

Lesson III: Overview

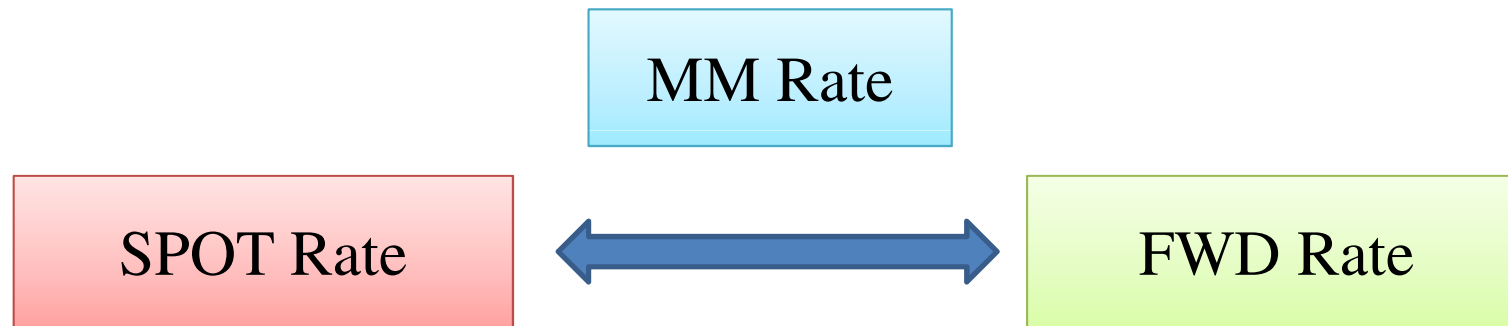
1. The relationship between spot, forward and money market rates
2. How to construct synthetic securities



The relationship between spot, forward and money rates



Spot, Fwd & MM Rates



Investment/borrowing decisions and currencies of denomination I

Assume you have some funds to place in the money market for 3 months: how to choose between domestic and foreign currency-denominated securities?



Relying exclusively on interest rate differentials might be seriously misleading: both **interest** and **exchange rates** should be taken into due account.



Investment/borrowing decisions and currencies of denomination II

1st option: invest in a USD-denominated security
(assuming \$ is the domestic currency)



At the end of a 3 months period, this would yield

$$\left(1 + \frac{r\$}{4}\right)$$

Investment/borrowing decisions and currencies of denomination III

2nd option: buy £ to invest in a GBP-denominated security and finally sell GBP forward after 3 months



1. Buy £: $\frac{1}{S_{\$/\pounds}}$

2. Invest $\frac{1}{S_{\$/\pounds}}$ in a GBP - denominated security

and get $\left(1 + \frac{r_{\pounds}}{4}\right) \frac{1}{S_{\$/\pounds}}$

3. Sell GBP forward to receive $\frac{F_{0.25\$/\pounds}}{S_{\$/\pounds}} \left(1 + \frac{r_{\pounds}}{4}\right) \text{USD}$

Investment/borrowing decisions and currencies of denomination IV

You will be indifferent between the two options only if

$$\left(1 + \frac{r\$}{4}\right) = \frac{F_{0.25\$/\pounds}}{S_{\$/\pounds}} \left(1 + \frac{r\pounds}{4}\right)$$



Investment/borrowing decisions and currencies of denomination V

Rearranging the terms we would get:

$$r_{\$} = r_{\pounds} + 4 \left[\frac{F_{0.25\$/\pounds} - S_{\$/\pounds}}{S_{\$/\pounds}} \right]$$

Annualised fwd premium/discount on £

Annualised GBP interest rate

Covered Interest Rate Parity I

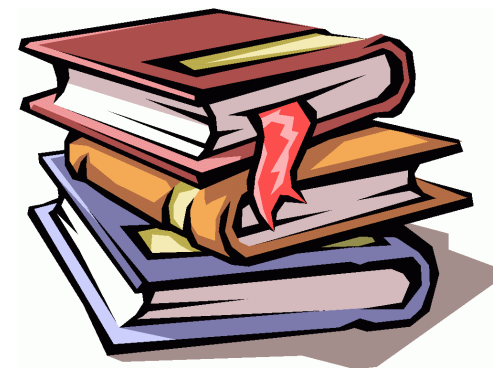
More generally, if we allow for compound interest, an investor/ borrower would be indifferent between domestic and foreign currency denominations of investment or debt if

$$(1 + r_D)^n = \frac{F({}_nD / F)}{S(D / F)} (1 + r_F)^n$$



Covered Interest Rate Parity II

When steps have been taken to **avoid foreign exchange risk** by use of **forward contracts** (hence the term “covered”), **rates of return on investments and costs of borrowing will be equal, irrespective of the currency of denomination** (*ceteris paribus*)



Covered Interest Rate Parity III

Delving with the *ceteris paribus* condition



There must be no frictions for the CIRP to hold, meaning no legal restrictions on the movement of K, no tax advantages among different countries...

Covered Interest Rate Parity IV

The CIRP links tightly together Spot and Forward rates → persistent deviations are **unlikely** to occur, because this would give rise to arbitrage opportunities (**No Free Lunch Principle**)

Deviations & arbitrage opportunities I

Suppose that

$$(1 + r_D)^n < \frac{F({}_nD/F)}{S(D/F)} (1 + r_F)^n$$



The best thing to do would be to borrow in your domestic currency and to invest simultaneously in a foreign currency-denominated security. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt (i.e. you will receive more domestic currency)

Deviations & arbitrage opportunities II

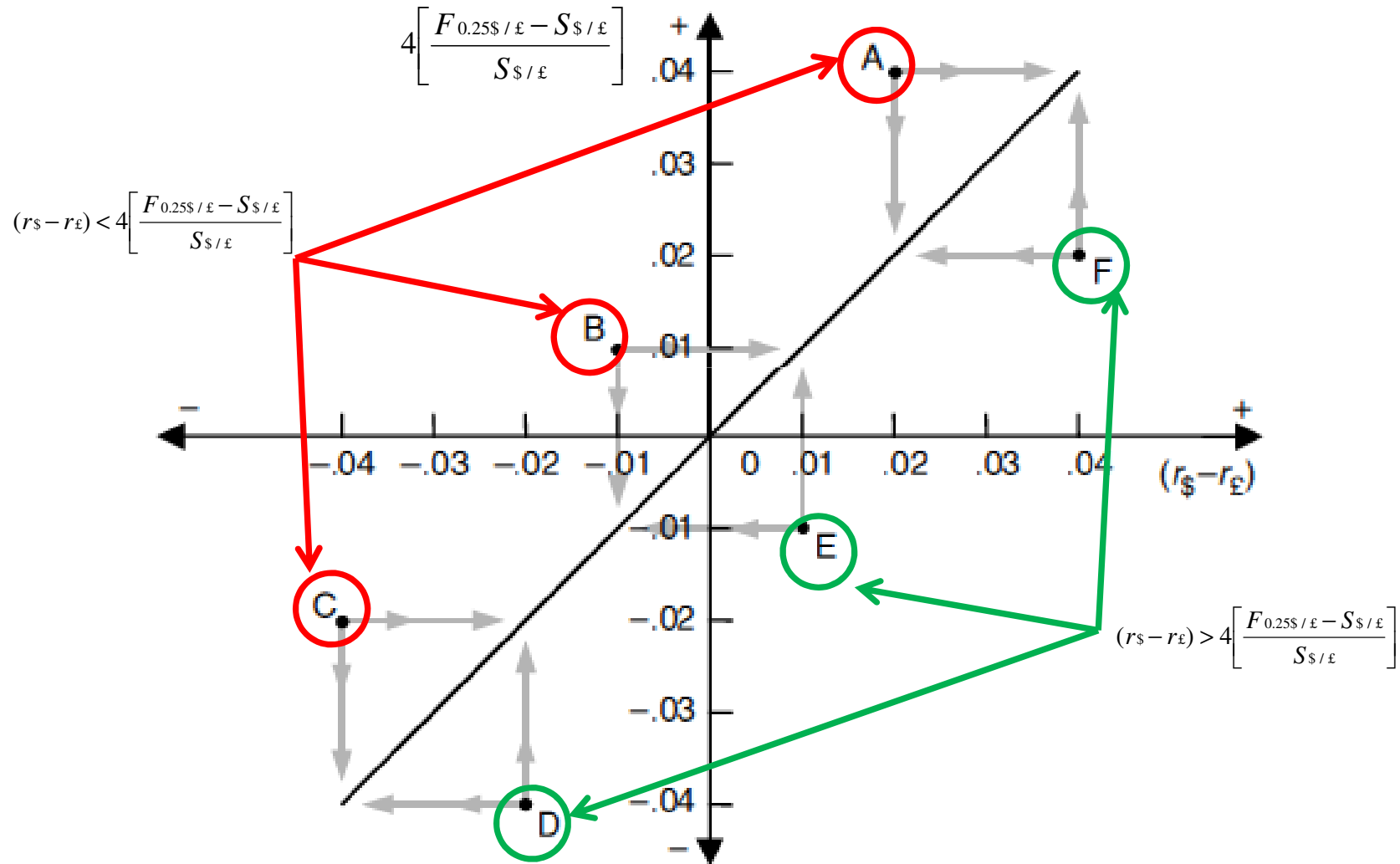
What if

$$(1 + r_D)^n > \frac{F({}_nD / F)}{S(D / F)} (1 + r_F)^n$$



The best thing to do would be to borrow foreign currency and to invest simultaneously in a domestic currency-denominated security. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt

CIRP: a graphical approach I



CIRP: a graphical approach II

For all the points lying above the 45° line (A, B and C), it must be that

$$(r_{\$} - r_{\pounds}) < 4 \left[\frac{F_{0.25\$/\pounds} - S_{\$/\pounds}}{S_{\$/\pounds}} \right]$$



This further implies:

- Covered investment in £ yields more than in \$;
- Borrowing in \$ is cheaper than covered borrowing in £

CIRP: a graphical approach III

The adjustment procedure driving A, B, and C down towards the 45° line works as follows:

1. Borrow \$, thus tending to increase $r_{\$}$;
2. Buy spot £ with the borrowed \$, thus tending to increase $S_{(\$/\pounds)}$;
3. Buy a £ security, thus tending to reduce r_{\pounds} ;
4. Sell the £ investment proceeds forward for \$, thus tending to reduce $F_{0.25}^{(\$/\pounds)}$.



Points 1 to 4 will all push **A**, **B** and **C** back down to the CIRP line

CIRP: a graphical approach IV

For all the points lying below the 45° line (D, E and F), it must be that

$$(r_{\$} - r_{\pounds}) > 4 \left[\frac{F_{0.2\$/\pounds} - S_{\$/\pounds}}{S_{\$/\pounds}} \right]$$



This further implies:

- Covered investment in \$ yields more than in £;
- Borrowing in £ is cheaper than covered borrowing in \$

CIRP: a graphical approach V

The adjustment procedure driving D, E, and F up towards the 45° line works as follows:

1. Borrow £, thus tending to increase $r\text{£}$;
2. Buy spot \$ with the borrowed £, thus tending to decrease $S(\text{\$/£})$;
3. Buy a \$ security, thus tending to reduce $r\text{\$}$;
4. Sell the \$ investment proceeds forward for £, thus tending to increase $F_{0.25}(\text{\$/£})$.



Points 1 to 5 will all push **D**, **E** and **F** back up to the CIRP line

Don't forget transaction costs I

Covered investment/borrowing involve two FX transactions
(one on the spot market and the other on the forward market)



Transaction costs have to be bore twice



There could be deviations from interest rate parity due to the
extra transaction costs of investing/borrowing in foreign
currency...

Is it always so?

Don't forget transaction costs II



It can be shown that transaction costs do not always contribute to deviations from CIRP.

Some preliminary notation:

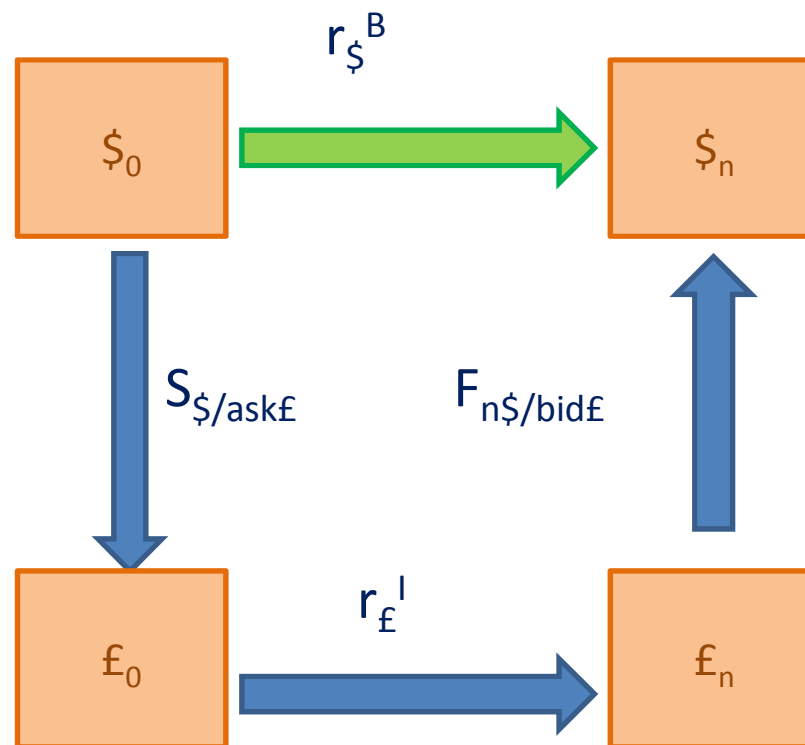
- $S(\$/\text{ask}\pounds)$, $S(\$/\text{bid}\pounds)$ = spot exchange rate when buying/selling \pounds with \$ respectively
- $F_n(\$/\text{ask}\pounds)$, $F_n(\$/\text{bid}\pounds)$ = n-year forward exchange rate when buying/selling \pounds with \$
- $r_{\I and r_{\pounds}^I = interest rates earned on USD/GBP-denominated investments
- $r_{\B and r_{\pounds}^B = interest rates due on USD/GBP-denominated borrowings

Don't forget transaction costs III

Case a) round-trip covered interest rate transaction

Suppose that, at time 0, you borrow $\$_0$. Assume further that the interest rate on such borrowing is $r_{\B ...

...the borrowed \$ are exchanged into £ at $S(\$/ask£)$...



... $£_n$ are sold at the forward rate $F(\$/bid£)$ stipulated at time 0, thus obtaining $\$_n$ (USD required to repay the debt).

...and placed in a £-denominated deposit yielding $£_n$ after n-months...

Don't forget transaction costs IV

Based on the CIRP, we could write

$$(1 + r_{\$}^B)^n = \frac{F_n(\$ / bid \pounds)}{S(\$ / ask \pounds)} (1 + r_{\pounds}^I)^n$$

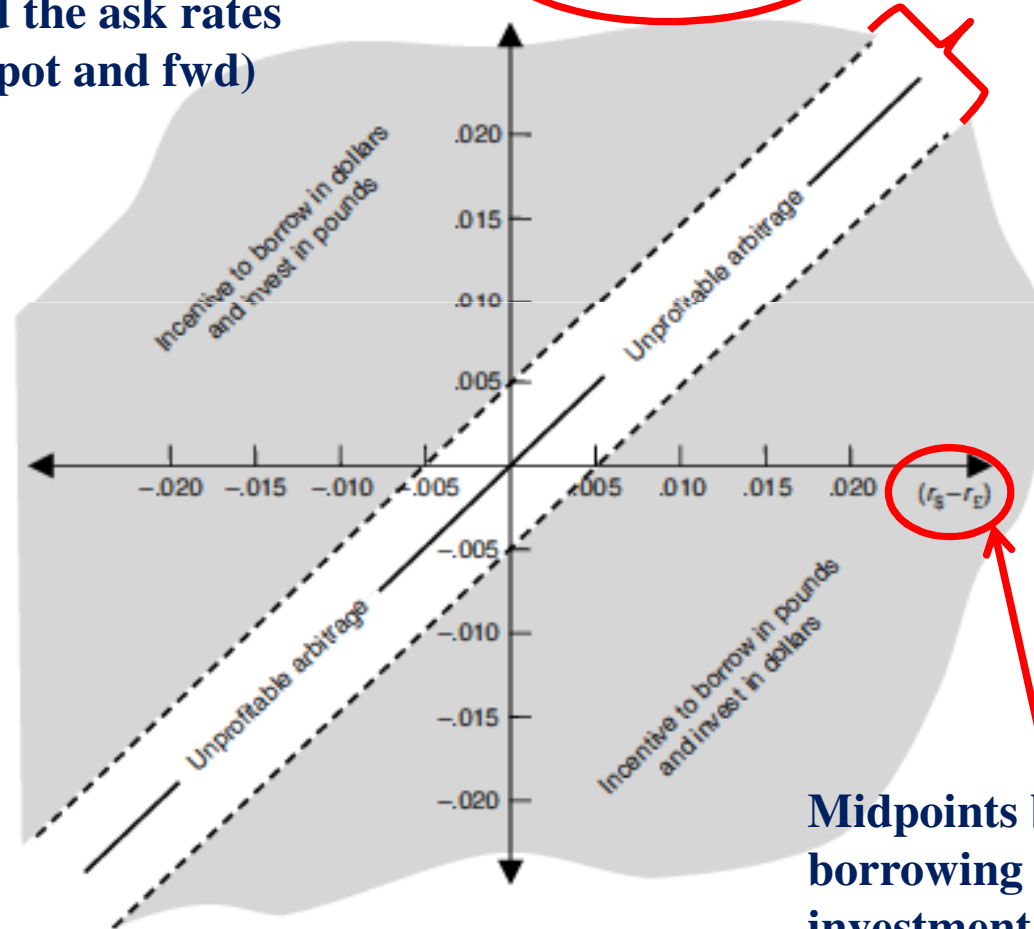
This is **NOT** a perfect 45°-line on the CIRP diagram, but more a “band” drawn around mid-rates. This is because of the transactions costs to be bore:

- Bid-ask spread = $(S(\$ / \text{ask} \pounds) - F_n(\$ / \text{bid} \pounds))$
- Borrowing-investment transaction costs = $(r_{\$}^B - r_{\pounds}^I)$

Don't forget transaction costs V

Midpoints between the bid and the ask rates (both spot and fwd)

$$\left[\frac{F_1(S/\pounds) - S(S/\pounds)}{S(S/\pounds)} \right] (1+r_{\pounds})$$



Round-trip arbitrages (and the related transaction costs) determine a “band” around the CIRP line, within which arbitrages would be unprofitable

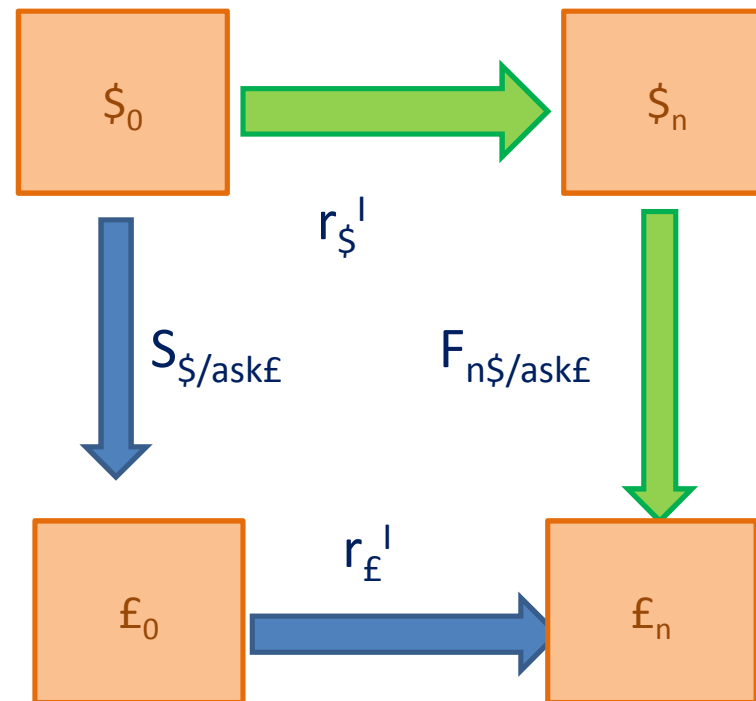
Midpoints between the borrowing and the investment rates

Don't forget transaction costs VI

Case b) one-way covered interest rate transaction

If you need \pounds_n sometime in the future and you have $\$0$ today, you could either...

...sell \$ for \pounds on the spot mkt at $S(\$/ask\pounds)$ and invest them in a \pounds -denominated deposit yielding \pounds_n when GBP are needed.



...or invest $\$0$ in a \$-denominated deposit and use the proceeds ($\$n$) to buy \pounds forward at $F_n(\$/ask\pounds)$, when GBP are needed.

Don't forget transaction costs VII

Based on the CIRP condition, we could write

$$(1 + r_{\$^I})^n = \frac{F_n(\$ / ask\pounds)}{S(\$ / ask\pounds)} (1 + r_{\pounds^I})^n$$

This would plot an exact 45° line in the CIRP diagram, given that there are virtually no transaction costs:

- Bid-ask spread = $(S(\$ / \mathbf{ask}\pounds) - F_n(\$ / \mathbf{ask}\pounds))$
- Borrowing-investment transaction costs = $(r_{\underline{\$}}^I - r_{\underline{\pounds}}^I)$

To sum up I

For round-trip arbitrages to be profitable, deviations from the CIRP line must be large enough to overcome **transaction costs...**

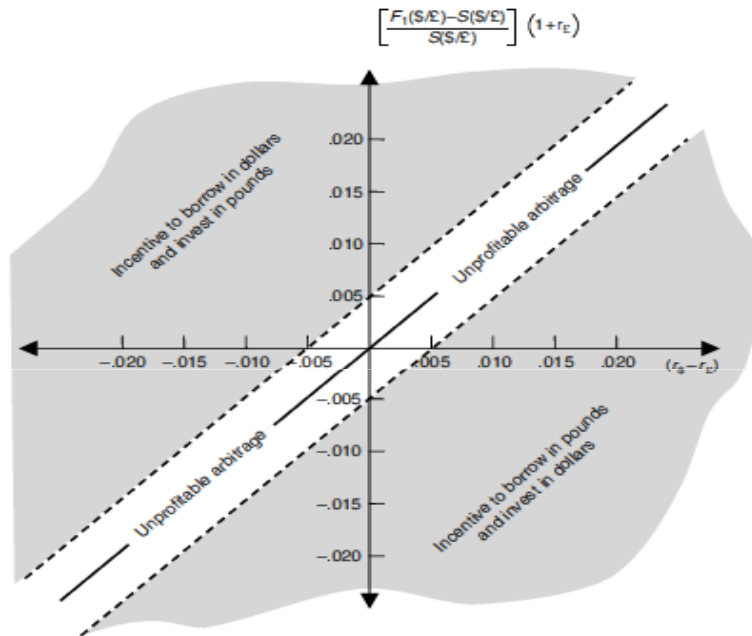


...and this will hardly ever occur in practice
(Could you explain why?)



To sum up II

Round-trip transactions

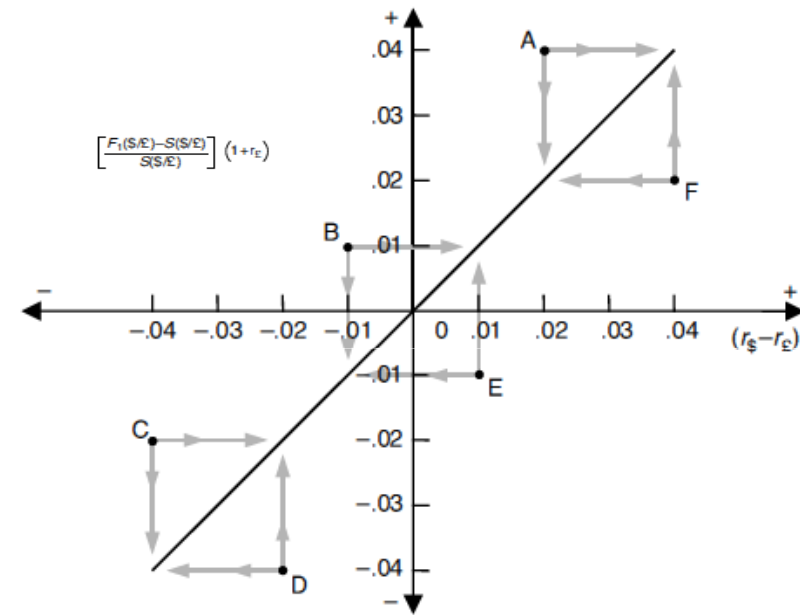


$$(1+r_s^B)^n = \frac{F_n(\$ / bid\pounds)}{S(\$ / ask\pounds)}(1+r_{\pounds}^I)^n$$

If this band were too large
and
... $(1+r_s^B)^n \neq \frac{F_n(\$ / bid\pounds)}{S(\$ / ask\pounds)}(1+r_{\pounds}^I)^n$
...



One-way transactions



$$(1+r_s^I)^n = \frac{F_n(\$ / ask\pounds)}{S(\$ / ask\pounds)}(1+r_{\pounds}^I)^n$$

... $(1+r_s^I)^n = \frac{F_n(\$ / ask\pounds)}{S(\$ / ask\pounds)}(1+r_{\pounds}^I)^n$ also will
not be satisfied, thus giving rise
to undue profit opportunities...

To sum up III

...for instance, if $(1+r_{\$}^I)^n > \frac{F_n(\$ / ask\pounds)}{S(\$ / ask\pounds)}(1+r_{\pounds}^I)^n$, nobody would invest in a £-

denominated deposit (as market players would rather put their money in a \$-denominated investment)



This would gradually drive $r_{\$}^I \downarrow$ and $r_{\pounds}^I \uparrow$, until equilibrium is restored again and **arbitrage opportunities are completely reabsorbed (No Free Lunch principle)**



Transaction costs do **not** bring about profitable arbitrage opportunities

How to construct synthetic securities with spot and forward contracts + borrowing and lending

Building synthetic securities I

The CIRP implies

$$(1 + r_D)^n = \frac{F_n(D/F)}{S(D/F)} (1 + r_F)^n$$

Rearranging the terms

$$F_n(D/F) = S(D/F) \frac{(1 + r_D)^n}{(1 + r_F)^n}$$

Building synthetic securities II

$$F_n(D/F) = S(D/F) \frac{(1+r_D)^n}{(1+r_F)^n}$$

An n-period synthetic forward can be constructed by combining...

...a spot contract with...

...fixed-rate, n-period borrowing and lending in the domestic and foreign currencies respectively.

Building synthetic securities III

$$(1 + r_D) = (1 + r_F) \frac{F_n(D / F)}{S(D / F)}$$

A synthetic domestic currency-denominated security can be obtained by combining...

...a foreign currency-denominated security with...

...a forward/spot swap.

Some lessons to learn

The **CIRP** is useful:

1. when trying to **understand the direction of K movements** → towards the currency with higher covered yield;
2. to **build/replicate a financial contract**;
3. to **hedge** a financial position



To put it into practice

Consider the following:

Spot rate: Currency₁ 0.64/Currency₂

$r_{1y_Currency1} = 5\%$

$r_{1y_Currency2} = 9\%$

1. Calculate the theoretical price of a one year forward contract.
2. What would you do if the forward price was quoted at Currency₁ 0.65/Currency₂ in the market place? Where would you borrow? Lend? Calculate the gain on a Currency₁ 100 million arbitrage transaction.
3. What would you do if the future price was quoted at Currency₁ 0.60/Currency₂ in the market place? Where would you borrow? Lend? Calculate the gain on a Currency₂ 100 million arbitrage transaction.