…

…sell \$ for £ on the spot mkt at S(\$/ask£) and invest them in a £-denominated deposit yielding £$_n$ when GBP are needed.



…or invest \$$_0$ in a \$-denominated deposit and use the proceeds (\$$_n$) to buy £ forward at F$_n$(\$/ask£), when GBP are needed.

When transaction costs are brought into the picture…VII

Based on the CIRP condition, we could write

$$(1+r_{\$}^{I})^{n} = \frac{F_n(\$/ask£)}{S(\$/ask£)}(1+r_{£}^{I})^{n}$$

This would plot an exact 45° line in the CIRP diagram, given that there are virtually no transaction costs:

- Bid-ask spread = (S($/**ask**£) - F_n($/**ask**£))
- Borrowing-investment transaction costs = ($r_{\$}^{I}$ - $r_{£}^{I}$)

To sum up I

For round-trip arbitrages to be profitable, deviations from the CIRP line must be large enough to overcome **transaction costs…**

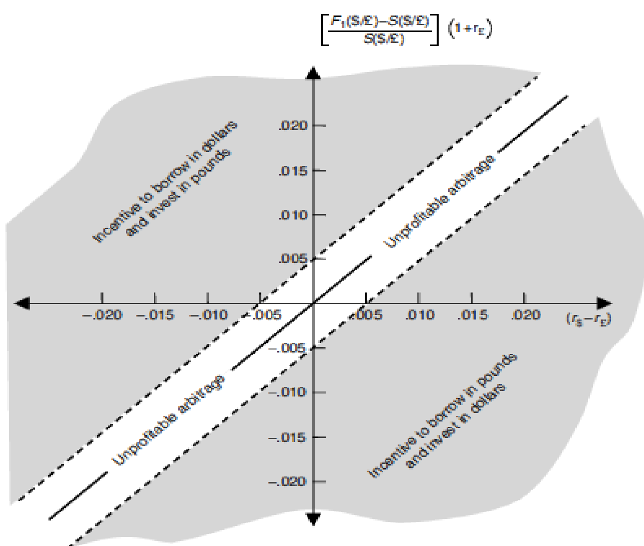
↓

…and this will hardly ever occur in practice (Could you explain why?)

↓

To sum up II

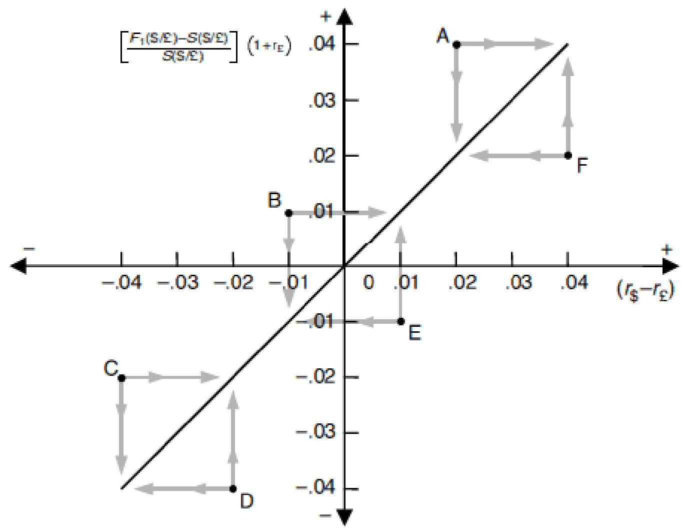
Round-trip transactions



$$(1+r_\$^{B})^n = \frac{F_n(\$/bid£)}{S(\$/ask£)}(1+r_£^{I})^n$$

If this band were too large and $(1+r_\$^{B})^n \neq \frac{F_n(\$/bid£)}{S(\$/ask£)}(1+r_£^{I})^n$ …

One-way transactions



$$(1+r_\$^{I})^n = \frac{F_n(\$/ask£)}{S(\$/ask£)}(1+r_£^{I})^n$$

… $(1+r_\$^{I})^n = \frac{F_n(\$/ask£)}{S(\$/ask£)}(1+r_£^{I})^n$ also will not be satisfied, thus giving rise to undue profit opportunities…

To sum up III

…for instance, if $(1+r_{\$}^{I})^{n} > \frac{F_{n(\$/ask£)}}{S_{(\$/ask£)}}(1+r_{£}^{I})^{n}$, nobody would invest in a £-denominated deposit (as market players would rather put their money in a \$-denominated investment)

⬇

This would gradually drive $r_{\$}^{I}\downarrow$ and $r_{£}^{I}\uparrow$, until equilibrium is restored again and **arbitrage opportunities are completely reabsorbed (No Free Lunch principle)**

⬇

Transaction costs do **<u>not</u>** bring about profitable arbitrage opportunities

How to construct synthetic securities with spot and forward contracts + borrowing and lending

How to construct synthetic securities I

The CIRP implies

$$(1+r_D)^n = \frac{F_n(D/F)}{S(D/F)}(1+r_F)^n$$

Rearranging the terms

$$F_n(D/F) = S(D/F)\frac{(1+r_D)^n}{(1+r_F)^n}$$

How to construct synthetic securities II

$$F_n(D/F) = S(D/F)\frac{(1+r_D)^n}{(1+r_F)^n}$$

An n-period synthetic forward can be constructed by combining…

…a spot contract with…

…fixed-rate, n-period borrowing and lending in the domestic and foreign currencies respectively.

How to construct synthetic securities III

$$(1 + r_D) = (1 + r_F)\frac{F_n(D/F)}{S(D/F)}$$

A synthetic domestic currency-denominated security can be obtained by combining…

…a foreign currency-denominated security with…

…a forward/spot swap.

Some lessons to learn

The **CIRP** is useful:

1. when trying to **understand the direction of K movements** → towards the currency with higher covered yield;
2. to **build/replicate a financial contract**;
3. to **hedge** a financial position

To put it into practice

Consider the following:

Spot rate: $Currency_1$ 0.64/$Currency_2$

$r_{1y\_Currency1} = 5\%$

$r_{1y\_Currency2} = 9\%$

1. Calculate the theoretical price of a one year forward contract.
2. What would you do if the forward price was quoted at $Currency_1$ 0.65/$Currency_2$ in the market place? Where would you borrow? Lend? Calculate the gain on a $Currency_1$ 100 million arbitrage transaction.
3. What would you do if the future price was quoted at $Currency_1$ 0.60/$Currency_2$ in the market place? Where would you borrow? Lend? Calculate the gain on a $Currency_2$100 million arbitrage transaction.