Lesson X: International Portfolio Investments

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Rusting Off...

- **Portfolio Expected Return**

$$E[r_p] = \Sigma_{i=1}^{n} x_i \cdot E[r_i]$$

- **Portfolio Variance**

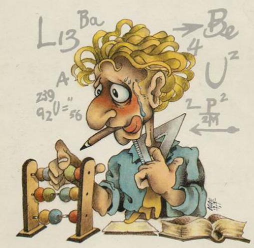
$$Var[r_p] = \Sigma_{i=1}^{n} x_i^2 \cdot \sigma_i^2 + \Sigma_{i=1}^{n} \Sigma_{j \neq i=1}^{n} x_i \cdot x_j \cdot \sigma(i,j)$$

Can you spot the diversification-related terms?

Looking for Diversification

The **benefits of diversification** for risk reduction (for a given E[r]) are to be found out in the **covariance-correlation term**.

$$\Sigma_{i=1}^{n}\Sigma_{j\neq i=1}^{n} x_i \cdot x_j \cdot \sigma(i,j)$$



The portfolio standard deviation is reduced if the correlation terms are **negative**, but, even when they are **positive**, the portfolio standard deviation is still less than the weighted average of the individual securities standard deviations

To Make Matters Explicit

	Stock a	Stock b
E[r]	0.08	0.055
Risk	0.15	0.1
Weights	0.75	0.25

ρ(a,b)	$E[r_p]$	$Risk_p$	$WRisk_p$
-1	0.07375	**0.0875**	0.1375
-0.5	0.07375	**0.1023**	0.1375
-0.2	0.07375	**0.1103**	0.1375
0	0.07375	**0.1152**	0.1375
0.2	0.07375	**0.12**	0.1375
0.5	0.07375	**0.1269**	0.1375
1	0.07375	**0.1375**	0.1375

A Few Key Points to Retain

Portfolios of **less than perfectly** correlated assets always offer better risk-return opportunities than the individual constituent securities on their own.

$$\Downarrow$$

What about **perfect positive** correlations?

The Risk-Return Framework

Assuming **risk aversion**, investors demand **higher returns for taking on higher risk**.

Remember: Risk relates to returns' **volatility - variability** over a given time period (generally defined as standard deviation of returns)$\Rightarrow$ Step back to Lesson IX

Portfolio Selection Criteria

How to select the most suitable combination of assets so as to **maximize portfolio return for a given level of risk**?

Focus on the triplet:
Risk-Return-Correlation

Portfolio Investment with 2 Risky Assets and Correl= -0.5

Suppose there are only 2 risky assets on the market (a and b, $\rho(a, b) = -0.5$) and assume further that:

Constituents	$E[r]$	$Risk$
a	0.08	0.15
b	0.055	0.1

Depending on the different weighting schemes below, would you be able to find the Expected Return and the Standard Deviation of the portfolio?

W_a	W_b	$E[r_p]$	$StDev_p$
1	0	....	....
0.75	0.25	....	....
0.5	0.5	....	....
0.25	0.75	....	....
0	1	....	....

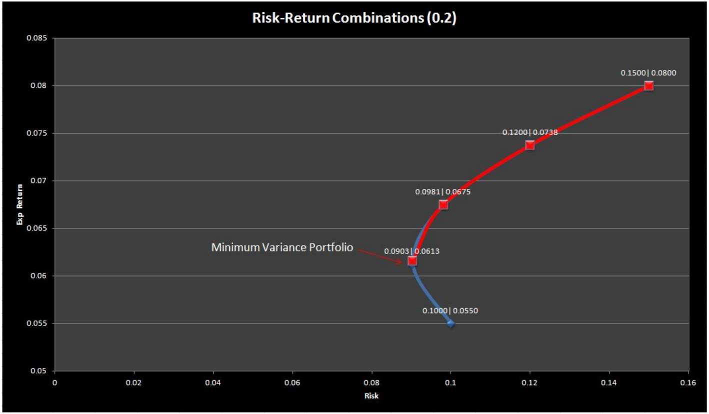
In Graphical Terms - Inter-Asset Correlation=-0.5

Portfolio Investment with 2 Risky Assets and Correl= 0.2

Assume now that $\rho(a, b) = 0.2$: given the different weighting schemes below, would you be able to find the Expected Return and the Standard Deviation of the portfolio?

W_a	W_b	$E[r_p]$	$StDev_p$
1	0	....	....
0.75	0.25	....	....
0.5	0.5	....	....
0.25	0.75	....	....
0	1	....	....

In Graphical Terms - Inter-Asset Correlation=0.2

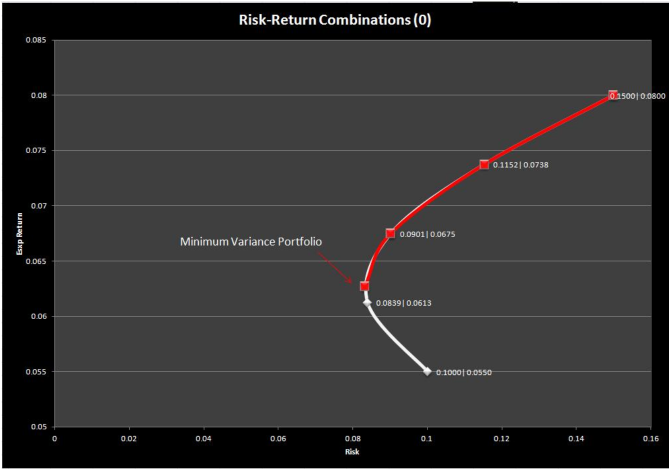


Portfolio Investment with 2 Risky Assets and Correl= 0

Assume now that $\rho(a, b) = 0$: given the different weighting schemes below, would you be able to find the Expected Return and the Standard Deviation of the portfolio?

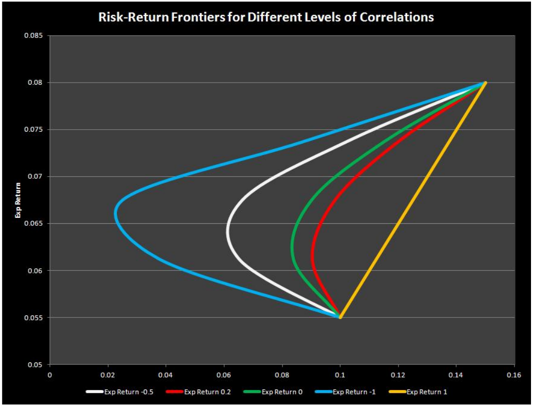
W_a	W_b	$E[r_p]$	$StDev_p$
1	0	....	....
0.75	0.25	....	....
0.5	0.5	....	....
0.25	0.75	....	....
0	1	....	....

In Graphical Terms - Inter-Asset Correlation=0



Wrapping Up

The **shape** of the frontier varies depending on **inter-assets correlations**.



The final portfolio selection will depend **exclusively** on individual risk appetite

What If We Added a Riskless Asset?

Suppose there are only 2 risky assets on the market (a and b, $\rho(a, b) = -0.5$ - step back to the previous section) and a riskless portfolio (made up of MM instruments and Govt Bonds), yielding 0.05.
How to determine which **optimal risky portfolio is to be best combined with the riskless** security basket?

Adopted Selection Criteria: **Max[REWARD to RISK]**

Digging a Little Deeper...

Assume you invest a proportion of your total wealth (α) in the risky portfolio ($E[r_{risky}]$) and the remaining portion of your investable K ($(1-\alpha)$) in the riskless asset (yielding $r_{riskless}$):

- **Portfolio Expected Return** $E[r_p] = \alpha E[r_{risky}] + (1-\alpha) r_{riskless} = r_{riskless} + \alpha(E[r_{risky}] - r_{riskless})$
- **Portfolio Standard Deviation** $StDev_p = \alpha StDev_{risky}$

Playing with Algebra I

Rearranging the StDev formula above, we would get

$$\alpha = \frac{StDev_p}{StDev_{risky}}$$

Let's now substitute α in the Expected Return formula:

$$E[r_p] = r_{riskless} + \frac{StDev_p}{StDev_{risky}}(E[r_{risky}] - r_{riskless})$$

Or equivalently

$$E[r_p] = r_{riskless} + StDev_p \frac{(E[r_{risky}] - r_{riskless})}{StDev_{risky}}$$

Playing with Algebra II

Notice that

$$E[r_p] = r_{riskless} + StDev_p \frac{(E[r_{risky}] - r_{riskless})}{StDev_{risky}}$$

is the equation of as straight line drawn in the Risk-Expected Return plan, with slope

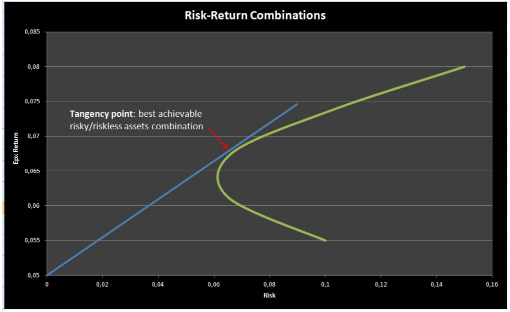
$$\frac{(E[r_{risky}] - r_{riskless})}{StDev_{risky}}$$

The ratio above technically goes under the name of **Sharpe Ratio**

$$\Downarrow$$

The best achievable combination riskless asset/risky portfolio is the one that **maximizes the Sharpe Ratio**

A Graphical Approach



- Where would you represent the risk-free portfolio? Why?
- Investors will combine the tangency portfolio with the risk-free asset to form their overall portfolio: the **allocation** they choose **depends on their preferences for risk**

A few points to stress

- The foregoing tangency line is known as **Capital Allocation Line**
- Depending on the proportions of your wealth that you decide to invest in the risky asset and in the riskless portfolio respectively, you will **move along the straight line**
- Assuming that α and $(1-\alpha)$ represent the proportions of your wealth invested in the risky portfolio and in the risk-free asset respectively, which point on the line represents $\alpha = 0$?
- Which point on the line represents $\alpha = 1$?

Finding the tangency portfolio I

Assume **T is a risky portfolio** belonging to the **efficient frontier** and r_f= risk free rate earned on the riskless asset

$$\Downarrow$$

If T is the tangency portfolio, then $\forall i, j$

$$\frac{E[r_i]-r_f}{Cov(r_i;r_T)} = \frac{E[r_j]-r_f}{Cov(r_j;r_T)}$$

with i and j= securities belonging to T
Remember that

$$Cov(z; Ax + By) = A \cdot Cov(z; x) + B \cdot Cov(z; y)$$

and assume T is made up of only two assets, so that

$$T = \omega r_i + (1 - \omega) r_j$$

Finding the tangency portfolio II

$Cov(r_i; r_T)$ and $Cov(r_j; r_T)$ can thus be restated as

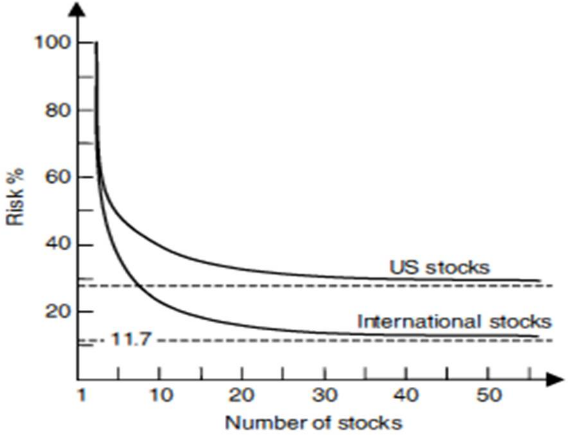
$$Cov(r_i; r_T) = \omega Cov(r_i; r_i) + (1-\omega) Cov(r_i; r_j) = \\ \omega Var(r_i) + (1-\omega) Cov(r_i; r_j)$$
$$Cov(r_j; r_T) = \omega Cov(r_i; r_j) + (1-\omega) Cov(r_j; r_j) = \\ \omega Cov(r_i; r_j) + (1-\omega) Var(r_j)$$

Let's substitue and solve for ω to determine the optimal (tangent) portfolio T to be best combined with the risk-free asset.

$$\frac{E[r_i]-r_f}{\omega Var(r_i)+(1-\omega)Cov(r_i;r_j)} = \frac{E[r_j]-r_f}{\omega Cov(r_i;r_i)+(1-\omega)Var(r_j)}$$

International Diversification - B.H. Solnik, 1974

The **benefits** of diversification **are even higher** when investment decisions are taken on an **international scale**.



The Benefits and the Drawbacks of International Diversification

- **Rewards**: Significant reduction in the volatility of the resulting portfolio
- **Risks**: An internationally-diversified portfolio is however subject to the risk of unexpected FX rate fluctuations

To Make Matters Explicit

When investing on an international scale

$$E[r_p] = r_{pF} + \Delta S_{\frac{F}{D}}$$

$$Var_p = Var(\Delta S_{\frac{F}{D}}) + Var(r_{pF}) + 2Cov(r_{pF}; \Delta S_{\frac{F}{S}})$$

- Var_{pF}: the variance of an internationally-diversified portfolio depends on...
- $Var(\Delta S_{\frac{F}{D}})$:...the variance of the FX rate...
- $Var(r_{pF})$:...the variance of the FC-denominated assets...
- $2Cov(r_{pF}; \Delta S_{\frac{F}{S}})$:...as well as on the **covariance** between them

Home-Equity Bias

Even though it would be beneficial (for risk reduction) to diversify on an international scale, the global **holding of foreign securities is largely sub-optimal**

$\Downarrow$

Home-Equity Bias

HEB: A Deeper insight

The major drivers of HEB:

- **Legal barriers** to foreign investments
- **Higher transaction costs** on foreign securities
- **Indirect barriers** to foreign investments (e.g. the difficulty in finding -and interpreting- information about foreign securities)
- **Additional risks** to be hedged (FX risk, country risk...)

CAPM: Main Assumptions

- Investors are purely **price-takers**
- Investments are **limited** to a universe of publicly traded financial assets
- **No** taxes and **no** transaction costs
- Investors are **rational mean-variance optimizers** and have the same investment horizon
- **Homogeneous expectations** (same views) and risk appetite

The Underlying Rationale

If all investors use **identical mean-variance** analysis, applied to the **same universe of securities**, for the **same time horizon** and use the **same information set**, they all must arrive at the same determination of the optimal risky portfolio on the efficient frontier...

$$\Downarrow$$

...however, if all the investors hold an identical risky portfolio, this has to be the **MARKET PORTFOLIO** (including all tradable assets)

Major Implications

The Risk-Reward Ratio for a generic asset j included within the market portfolio must be identical to the Risk-Reward Ratio for the market portfolio itself:

$$\frac{E[r_j]-r_f}{Cov(r_j;r_m)} = \frac{E[r_m]-r_f}{Var(r_m)}$$

with:

- $E[r_j]$: expected return on the j^{th} asset
- r_f: risk-free rate of return
- $E[r_m]$: expected return on the market portfolio
- $Cov(r_j;r_m)$: covariance between asset j and the market portfolio
- $Var(r_m)$: variance of the market portfolio

Playing with Algebra

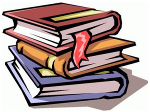
Rearranging the terms, we would get:

$$r_j - r_f = \beta(r_m - r_f)$$
$$\beta = \frac{Cov(r_j;r_m)}{Var(r_m)}$$

- $r_j - r_f$: The risk premium is linearly related to...
- $\frac{Cov(r_j;r_m)}{Var(r_m)}$:...the risk that the single asset contributes to the mkt as a whole $\Rightarrow$ **SYSTEMATIC RISK**

Portfolio Investment

Investment where the investor's holding is too small to provide any effective control
(Just to revise, could you define what a FDI is? Hint: step back to Lesson I...)



Diversification

Diversification means building **multi-asset portfolios**, such that only a portion of total wealth is invested in each individual asset. This allows in turn to **spread out exposure to security-specific factors**, so as to reduce the overall level of risk.

$$\Downarrow$$

Even common wisdom suggests that putting all eggs in one basket can be very risky!

Asset-Specific vs Mkt Risk

- **Asset-Specific Risk = Non-Systematic Risk = Diversifiable Risk**: risk that can be diversified away.
- **Mkt Risk = Systematic Risk = Non Diversifiable Risk**: risk that remains even after extensive diversification

Systematic vs Systemic Risk

- **Systematic risk**: risk that **cannot be diversified** away
- **Systemic risk**: risk of collapse of an **entire financial system** or entire market

Efficient Frontier

Optimal set of portfolios that offer the highest expected return for a specific level of risk (Markowitz, 1952)

Riskless assets

Financial instruments that have a **certain** future return (MM securities, Government bonds...)

$$\Downarrow$$

Are they truly (and completely) riskless in practice?

Sharpe Ratio

Sharpe Ratio: measure for calculating **risk-adjusted returns**. In more quantitative terms, it can be defined as the **average return earned in excess of the risk-free rate per unit of volatility**

Integrated vs Segmented Capital Markets

- **Integrated Capital Markets**: the connection among international capital markets is seamless
- **Segmented Capital Markets**: implicit or explicit factors inhibit the free movement of capital among the various countries

WATCH OUT: HEB is the most obvious example of capital market segmentation

To Put It into Practice I

10.1 Given the data here below, please find the Expected Return and the Variance of both portfolios. Which one would you choose? Why?

- **Portfolio 1**

Constituents	**Weight**	**E(r)**	**Var(r)**	**Cov(a,b)**
Stock a	0.6	0.15	0.19	0.4
Stock b	0.4	0.07	0.25	

- **Portfolio 2**

Constituents	**Weight**	**E(r)**	**Var(r)**	**Cov(c,d)**
Stock c	0.3	0.1	0.23	0.3
Stock d	0.7	0.15	0.12	

To Put It into Practice II

- **9.2** Suppose that the risk premium on the market portfolio is estimated at 0.08 with a standard deviation of 0.22. What is the risk premium on a portfolio invested 0.25 in Apple and 0.75 in Google, if they have $\beta = 1.10$ and 1.25, respectively?
- **9.3** Stock ABC has an expected return of 0.12 and $\beta = 1$. Stock XYZ has expected return of 0.13 and $\beta = 1.5$. The market's expected return is 0.11 and $r_f = 0.05$. According to the CAPM, which stock is a better buy? Why?

To Put It into Practice III

9.4 Given the data here below, please find the Expected Return and the Variance of both portfolios. Which one would you choose? Why?

- **Portfolio 1**

Constituents	**Weight**	**E(r)**	**StDev(r)**
Stock a	0.3	0.14	0.2
Stock b	0.3	0.08	0.12
Stock c	0.3	0.02	0.3

- **Portfolio 2**

Constituents	**Weight**	**E(r)**	**StDev(r)**
Stock d	0.3	0.2	0.28
Stock e	0.3	0.18	0.33
Stock f	0.3	0.33	0.4

To Put It into Practice IV

- **Portfolio 1**

Correlations	a	b	c
a	1	0.5	0.2
b	0.5	1	0.4
c	0.2	0.4	1

- **Portfolio 2**

Correlations	d	e	f
d	1	0.3694	0.1539
e	0.3694	1	0.2148
f	0.1539	0.2148	1