(1) X = numero settimane (su 7) in cui il dollaro cresce

$X \sim Bin(7, 0.4)$

A) $P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7) = 0.290$

B) $P(X=1) = 0.131$

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(2) X = numero caldaie (sulle 6 installate) che richiedono messa a punto

$X \sim Bin(6, 0.15)$

A) $P(X=0) = 0.377$

B) $P(X>2) = 1 - P(X=0) - P(X=1) - P(X=2) = 0.047$

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(3) X = numero auto restituite (sulle 50 vendute)

$X \sim Bin(50, 0.1)$

A) $E(X) = n \cdot p = 50 \cdot (0.1) = 5$

B) $\sigma(X) = \sqrt{Var(X)} = \sqrt{np(1-p)} = \sqrt{50 \cdot (0.1) \cdot (0.9)} = \sqrt{4.5} = 2.121$

C) Y = costi complessivi

$Y = 500 \cdot X$

$E(Y) = 500 \cdot E(X) = 500 \cdot 5 = 2500$

$Var(Y) = 500^2 \cdot Var(X) = 250000 \cdot (4.5) = 1125000$

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(4) X = numero fatture (sulle 20) con errori

$X \sim Bin(20, 0.12)$ $\quad Y = X/20$ = proporz. fatture errate

A) $P(Y < 0.1) = P(X/20 < 0.1) = P(X < 2) = P(X=0) + P(X=1) = 0.289$

B) $E(Y) = E\left(\frac{1}{20} X\right) = \frac{1}{20} E(X) = \frac{1}{20} \cdot 20 \cdot (0.12) = 0.12$

$\sigma(Y) = \sqrt{Var(Y)} = \sqrt{\frac{1}{400} Var(X)} = \sqrt{\frac{20 \cdot (0.12)(0.88)}{400}} = 0.073$

(5) X = numero impiegati che arrivano in 5 minuti

$X \sim Poisson(2)$

Y = numero impiegati che arrivano in 10 minuti

$Y \sim Poisson(4)$

A) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.677 = 0.323$

B) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.406 = 0.594$

C) $P(Y < 4) = P(Y \leq 3) = 0.433$

D) $P(Y = 3) = P(Y \leq 3) - P(Y \leq 2) = 0.433 - 0.238 = 0.195$

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(6) X = numero prenotazioni al minuto

Y = numero prenotazioni ogni 10 minuti

$X \sim Poisson(0.3)$ $\quad Y \sim Poisson(3)$

A) $P(X \geq 1) = 1 - P(X \leq 0) = 1 - 0.741 = 0.259$

B) $P(Y \geq 1) = 1 - P(Y \leq 0) = 1 - 0.050 = 0.950$

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(7) X = numero palline gialle estratte

A) $X \sim Bin(5, 0.3)$

B) $Y = X/5$

$$p_Y(y) = P(Y = y) = P(X/5 = y) = P(X = 5y) =$$

$$= \begin{cases} 0.168 & y = 0 \\ 0.360 & y = 1/5 \\ 0.309 & y = 2/5 \\ 0.132 & y = 3/5 \\ 0.028 & y = 4/5 \\ 0.002 & y = 1 \end{cases}$$

C) $E(Y) = \frac{1}{5} E(X) = \frac{1}{5} \cdot 5 \cdot (0.3) = 0.3$

$Var(Y) = \frac{1}{25} Var(X) = \frac{1}{25} \cdot 5 \cdot (0.3) \cdot (0.7) = 0.042$

8

X = errore misurazione

$X \sim N(0, 100)$

A) $P(X > 10) = P\left(\frac{X-0}{10} > \frac{10-0}{10}\right) = P(Z > 1) = 1 - P(Z \leq 1) =$

$= 1 - 0.8413 = 0.1587.$

B) $P(-5 < X < 10) = P\left(-\frac{5}{10} < Z < \frac{10}{10}\right) = P(-0.5 < Z < 1) =$

$= P(Z < 1) - P(Z < -0.5) = P(Z < 1) - [1 - P(Z < 0.5)] =$

$= 0.8413 - [1 - 0.6915] = 0.5328$

C) Y = numero misurazioni (sulle 10) con errore > 10

$Y \sim Bin(10, 0.1587)$ [0.1587 è stato calcolato in A]

$P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y=0) - P(Y=1) =$

$= 1 - 0.018 - 0.335 = 0.647$

9

X rend. A $\quad X \sim N(0, 0.01^2)$

Y rend. B $\quad Y \sim N(-0.01, 0.02^2)$

A) $E(X) > E(Y)$ $\quad X$ ha, in media, rend. maggiore

B) $Var(Y) > Var(X)$ $\quad Y$ ha maggiore rischio

C) $P(X > 0.02) = P\left(Z > \frac{0.02-0}{0.01}\right) = P(Z > 2) = 0.0228.$

$P(Y > 0.02) = P\left(Z > \frac{0.02+0.01}{0.02}\right) = P(Z > 1.5) = 0.0668.$

D) T = prezzo finale

$T = e^{Y} \cdot 10$

$P(T > 9) = P(10e^{Y} > 9) = P(e^{Y} > 0.9) = P(Y > \log(0.9)) =$

$= P\left(Z > \frac{\log(0.9) + 0.01}{0.02}\right) = P(Z > -4.76) \simeq 1$

10 $X \sim Bin(3, 0.5)$ $\quad (n=3,\ p=5)$

A) $Y = X/3$ $\quad S_Y = \{0, 1/3, 2/3, 1\}$

$$p_Y(y) = P(Y=y) = P(X/3 = y) = P(X = 3y) =$$

$$= \begin{cases} p_X(0) = \frac{3!}{0!\,3!}(0.5)^0(0.5)^3 = 0.125 & y=0 \\ p_X(1) = \frac{3!}{1!\,2!}(0.5)^1(0.5)^2 = 0.375 & y=1/3 \\ p_X(2) = \frac{3!}{2!\,1!}(0.5)^2(0.5)^1 = 0.375 & y=2/3 \\ p_X(3) = \frac{3!}{3!\,0!}(0.5)^3(0.5)^0 = 0.125 & y=1 \\ 0 & \text{altrove} \end{cases}$$

B) $T = e^X$ $\quad S_T = \{e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.08\}$

$$p_T(t) = \begin{cases} 0.125 & t=1 \\ 0.375 & t=2.72 \\ 0.375 & t=7.39 \\ 0.125 & t=20.08 \\ 0 & \text{altrove} \end{cases}$$

---

11 $X \sim N(2, 9)$

A) $Y = e^X$ $\quad P(Y<8) = P(e^X < 8) = P(X < \log 8) =$

$$= P(X < 2.08) = P\left(Z < \frac{2.08-2}{3}\right) = P(Z < 0.03) = 0.5120$$

B) DISTRIBUZIONE LOG-NORMALE (NON IN PROGRAMMA NELLA PRIMA PARTE DEL CORSO)

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12) $X \sim Bern(1/3)$ $\quad p_X(x) = \begin{cases} 1/3 & x=1 \\ 2/3 & x=0 \\ 0 & \text{altrove} \end{cases}$

$Y = \quad p_Y(y) = \begin{cases} 1/3 & x=1,\ x=2,\ x=3 \\ 0 & \text{altrove} \end{cases}$

X \ Y	1	2	3	
0	2/9	2/9	2/9	2/3
1	1/9	1/9	1/9	1/3
	1/3	1/3	1/3	

X, Y indipendenti

A) $P(X=1, Y<3) = P(X=1)\cdot P(Y<3) = \frac{1}{3}\cdot\frac{2}{3} = 2/9$

B) $P(X+Y=2) = P(X=1, Y=1) + P(X=0, Y=2) =$
$= \frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3} = 1/9 + 2/9 = 1/3$

C) $\rho(X,Y) = 0$ (perché X e Y sono indipendenti)

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13)

X \ Y	0	1	2	
0	4/16	5/16	0	9/16
1	0	3/16	3/16	6/16
2	0	0	1/16	1/16
	4/16	8/16	4/16	

$S = X+Y \qquad T = (X+Y)/2$

E) $p_S(s) = \begin{cases} 4/16 & s=0 \\ 5/16 & s=1 \\ 3/16 & s=2,\ s=3 \\ 1/16 & s=4 \\ 0 & \text{altrove} \end{cases}$

$p_T(t) = \begin{cases} 4/16 & t=0 \\ 5/16 & t=1/2 \\ 3/16 & t=1,\ t=3/2 \\ 1/16 & t=2 \\ 0 & \text{altrove} \end{cases}$

A) $p_X(x) = \begin{cases} 1/16 & x=2 \\ 6/16 & x=1 \\ 9/16 & x=0 \\ 0 & \text{altrove} \end{cases}$ $\qquad X \sim Bin(2, 1/4)$

B) $p_Y(y) = \begin{cases} 4/16 & x=0,\ x=2 \\ 8/16 & x=1 \\ 0 & \text{altrove} \end{cases}$ $\qquad X \sim Bin(2, 1/2)$

C) Non sono indipendenti (ad esempio,
$P(X=1, Y=0) \neq P(X=1)\cdot P(Y=0)$)

D) $cov(X,Y) = 5/16 \qquad \rho(X,Y) = 0.722$

(14) $X \sim N(20, 9)$ $\quad Y \sim N(10, 25)$

$\rho(X,Y) = 0.1$

a) $P(X > K) = 0.65$ quindi $P\left(Z > \frac{K-20}{3}\right) = 0.65$

quindi $P\left(Z \le \frac{K-20}{3}\right) = 0.35$ quindi $P\left(Z \ge -\frac{K-20}{3}\right) = 0.35$

quindi $P\left(Z < -\frac{K-20}{3}\right) = 0.65$.

Dalle tavole: $-\frac{K-20}{3} = 0.39$, cioè $K = 18.83$.

b) $T = X + Y$ = tempo totale $\quad T \sim N(30, 37)$

$E(T) = 20 + 10 = 30$

$Var(T) = 9 + 25 + 2\,cov(X,Y) = 34 + 2 \cdot (3 \cdot 5 \cdot (0.1)) = 37$

c) $P(T > 35) = P\left(Z > \frac{35-30}{\sqrt{37}}\right) = P(Z > 0.82) = 0.2061$.

d) $T_1, T_2, \dots, T_{30}$ i.i.d. $\sim N(30, 37)$

$\overline{T} = \frac{T_1 + T_2 + \dots + T_{30}}{30}$ media campionaria

$\overline{T} \sim N(30, 37/30) = N(30, 1.233)$

e) Se $\rho(X,Y) = 0$, $Var(T)$ sarebbe stata minore (vedi punto b)), quindi anche $Var(\overline{T})$ sarebbe stata minore.