Università Carlo Cattaneo – Castellana (LIUC)
Faculty of Economics

Quantitative Methods for Economics, Finance and Management

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Sample Questions and Exercises

The exam is written and will typically consist of three questions, two from Group 1 (Theory) and one from Group 2 (Applications). Time allowed: 1h and 30m.

Group 1: Theory

Question 1

Consider the following model:

$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + u_i$	(1)

where i=1,…,N; u_i is a classical error term; x_{i2} and x_{i3} are non-stochastic regressors. Notice that $x_{i4} = x_{i4}^* + v_i$, where x_{i4}^* is a non-stochastic variable and v_i is a classical error term.
I) Explain why the OLS estimator of the parameters in model (1) is inconsistent.
II) Assume that an instrument for x_{i4}, say x_{i5}, exists. Which properties has x_{i5} to satisfy in order to be considered a valid instrument?
III) Describe a procedure to obtain consistent estimates of the parameters in model (1).

Question 2

Assume that the classical linear regression model:

$y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + u_t$	(1)

is correct.
I) Explain why the OLS estimator of y_t on x_{t2} only is biased.
II) Calculate the bias.
III) Under which conditions the OLS estimator described at point **I**) is unbiased?

Question 3

Consider the linear regression model:

$y_t = \alpha + \beta x_t + u_t$	(1)
$u_t = \phi_1 u_{t-1} + \varepsilon_t$	(2)

t=2,...,T, where ε_t is a classical error term.
I) Illustrate an appropriate test to check the null hypothesis of absence for error first-order autocorrelation.
II) Apply the quasi-difference transformation to model (1)-(2).
III) Which is the advantage of such a transformation with respect to the GLS estimator?

Question 4
Consider the following linear regression model:

$y_i = \alpha + \beta x_i + u_i$	(1)
$Var(u_i) = \sigma^2 \exp(z_i)$	(2)

$i=1,...,N$, where x_i and z_i are observed non-stochastic variables.
I) Illustrate how to test the null hypothesis $Var(u_i) = \sigma^2$.
II) Explain how you should estimates the parameters of equation (1).

Question 5
You are given the following models:

$$M_U: y_t = b_1+b_2x_t + b_3z_t + b_4w_t + u_t$$
$$M_R: y_t = b_1 +b_2x_t + b_3z_t + e_t$$

where t=1,…,T, M_U is the unrestricted model and M_R is the restricted model.
I) Define and interpret the goodness-of-fit indicator R^2 (R-squared).
II) Explain why $R^2_U \geq R^2_R$, irrespective of the statistical significance of the regressors.

Question 6
Assume that the error term ut of model M_U in Question 5 is given by:

$$u_t = r_1u_{t-1} + r_2u_{t-2} + v_t$$

where v_t is a classical error term.
I) Under these conditions, which are the statistical properties of the OLS estimator of parameters b_1, b_2, b_3 and b_4 in model M_U?
II) Which transformation should you use on model M_U in order to obtain transformed classical error terms?
III) Assume now that the error terms of model M_U are given by $u_t = f_1y_{t-1} + f_2x_{t-1} + f_3z_{t-1} + f_4w_{t-1} + v_t$. Which transformation should you use on model M_U in order to obtain transformed classical error terms?

Question 7
You are given the following model:

$$y_i = b_1 + b_2x_i + u_i, \; i=1,…,N$$

where $Var(u_i) = s^2z^2_i$, s^2 being constant, whereas z_i varies according to i.
I) Illustrate a test of homoskedasticity which is appropriate in this specific context.
II) How should you estimate the parameters b_1 and b_2?

Question 8
You are given the following model:

$$y_t = \beta_1 + \beta_2x_t + \beta_3z_t + e_t$$

where the error term et is determined by the process:

$$e_t = \alpha + \gamma x^2_t + \delta z^3_t + v_t$$

v_t being a classical error term.

I) Under these conditions, which are the statistical properties of the OLS estimator of parameters β_1, β_2 and β_3?

II) How should you estimate parameters α, γ and δ?

III) Which transformation should you use on the model for *yt* in order to obtain transformed error terms which satisfy the classical assumptions?

Question 9

Consider the following model:

$$y_i = \beta_1 + \beta_2 x_i + u_i,\ i=1,\ldots,N$$

where $Corr(x_i,ui) \neq 0$. Assume that variables z_i and w_i are observed. These variables have the following properties: $Corr(z_i,u_i)=0$; $Corr(w_i,u_i)=0$; $Corr(z_i,x_i) \neq 0$; $Corr(w_i,x_i) \neq 0$.

I) Which are the statistical properties of the OLS estimator for β_1 and β_2?

II) Illustrate an alternative estimator for β_1 and β_2 which uses both variables z_i and w_i.

Question 10

You are given the model:

$$y_t = b_1 + b_2 x_t + b_3 z_t + b_4 w_t + u_t \qquad (1)$$

Assume that the regressors z_t and w_t in model (1) are correlated with the error term u_t.

I) Which are the effects of this assumption on the statistical properties of the OLS estimator of the parameters in model (1)?

II) Assume that the explanatory variables x_t, z_t and w_t are not correlated with u_t. Illustrate how to obtain the Two-Stage Least Squares (2SLS) estimator of b_1, b_2, b_3 and b_4 in model (1).

Question 11

Consider the model:

$$y_i = b_1 + b_2 x_i + u_i,\ i=1,\ldots,N$$

where $Var(u_i) = s^2(z_i)^{0.5}$, $z_i>0$, $s^2=1$.

I) Show that the OLS estimator of parameters b_1 and b_2 is unbiased, but it is no longer a minimum variance estimator.

II) Derive an appropriate estimator for b_1 and b_2, which is unbiased and with minimum variance.

Question 12

You are given the following classical linear regression model:

$$y_t = \alpha + \beta x_t + u_t \qquad (1)$$

$t=1,...,T+H$.

I) Derive a test for the stability of parameters α and β of model (1) from the subsample t=1,…,T to the subsample t=T+1,…,T+H.
II) Assume that the errors u_t are normally distributed with zero mean and constant variance σ^2. Illustrate an appropriate test for homosckedasticity.

Question 13
Consider the model:

$y_t = \beta_1 + \beta_2 x_{t2} + ... + \beta_K x_{tK} + u_t$	(1)

I) Illustrate an appropriate procedure to test the null hypothesis of absence of error autocorrelation against the alternative hypothesis of second-order error autocorrelation.
II) Assuming the presence of second-order error autocorrelation, transform model (1) to obtain classical transformed error terms.
III) Suppose you have estimated the transformed model at point II) and that its residuals are still autocorrelated. Which conclusions can you draw from this empirical finding?

Question 14
I) Illustrate the effects on the OLS estimator of omitting one relevant explanatory variable.
II) Illustrate the effects on the OLS estimator of including one irrelevant explanatory variable.

Question 15
You are given the linear regression model: $y_i = \beta_1 + \beta_2 x_i + u_i$, where $Var(u_i) = \sigma^2 x_i^2$, i=1,…,N.
I) How would you test the null hypothesis of homoskedasticity in this case?
II) Illustrate how to implement the Weighted Least Squares (WLS) estimator for β_1 and β_2?

Question 16
Consider the classical linear regression model:

$Y_t = \beta_1 + \beta_2 X_{t2} + \beta_3 X_{t3} + \beta_4 (X_{t2} - X_{t3}) + \beta_5 X_{t5} + u_t$	(1)

t=1,...,T.
I) How many parameters in model (1) can be estimated with OLS? Explain.
II) Illustrate with an example the conceptual distinction between perfect and quasi-perfect multicollinearity.

Question 17
Consider the following equations:

$\log Y_i = \beta_1 + \beta_2 \log W_i + \beta_3 S_i + u_i$	(1)
$\log(Y_i / W_i) = \gamma_1 + \gamma_2 \log W_i + \gamma_3 S_i + u_i$	(2)

i=1,...,N. Variable Y_i indicates the i-th individual's annual income, W_i is the number of weeks worked by the i-th individual, whereas S_i is the number of year of schooling for the i-th individual.
I) The OLS estimator is defined as the solution of the problem of minimizing the sum of squared errors. Write down this minimization problem for equations (1) and (2).

II) Show that the OLS estimation of models (1) and (2) yields the following results: $\hat{\gamma}_1 = \hat{\beta}_1$; $\hat{\gamma}_3 = \hat{\beta}_3$; $\hat{\gamma}_2 = (\hat{\beta}_2 - 1)$.
III) Explain why the residuals from the two regressions (1) and (2) are identical.

Question 18
You are given the following models:

$Y_t = \alpha + \beta X_t + u_t$	(1)
$Y_t - \bar{Y} = \beta\left(X_t - \bar{X}\right) + u_t$	(2)

t=1,...,T, where $\bar{Y}$ and $\bar{X}$ indicate the sample means of Y and X, respectively.
I) Derive the expression of the OLS estimator for β, $\hat{\beta}$, in equation (1)
II) Under which conditions the R^2 of regression (1) can take either the value of 0 or the value of 1?
III) Show that the expression of $\hat{\beta}$ at point I) is identical to the expression of $\hat{\beta}$ which can be derived from the OLS estimation of β in model (2).

Question 19
Consider the following macroeconomic consumption function:

$C_t = \beta_1 + \beta_2 Y_t + u_t$, t=1,…,T

where C_t is consumption, Y_t is income and u_t is a classical error term. Income Y_t is defined as the accounting identity: $Y_t \equiv C_t + I_t$, where I_t is aggregate investment.
I) Show that Y_t is a stochastic variable, correlated with u_t.
II) Which are the effects of the correlation between Y_t and u_t on the OLS estimator of β_1 and β_2?
III) Assume that I_t is correlated with Y_t but not with u_t. Illustrate how to implement the Two Stage Least Squares (2SLS) estimator for β_1 and β_2.

Question 20
You are given the linear regression model: $y_i = \beta_1 + \beta_2 x_i + u_i$, where $Var\left(u_i\right) = \sigma^2\left(\alpha_0 + \alpha_1 w_i\right)^2$, i=1,…,N.
I) How should you test the null hypothesis of homoskedasticity in this case?
II) Illustrate an appropriate estimator of parameters β_1 and β_2.

Question 21
You are given the linear regression model: $y_t = \beta_1 + \beta_2 x_t + \beta_3 z_t + \beta_4 w_t + u_t$, t=1,…,T. Suppose you would like to test the joint null hypothesis: β_2+β_3=1, β_4=0.
I) Write down the restricted model.
II) Write down the expression for the F-test of the null hypothesis at point I) and clearly indicate which are the degrees of freedom at the numerator and at the denominator.
III) Is it possibile to write down the expression of the F-test in terms of R^2 in this case?

Group 2: Applications

Question 1

A researcher has estimated with OLS the linear regression model reported on Table 1, where the oil spot price (LOG(S_WTI)) is regressed on a constant term (C), the percentage change in oil world demand (D(LOG(Q_TOT))), the level of oil world demand (LOG(Q_TOT)), the 3-month oil futures price (LOG(F3_WTI)), the percentage change in the 3-month oil futures price (D(LOG(F3_WTI))), the percentage change in industry oil stocks (D(LOG(Q_IS))) and the level of industry oil stocks (LOG(Q_IS)). All variables are expressed in logs and have a quarterly frequency, from the second quarter of 1993 to the third quarter of 2005.

Table 1. Oil price

Dependent Variable: LOG(P_WTI)
Method: Ordinary Least Squares
Sample: 1993Q2 2005Q3
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	9.101826	0.869823		0.0000
D(LOG(Q_TOT))	0.429238	0.157646		0.0093
LOG(Q_TOT)	0.011383	0.076692		0.8827
LOG(F3_WTI)	0.965532	0.015095		0.0000
D(LOG(F3_WTI))	0.089740	0.028369		0.0029
D(LOG(Q_IS))	0.777185	0.160660		0.0000
LOG(Q_IS)	-1.149918	0.111427		0.0000

R-squared		Mean dependent var	3.174285
		S.D. dependent var	0.368448
S.E. of regression	0.018954	Akaike info criterion	-4.964459
Sum squared resid	0.015447	Schwarz criterion	-4.696776
Log likelihood	131.1115	F-statistic	3078.935
Durbin-Watson stat	1.674077	Prob(F-statistic)	0.000000

a) Complete the column entitled t-Statistic and indicate which coefficients are statistically significant at the 1% significance level.
b) Provide an economic interpretation of the coefficients of LOG(Q_TOT), LOG(F3_WTI) and LOG(Q_IS).
c) Using the results reported on Table 1, calculate the value of the residual autocorrelation coefficient ϕ within the model: $\hat{u}_t = \phi \hat{u}_{t-1} + \varepsilon_t$, where $\hat{u}_t$ indicates the residuals of the model estimated in Table 1. Which is the statistical meaning of this coefficient?
d) Using the results reported on Table 1, calcolate the value of R^2.

Question 2

A researcher has estimated with OLS the following model:

$$\hat{D}_t = \underset{(2.97)}{15.08} - \underset{(0.86)}{2.15} V_t \qquad (1)$$

where D_t is the average period of unemployment and V_t is the number of new job positions. Model (1) has been estimated on annual data, from 1949 to 1972 included. Standard errors are reported in brackets. Explained Sum of Squares (ESS) is 52.198; Residual Sum of Squares (RSS) is 77.262; the calculated Durbin-Watson test is 0.64.

a) Without using any statistical table, indicate which coefficients are statistically significant at 5% significance level.
b) Calculate and comment the value of R^2.
c) Which indications does the Durbin-Watson test provide in the present context? What are the implication of the value of the Durbin-Watson test on the estimation of model (1)?

The researcher finds up that in 1966 the Government has introduced a set of policy measures to contrast unemployment. He/she thinks that such a policy could have caused a structural break in the time series of D_t and V_t. For this reason, he/she estimates two additional regressions using the same specification (1), on two different subsamples. The first subsample goes from 1949 to 1965 included and yields to RSS1 = 43.11. The second subsample starts in 1966 and ends up with 1972 included, yielding RSS2 = 5.987.

d) Calculate the value of the test for stability of the parameters in model (1) between the first and the second subsample.

Question 3

A researcher estimates with OLS an Environmental Kuznets Curve (EKC) for New Zealand using the following quadratic specification:

$$CO2_t = b_1 + b_2 GDP_t + b_3 GDP^2{}_t + u_t,\ t=1,\ldots, 43 \qquad (1)$$

where $CO2_t$ is the logarithm of the CO2 emissions in New Zealand in year t, GDP_t is the log of per capita gross domestic product within New Zealand in year t and $GDP2_t$ is the log of the square of per capita GDP in year t.

Table 1. OLS estimation of quadratic EKC for New Zealand

Dependent Variable: CO2
Method: Least Squares
Sample: 1 43

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.408974	0.552876		0.1000
GDP	3.610349	0.400781		0.0090
GDP2	-0.520080	0.072239		0.0010

R^2	0.867	F-statistic	
Durbin-Watson stat	2.015	Prob(F-statistic)	

a) For each estimated coefficient reported in Table 1, calculate the corresponding t-test and indicate, at 1% significance level, if the null hypothesis b_j=0 , j=1,2,3, is rejected.
b) What is the interpretation of the R^2 value reported in Table 1?
c) Using the R^2 value reported in Table 1, calculate the value of the F-test for "zero slopes". What is the interpretation of this value?
d) Using the value of the Durbin-Watson statistic reported in Table 1, is model (1) affected by residual autocorrelation? Which error autocorrelation order does the Durbin-Watson statistic test against? Write down the null hypothesis of the Durbin-Watson test.

Question 4

The following individual demand function has been estimated with OLS:

$$LQB_t = \beta_1 + \beta_2 LPB_t + \beta_3 LPL_t + \beta_4 LPR_t + \beta_5 LM_t + u_t,\ t=1,\dots,T=30 \qquad (1)$$

where LQB = log of the demanded quantity of good B; LPB = log of the price of good B; LPL = log of the price of good L; LPR = log of the consumers' price index; LM = log of the consumer's money income. Estimation results are reported in Table 1.

Table 1. Estimated demand function (unrestricted)
Dependent Variable: LQB
Method: Least Squares
Sample: 1 30
Included observations: 30

Variable	Coefficient	Std. Error	t-Statistic
C	-3.243238	3.743000	-0.866481
LPB	-1.020419	0.239042	-4.268787
LPL	-0.582934	0.560150	-1.040674
LPR	0.209545	0.079693	2.629415
LM	0.922864	0.415514	2.221016
R-squared	0.825389		

a) Which explanatory variables are statistically significant and why?
b) Calculate the F-test of the null hypotesis: $\beta_2=\beta_3=\beta_4=\beta_5=0$.
c) Are goods B and L substitutes, complements, or uncorrelated? Motivate your answer.

The following restricted version of model (1) has been estimated:

Table 2. Estimated demand function (restricted)
Dependent Variable: LQB
Method: Least Squares
Sample: 1 30
Included observations: 30

Variable	Coefficient	Std. Error	t-Statistic

C	-4.797798	3.713905	-1.291847
LPB-LPR	-1.299386	0.165738	-7.840022
LPL-LPR	0.186816	0.284383	0.656916
LM-LPR	0.945829	0.427047	2.214813
R-squared	0.807949		

d) Write down which restriction(s) on parameters β_j, j=1,…,5, are incorporated by the restricted model.
e) Calculate a test for the validity of the restrictions presented at point d).

Question 5

A researcher estimates with OLS the Environmental Kuznets Curve (EKC) for US using the following cubic specification:

$$CO2_t = b_1 + b_2 GDP_t + b_3 GDP^2{}_t + b_4 GDP^3{}_t + u_t, \; t=1,..., 43 \qquad (1)$$

where $CO2_t$ is the logarithm of the CO2 emissions in US in year t, GDP_t is the log of per capita gross domestic product within US in year t, $GDP2_t$ is the log of the square of per capita GDP in year t and $GDP3_t$ is the log of the cube of per capita GDP in year t.

Table 1. OLS estimation of cubic EKC for US
Dependent Variable: CO2
Method: Least Squares
Sample: 1 43

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.408974	3.552876		
GDP	3.610349	0.400781		
GDP2	-0.520080	0.072239		
GDP3	0.789357	1.785432		
R^2		F-statistic		45.23478
Durbin-Watson stat	0.156728	Prob(F-statistic)		0.000000

a) For each estimated coefficient reported in Table 1, calculate the corresponding t-test and indicate, at 1% significance level, if the null hypothesis $b_j=0$, j=1,2,3,4 is rejected.
b) What is the interpretation of the R^2 value reported in Table 1?
c) Using the R^2 value reported in Table 1, calculate the value of the F-test for "zero slopes". What is the interpretation of this value?
d) Using the value of the Durbin-Watson statistic reported in Table 1, calculate the value of the autocorrelation coefficient r in the model $\hat{u}_t = r\hat{u}_{t-1} + e_t$, where $\hat{u}_t$ are the residuals from model (1). Which indications does the value of $\hat{r}$ provide?

<u>Question 6</u>

A researcher would like to analyze a dataset formed by 52 weekly observations on the supermarket sales of canned tuna. Specifically, the available variables are: number of sold units of canned tuna of brand 1 (*SAL1*); unit price of canned tuna of brand 1 (*APR1*); unit price of canned tuna of brand 2 (*APR2*); unit price of canned tuna of brand 3 (*APR3*); dummy variable, equal to 1 if the supermarket has advertised the canned tuna of brand 1 within the t-th week, but no other forms of advertising have been used, equal to 0 otherwise (*DISP*); dummy variable, equal to 1 if the supermarket and other forms of advertising have advertised the canned tuna of brand 1 within the t-th week, equal to 0 otherwise (*DISPAD*).

The researcher estimates the following model (see Table 1):

$$log(SAL1_t) = \beta_1 + \beta_2 APR1_t + \beta_3 APR2_t + \beta_4 APR3_t + \beta_5 DISP_t + \beta_6 DISPAD_t + u_t \qquad (1)$$

a) Which estimated coefficients are statistically significant and why?
b) Which indications in the present context does the F-test for "zero slopes" provide?
c) Discuss the economic interpretation of the estimated coefficients β_2, β_3 and β_4.
d) Is the sign of the estimated coefficients β_5 and β_6 coherent with economic theory?
e) Provide an economic justification of and explain how to test the following null hypotheses:

H'_0: $\beta_5 = \beta_6 = 0$ contro H'_1: $\beta_5 \neq 0$ oppure $\beta_6 \neq 0$

H''_0: $\beta_6 \geq \beta_5$ contro $H''1$: $\beta_6 < \beta_5$

Table 1. OLS estimation of model (1)

Dependent Variable: LOG(SAL1)
Method: Least Squares
Sample: 1 52

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8.984812	0.646366	13.90050	0.0000
APR1	-3.746301	0.576518	-6.498155	0.0000
APR2	1.149495	0.448643	2.562160	0.0137
APR3	1.288048	0.605325	2.127864	0.0387
DISP	0.423744	0.105209	4.027624	0.0002
DISPAD	1.431253	0.156163	9.165152	0.0000

R-squared	0.842812	Mean dependent var	8.437187
Adjusted R-squared	0.825726	S.D. dependent var	0.813654
S.E. of regression	0.339669	Akaike info criterion	0.786477
Sum squared resid	5.307252	Schwarz criterion	1.011620
Log likelihood	-14.44839	F-statistic	49.32855
Durbin-Watson stat	1.675895	Prob(F-statistic)	0.000000

Question 7

The following model has been estimated:

$y_t = \beta_1 + \beta_2 {*} x_t + \beta_3 {*} z_t + u_t$	(1)
$u_t = \phi\, u_{t-1} + \varepsilon_t$	(2)

where ε_t is a classical error term. The estimation results are: $\hat{\beta}_1 = 7.5$ (8.2); $\hat{\beta}_2 = 0.8$ (0.002); $\hat{\beta}_3 = -0.45$ (0.05); $\hat{\phi} = 0.75$ (0.01). Standard errors are reported in brakets.

a) Which procedure has been used to estimate the parameters in models (1) and (2)?
b) Which coefficients in model (1) are statistically significant and why?
c) Is it possibile to conclude that the coefficient in model (2) is statistically significant?
d) Calculate the Durbin-Watson (DW) test for first-order autocorrelation of the errors in model (1). Without any statistical table, does the calculated value of the DW test allow the researcher to understand whether the errors of model (1) are affected by first-order autocorrelation?

Question 8

A researcher has estimated with OLS the following model:

$$\hat{Y}_t = \underset{(0.702)}{2.417} + \underset{(0.081)}{0.724}\, X_t \qquad (1)$$

Model (1) has been estimated on quarterly data, from 1963 (first quarter) to 1972 (fourth quarter) included. Standard errors are reported in parentheses. Residual Sum of Squares (RSS) is 137.21, whereas the value of R^2 is 0.678.

a) Without any statistical table, indicate which coefficients within regression (1) are statistically significant at 5% significance level.
b) Calculate the standard deviation of the dependent variable.

The researcher suspects that the relationship between Y and X is not constant and estimates the following regression:

$$\hat{Y}_t = \underset{(0.719)}{1.932} - \underset{(0.028)}{0.044}\, X_t + \underset{(0.017)}{0.024}\, X_t \cdot t \qquad (2)$$

where t=1,...T is a linear trend starting from the second quarter of 1960.

c) What kind of empirical evidence does regression (2) provide about the temporal variation of the relationship between Y and X?

Our researcher estimates two additional regressions of type (1) on the two subperiods 1963 (first quarter) - 1966 (third quarter) and 1966 (fourth quarter) - 1972 (fourth quarter). The first subperiod yields a RSS equal to 33.04, while the second subperiod yields a RSS of 68.19.

d) Test the null hypothesis of parameter stability between the two subsamples.

e) Compare the results obtained at point d) with your comments at point c). Are you able to make a more precise comment now about the relationship between *Y* and *X*?

Question 9

The following microeconomic consumption function has been investigated:

$$CONSUMPTION_i = \beta_1 + \beta_2 INCOME_i + u_i,\ i=1,..., 40$$

where $CONSUMPTION_i$ is the i-th individual's consumption, while $INCOME_i$ indicates the i-th individual's income. We suspect that the error terms u_i are affected by heteroskedasticity of the following form: $Var(u_i) = f^+(REDDITO_i)$, where f^+ is a monotonically increasing function of income. In order to test the null hypothesis of homoskedasticity we order the observations on $INCOME_i$ increasingly (i.e. starting from the lowest value and ending with the largest value) and we estimate with OLS the following two regression models:

Table 1

Dependent Variable: CONSUMPTION
Sample: 1 18
Included observations: 18

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.299095	5.583747		0.3567
INCOME	0.266717	0.103905		0.0207
R-squared	0.291696	S.D. dependent var		5.291600

Table 2

Dependent Variable: CONSUMPTION
Sample: 23 40
Included observations: 18

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	12.43306	14.27373		0.3966
INCOME	0.175269	0.163057		0.2984
R-squared	0.067348	S.D. dependent var		9.050760

a) For each estimated coefficient reported on Tables 1 and 2, indicate, at the significance level of 5%, if the null hypothesis $\beta_j=0$, $j=1,2$, is rejected or not.
b) With reference to the first regression (see Table 1), calculate the t-statistics of the null hypothesis $\beta_1 = 5$ and of the null hypothesis $\beta_2 = 0.3$.
c) Calculate the values of RSS relative to the first (see Table 1) as well as to the second (Table 2) estimated regression.
d) On the basis of your answer to point c), can you understand whether the microeconomic consumption function is affected by heteroskedasticity?

Question 10

Using annual data from 1930 to 1978 a researcher has estimated with OLS the following model for cigarette demand:

Dependent Variable: LNC
Method: Least Squares
Sample: 1930 1978
Included observations: 49

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.972900	1.760509		0.2685
LNY	1.236227	0.083966		0.0000
LNP	-0.609198	0.324479		0.0671
LNA	0.049176	0.069216		0.4812
D64	-0.277694	0.077386		0.0008
		S.D. dependent var		0.391376
S.E. of regression	0.124051			

where, relative to year *t*, LNC is the log of per capita cigarette consumption, LNY is the log of per capita income, LNP is the log of the cigaratte price, LNA is the log of the stock of per capita advertising and D64 is a *dummy*, equal to 1 from 1964 (starting year of an aggressive anti-smoking government policy) to 1978.

a) Calculate the F-test for "zero slopes".
b) Which estimated coefficient are statistically significant and why?.
c) Provide an economic interpretation of the coefficient on D64.
d) The variable LNP is potentially correlated with the errors of the estimated cigarette demand function. The researcher has the variable LNPRTOB, i.e. the log of the tobacco price, as a possibile instrument. What are the properties of a valid instrument? Is LNPRTOB a valid instrument?
e) Assuming that LNPRTOB is a valid instrument, indicate the procedure to obtain Two-Stage Least Squares estimates of the coefficients of the cigarette demand function.