# Lesson III: The Relationship among Spot, Fwd and Money Mkt Rates

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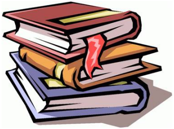
# Getting Started

# Investing on an International Scale

Assume you have some funds to place in the money market for 3 months: how to choose between **domestic and foreign currency-denominated** securities?

# Watch out!

Relying **exclusively** on interest rate differentials might be seriously misleading: **both** interest and exchange rates should be taken into due account



# Domestic-Currency Denominated Investment

If you decide to invest in a USD-denominated security (assuming the USD is the domestic currency), at the end of the investment period you would get

$$1 + \frac{r_{USD}}{4}$$

# Foreign-Currency Denominated Investment

If you conversely decide to invest in a foreign-currency denominated security (assume GBP), you would have to:

- **Buy GBP**, thus getting
$$\frac{1}{S_{\frac{USD}{GBP}}}$$
- **Invest the amount above in a GBP-denominated asset** and get (at maturity)
$$\frac{1}{S_{\frac{USD}{GBP}}} \cdot \left(1 + \frac{r_{GBP}}{4}\right)$$
- **Sell GBP forward** in order to receive
$$\frac{F_{0.25\,\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}} \cdot \left(1 + \frac{r_{GBP}}{4}\right)$$

# The Investor's Dilemma

You will be indifferent between the two options only if

$$1 + \frac{r_{USD}}{4} = \frac{F_{0.25\,\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}} \cdot \left(1 + \frac{r_{GBP}}{4}\right)$$

# Playing with Algebra

Rearranging the terms we would get:

$$r_{USD} = r_{GBP} + 4 \cdot \frac{F_{0.25\,\frac{USD}{GBP}} - S_{\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}}$$

With

- **Annualised GBP interest rate**: $r_{GBP}$
- **Annualised fwd premium/discount on GBP**: $4 \cdot \frac{F_{0.25\,\frac{USD}{GBP}} - S_{\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}}$

# CIRP: Definition

More generally, if we allow for compound interest, an investor/ borrower would be **indifferent** between domestic and foreign currency denominations of investment or debt if

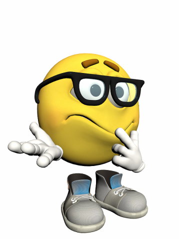
$$(1 + r_D)^n = \frac{F_{n\frac{D}{F}}}{S_{\frac{D}{F}}}(1 + r_F)^n$$

# In Simpler Terms...

When steps have been taken to avoid foreign exchange risk by use of forward contracts (hence the term "covered"), rates of return on investments and costs of borrowing will be equal, irrespective of the currency of denomination (**ceteris paribus**)

# Lifting the Curtain on the Ceteris Paribus Condition

There must be **no** frictions for the CIRP to hold perfectly, meaning **no** legal restrictions on the movement of K, **no** tax advantages among different countries...



# Deviation from Equilibrium and Arbitrage Opportunities I

Suppose that

$$(1 + r_D)^n < \frac{F_{n\frac{D}{F}}}{S_{\frac{D}{F}}}(1 + r_F)^n$$

The best thing to do would be **to borrow in your domestic currency** and **to invest simultaneously in a foreign currency-denominated security**. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt (i.e. you will receive more domestic currency)

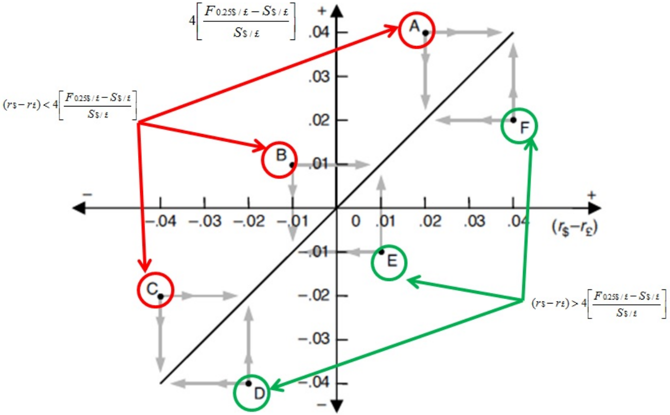
# Deviation from Equilibrium and Arbitrage Opportunities II

If, conversely,

$$(1 + r_D)^n > \frac{F_{n\frac{D}{F}}}{S_{\frac{D}{F}}}(1 + r_F)^n$$

The best thing to do would be **to borrow foreign currency and to invest simultaneously in a domestic currency-denominated security**. At the end of the investment period, the hedged transaction will allow you to get more than required to repay the initial debt

# Deviations from Equilibrium: a Graphical Approach



# What Happens above the CIRP Line? I

For all the **points lying above the equilibrium line** (A,B and C), it must be that

$$(r_{USD} - r_{GBP}) < 4 \cdot \frac{F_{n\frac{USD}{GBP}} - S_{\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}}$$

This further implies:

- Covered **investment** in **GBP yields more than in USD**
- **Borrowing in USD is cheaper** than covered borrowing in **GBP**

# What Happens above the CIRP Line? II

The **adjustment procedure** driving A, B, and C down towards the equilibrium line works as follows:

- **Borrow USD**, thus tending to increase $r_{USD}$
- **Buy spot GBP with the borrowed USD**, thus tending to increase $S_{\frac{USD}{GBP}}$
- **Buy a GBP-denominated security**, thus tending to reduce $r_{GBP}$
- **Sell the GBP investment proceeds forward for USD**, thus tending to reduce $F_{0.25\,\frac{USD}{GBP}}$

Points 1 to 4 will all push A, B and C **back down to the CIRP** line

# What Happens below the CIRP Line? I

For all the **points lying below the equilibrium line** (D, E and F), it must be that

$$(r_{USD} - r_{GBP}) > 4 \cdot \frac{F_{n\frac{USD}{GBP}} - S_{\frac{USD}{GBP}}}{S_{\frac{USD}{GBP}}}$$

This further implies:

- Covered **investment** in **USD yields more than in GBP**
- **Borrowing in GBP is cheaper** than covered borrowing in **USD**

# What Happens below the CIRP Line? II

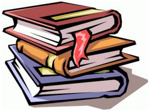
The **adjustment procedure** driving D, E, and F up towards the equilibrium line works as follows:

- **Borrow GBP**, thus tending to increase $r_{GBP}$
- **Buy spot USD** with the borrowed GBP, thus tending to decrease $S_{\frac{USD}{GBP}}$
- **Buy a USD-denominated security**, thus tending to reduce $r_{USD}$
- **Sell the USD investment proceeds forward** for USD, thus tending to increase $F_{0.25\frac{USD}{GBP}}$

Points 1 to 5 will all push D, E and F **back up to the CIRP** line

# Empirical Findings

Persistent deviations from the CIRP are **unlikely** to occur, because this would give rise to arbitrage opportunities (**No Free Lunch Principle**)



# And What If We Included TC?

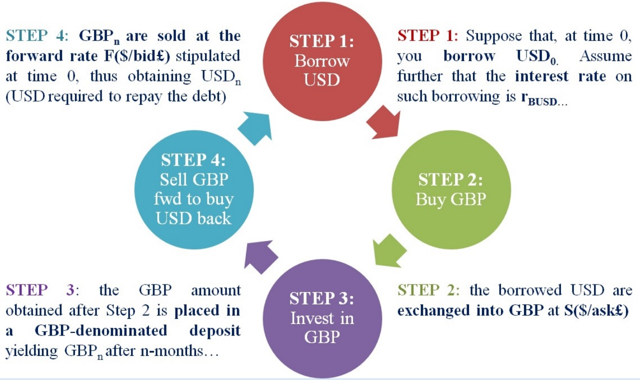
Covered investment/borrowing involve **two FX transactions** (one on the spot market and the other on the forward market).

**Transaction costs** have to be faced **twice**.

One may be lead to think there **could be deviations** from interest rate parity due to the **extra transaction costs** of investing/borrowing in foreign currency...

**Is it always and necessarily so?**

# Case 1: Round-Trip Transactions



# Round-Trip Transactions and CIRP



Based on the CIRP,

$$(1 + r_{BUSD})^n = \frac{F_{n\frac{USD}{bidGBP}}}{S_{\frac{USD}{askGBP}}} \cdot (1 + r_{IGBP})^n$$

This is **NOT** a perfect equilibrium line on the CIRP diagram, but more a "band" drawn around mid-rates. This is because of the transactions costs to be faced:

- **Bid/Ask spread**: $S_{\frac{USD}{askGBP}} - F_{n\frac{USD}{bidGBP}}$
- **Borrowing/Investment spread**:$(r_{BUSD} - r_{IGBP})$

# Case 2: One-Way Transactions I

If you need $GBP_n$ sometime in the future and you have $USD_0$ today, you could:

- **Alternative 1**: invest the USD you have in USD-denominated security and use the proceeds of the foregoing investment to buy GBP fwd (when they are needed)
- **Alternative 2**: sell the USD you have to buy GBP and invest them in a GBP-denominated security, yielding the GBP amount you need at maturity

# Case 2: One-Way Transactions II

**STEP 1: invest $USD_0$ in a USD-denominated deposit**

**STEP 2:** Use the proceeds ($USD_n$) to **buy GBP forward at $F_n$($/ask£)**, when GBP are needed.

**STEP 1:** Invest in USD → **STEP 2:** Use the proceeds to buy GBP fwd

**STEP 1: sell USD for GBP** on the spot mkt at **S($/ask£)**

**STEP 2: invest** the GBP in a **GBP-denominated deposit** yielding $GBP_n$ when GBP are needed.

**STEP 1:** Sell USD to buy GBP → **STEP 2:** Invest in GBP

# One-Way Transactions and CIRP

Based on the CIRP,

$$(1 + r_{IUSD})^n = \frac{F_{n\frac{USD}{askGBP}}}{S_{\frac{USD}{askGBP}}}(1 + r_{IGBP})^n$$

This would plot an **exact** line in the CIRP diagram, given that there are virtually **no** transaction costs:

- **Bid/Ask spread**: $S_{\frac{USD}{askGBP}} - F_{n\frac{USD}{askGBP}}$
- **Borrowing/Investment spread**:$(r_{IUSD} - r_{IGBP})$

# Profit Opportunities are more Apparent than Real...

For round-trip arbitrages to be profitable, deviations from the CIRP line must be large enough to overcome transaction costs...and this will hardly ever occur in practice (Could you explain why?)

Transaction costs do **not** bring about profitable arbitrage opportunities

# Synthetic Fwd I

Rearranging the CIRP...

$$F_{n\frac{D}{F}} = S_{\frac{D}{F}} \cdot \frac{(1+r_D)^n}{(1+r_F)^n}$$

# Synthetic Fwd II

- An n-period **synthetic forward**

$$F_{n\frac{D}{F}}$$

- ...can be constructed by **combining a spot** contract

$$S_{\frac{D}{F}}$$

- ...with fixed-rate, n-period **borrowing and lending** in the domestic and foreign currencies respectively.

$$\frac{(1+r_D)^n}{(1+r_F)^n}$$

# Synthetic DC-denominated security

$$(1 + r_D)^n = (1 + r_F)^n \cdot \frac{F_{n\frac{D}{F}}}{S_{\frac{D}{F}}}$$

- A **synthetic domestic currency-denominated security**

$$(1 + r_D)^n$$

- ... can be obtained by combining a **foreign currency-denominated security**

$$(1 + r_F)^n$$

- ...with a **forward/spot swap**

$$\frac{F_{n\frac{D}{F}}}{S_{\frac{D}{F}}}$$

# Some Lessons to Learn

The **CIRP** is useful:

- when trying to **understand the direction of K movements** (towards the currency with higher covered yield)
- to **build/replicate** a financial contract
- to **hedge** a financial position

# Synthetic Securities

**Synthetic Security**: financial instrument that is created artificially by combining the features of a collection of other assets

# Round-Trip and One-Way Transactions

- **Round-Trip Transaction**: Borrowing in one currency, lending in another, and then selling the second currency back into the first so as to end up back in the first currency (*id est*, you start with a currency and you end up with the same one).
- **One-Way Transaction**: The process of choosing the best way to exchange one currency for another or choosing the best currency in which to invest or borrow (*id est*, you start with a currency and you end up with a different one).

# To Put It into Practice I



**3.1**: Consider the following rates:

| | |
|---|---|
| $\mathbf{S}_{\frac{C_1}{C_2}}$ | 0.64 |
| $\mathbf{r}_{1y-C_1}$ | 0.05 |
| $\mathbf{r}_{1y-C_2}$ | 0.09 |

- Calculate the theoretical price of a one year forward contract
- What would you do if the forward price was quoted at $F_{1\frac{C_1}{C_2}}$=0.65 in the market place? Where would you borrow? Lend? Calculate the gain on a $C_1$ 100 million arbitrage transaction
- What would you do if the forward price was quoted at $F_{1\frac{C_1}{C_2}}$=0.6 in the market place? Where would you borrow? Lend? Calculate the gain on a $C_2$ 100 million arbitrage transaction

# To Put It into Practice II

**3.2**: The following exchange rates and one-year interest rates exist.

| | **Bid** | **Ask** |
|---|---|---|
| $\mathbf{S}_{\frac{A}{B}}$ | 1.52 | 1.63 |
| $\mathbf{F}_{1\frac{A}{B}}$ | 1.42 | 1.53 |

| | **Deposit** | **Loan** |
|---|---|---|
| $\mathbf{r}_A$ | 0.04 | 0.09 |
| $\mathbf{r}_B$ | 0.05 | 0.1 |

You have 100 A to invest for 1 year. Would you benefit from engaging in covered interest arbitrage?