WHY DO INTEREST RATES CHANGE?

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02/29/2015
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TODAY'S AGENDA

- Debt and Bonds
- Changes in interest rates
- Supply and demand in the bond market
- Yield curve
- Spot and forward contracts
- Bond pricing

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Debt

If a company wants to finance its business, it can alternatively:

- Issue new equity.
- **Borrow money**, promising to make regular interest payment and to repay the principal.

Debt

Who can be the lender?

- A financial institution.
- A bank
- The households.

What type of instruments can be used?

- Mortgages
- Bonds

Bonds

What are bonds?

Bonds are debt instruments* that represent cash flows payable during a specified time period.

The cash flow they represent are:

- Interest payment on the loan;
- The redemption of the loan.

*Bonds are portion of a single operation of indebtedness.

Bonds vs shares

Bonds are debt instruments, whereas shares are fraction of equity.

- By purchasing equity (shares).

  Investor becomes the owner of the corporation.

- By purchasing debt (bonds).

  Investor becomes a company's creditor.

While shares are fraction of equity, bonds are fraction of debt (credit).

Bonds vs shares

Bonds are debt, whereas shares are equity.

- The primary advantage of being a creditor is that you have a higher claim on assets than shareholders do: that is, in the case of bankruptcy, bondholders will be paid before shareholders.
- However, bondholders do not share in the profits if a company does well – they are entitled only to the principal plus interest.

Bonds key features

- **Maturity:**

  The maturity of a bond refers to the date that the debt will cease to exist.

- **Term to maturity:**

  The term to maturity of a bond is the number of years after which the issuer will repay the obligation.

Bonds key features

- Face value (a.k.a. Redemption value/ Par value):

  The face value is the amount that the issuer agrees to repay the bondholder on the maturity date.

- Coupon Rate:

  The coupon rate is the interest rate used to compute the coupon that the issuer agrees to pay each period. It can vary or be fixed.

Focus: T-bill and Money Market

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- The money market consists of very short term debt securities that usually are highly marketable.
- The most marketable securities of the money markets are the US Treasury-Bills (or simply T-bills).
- T-bills are issued at initial maturity of 1 month, 3 months, 6 months and 1 year.

Bond categories

With respect to the issuer, bonds can be classified into:

- ✓ Sovereign governments B;
- ✓ Local government authorities B;
- ✓ Supranational bodies B (e.g. the World Bank);
- ✓ Corporate B

Bond categories

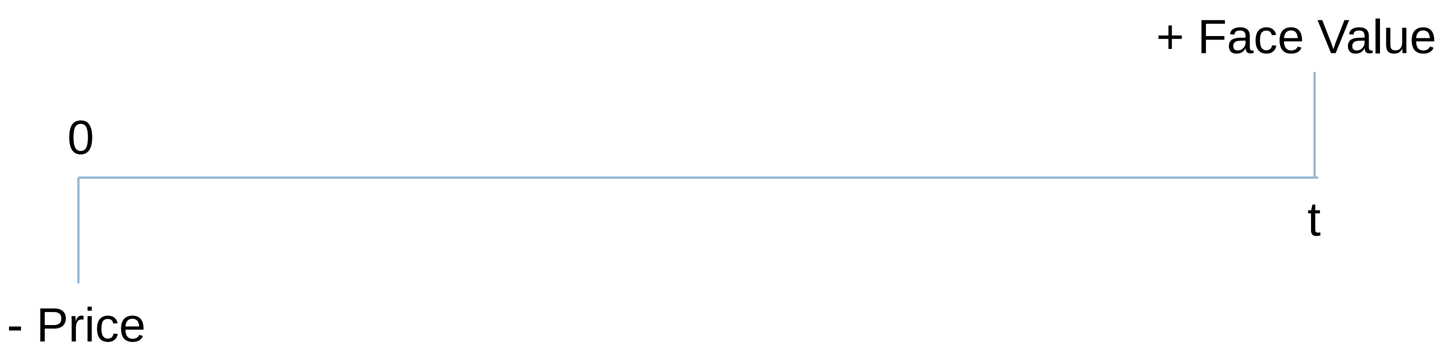
According to the interest payment scheme, bond can be classified into:

- ZCB – Zero Coupon Bond
- CB – Coupon Bond, in turn classified into:
  - ✓ Fixed income bond which, in all period, pay the same coupon;
  - ✓ Floating Rate Notes which have, for each period, a different interest rate depending on a specific "underlying" interest rate.

Zero Coupon Bond

The Zero Coupon Bond is a debt instruments which grants the lender a profit originated by the difference between:

- Issuing Price
- Face/Nominal Value



Valuation of a ZCB

Since the interest rate of a Zero Coupon Bond is implicit in the difference between the issuing price and the par value…

EG, the interest rate of a 1Y ZCB.

$$r_{0,1} = \frac{FV}{P} - 1$$

Fixed Income Bonds

- The fixed income bond grants the owner
  - the payment of a certain amount of interest each period and
  - the repayment of the principal at the maturity.
- In all periods (e.g. quarters, years…) the interest is computed on the same interest rate which is fixed in all the time contract.

Valuation of Fixed-Income Bonds

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$$P = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C+FV}{(1+r)^n}$$

Where,

- P is the fair price of the bonds
- C is the coupon
- FV is the par (or face) value
- r is the interest rate.

Fixed Income Bond

Compute the fair price of the following three bonds:

- Interest paid on a yearly basis;
- Face value: 100$.
- Time to maturity: 3 years
- Interest rate, r=5%.
- Coupon rate:
  - 7% on annual basis, for the 1^{st} bond
  - 5% on annual basis, for the 2^{st} bond
  - 3% on annual basis, for the 3^{st} bond

Bonds fair price

The fair price of a bond depends on two rate:

- Discount rate, *r*
- Coupon rate, c

More precisely, if

- $$r > c \Rightarrow P < FV$$
- $$r = c \Rightarrow P = FV$$
- $$r < c \Rightarrow P > FV$$

Bonds fair price

- To discount the diferrent cash flows granted to the bondholder we used the same interest rate, r.
- However it is important to know that:
  - Interest rates may, and usually do, vary over time;
  - The 1y interest rate differs from the 2y interest rate. That is to say, each maturity has its interest rate.

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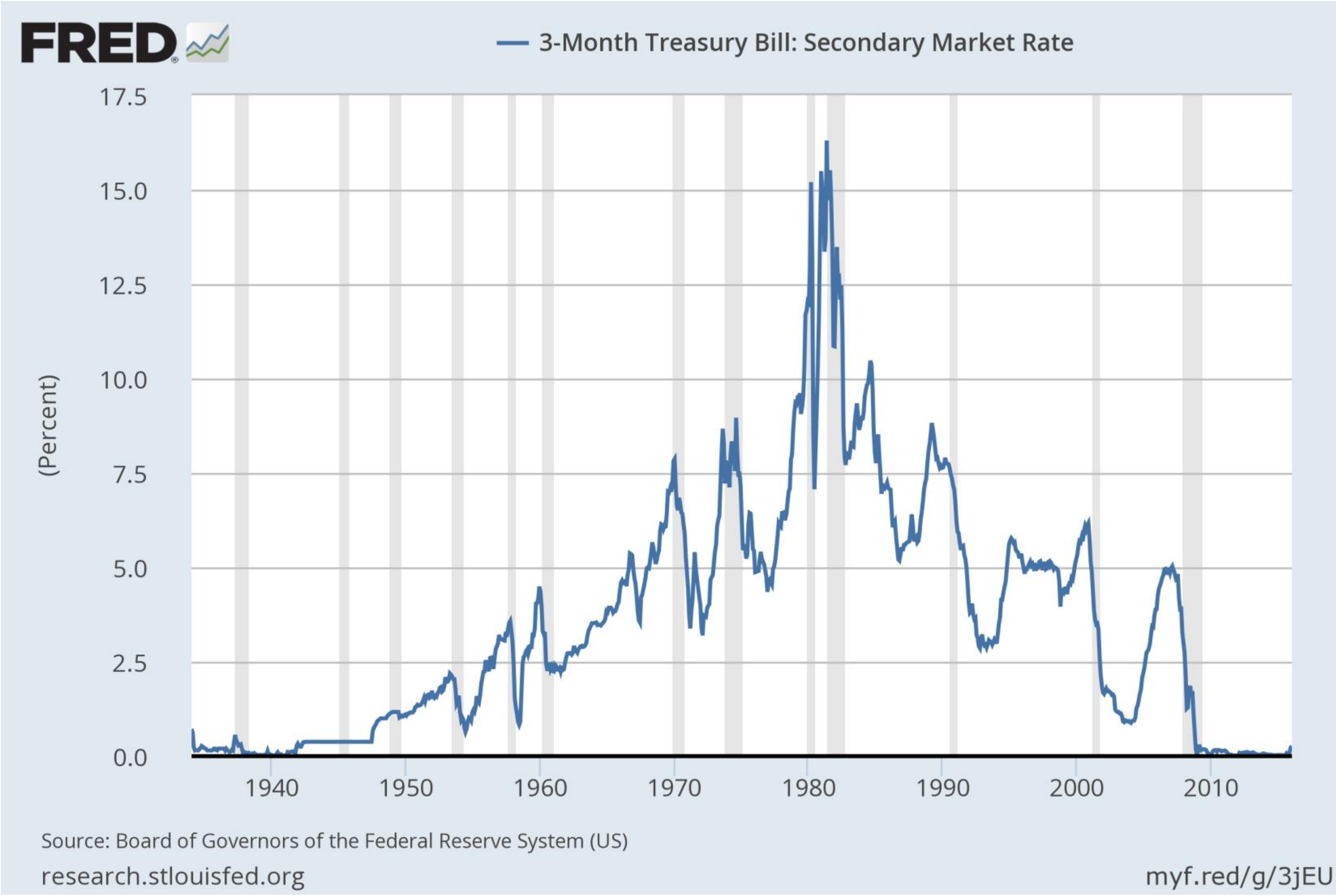
Why do interest rates change?



Why do interest rates change?

- We have seen that bond prices determine the level of the interest rates.
- Changes in bond prices determine changes in interest rates.
- If we explain why bond prices change, we can also explain why interest rates change.

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Determinants of Asset Demand

- An asset is a piece of property that is a store of value. An individual who want to purchase an asset must take into account:
  - Wealth
  - Expected Return
  - Risk
  - Liquidity

Determinants of Assets Demand

Wealth

- Should the wealth increase, the demand for asset may increase, due to the more resources available with which to purchase assets.
- All else equal, the greater is the wealth the greater will be the quantity demanded for an asset.

Determinants of Assets Demand

Expected Returns

- When we make a decision to buy an asset, we are influenced by what we expect the return on that asset to be.
- If a Walt Disney bond has a return of 15% half of the time and 5% the other half, its expected return is…

Determinants of Assets Demand

Expected Returns

- When we make a decision to buy an asset, we are influenced by what we expect the return on that asset to be.
- If a Walt Disney bond has a return of 15% half of the time and 5% the other half, its expected return is… 10%

Determinants of Assets Demand

Expected Returns

- You can imagine the expected return as the weighted average of all possible returns.
- The weights are the probabilities of occurrence

$$R^e = p_1 * r_1 + p_2 * r_2 + \cdots + p_n * r_n$$

where R^e is the expected return, n the number of possible outcomes, r_i the return in the i-th state of nature, and p_i the probability of occurrence of the i-th return

Determinants of Assets Demand

Expected Returns

- An increase in an asset's expected return relative to that of an alternative assets, holding everything else unchanged, raises the quantity demanded of the asset. It happens if:
  - The expected return on the asset increase, while the ones of the other assets remain the same.
  - When the expected return on alternative asset falls while the return on the asset remain constant.

Determinants of Assets Demand

Risk

- The degree of risk (or uncertainty) of an asset's returns also affects the demand for the asset.
- Consider the two alternative assets:
  - A – $r_1 = 15\%, p_1 = 50\%; r_2 = 5\%, p_2 = 50\%$
  - B – $r_1 = 10\%, p_1 = 100\%;$

Determinants of Assets Demand

Risk

- A has an expected return of 10%; B has a fixed certain return of 10%.
- However, A has uncertainty associated with its returns and so a greater risk than B.
- To evaluate the asset's riskiness, we will use the standard deviation.

Determinants of Assets Demand

Risk

- The standard deviation (sd or σ) is a statistical measure that summarize how far are the observation from the mean value.
- We compute the sd in the following way:

$$\sigma = \sqrt{p_1 * (r_1 - R^e)^2 + p_2 * (r_2 - R^e)^2 + \cdots + p_n * (r_n - R^e)^2}$$

- The higher the standard deviation, the greater the risk of an asset.

Determinants of Assets Demand

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Risk

- A risk-averse person prefer less risky assets;
- On the contrary, a risk-lover person prefer most risky assets.
- The majority of people are risk-averse, especially when evaluating financial decisions.

Determinants of Assets Demand

Risk - Exercise

- Consider once again the two investor A and B:
  - A – $r_1 = 15\%, p_1 = 50\%; r_2 = 5\%, p_2 = 50\%$
  - B – $r_1 = 10\%, p_1 = 100\%;$
- Of the two assets, which is riskier?

Determinants of Assets Demand

Risk

- The Expected Return of A is:

$$R_A^e = 50\% * 15\% + 50\% * 5\% = 10\%$$

- Its Standard Deviation is:

$$\sigma_A^e = \sqrt{50\%(15\% - 10\%)^2 + 50\%(5\% - 10\%)\text{^}2}$$

$$\sigma_A^e = \sqrt{0.25\%} = 5\%$$

Determinants of Assets Demand

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Risk

- The Expected Return of B is:

$$R_B^e = 100\% * 10\% = 10\%$$

- Its Standard Deviation is:

$$\sigma_B^e = \sqrt{100\%(10\% - 10\%)^\wedge 2}$$

$$\sigma_B^e = \sqrt{0\%} = 0\%$$

Determinants of Assets Demand

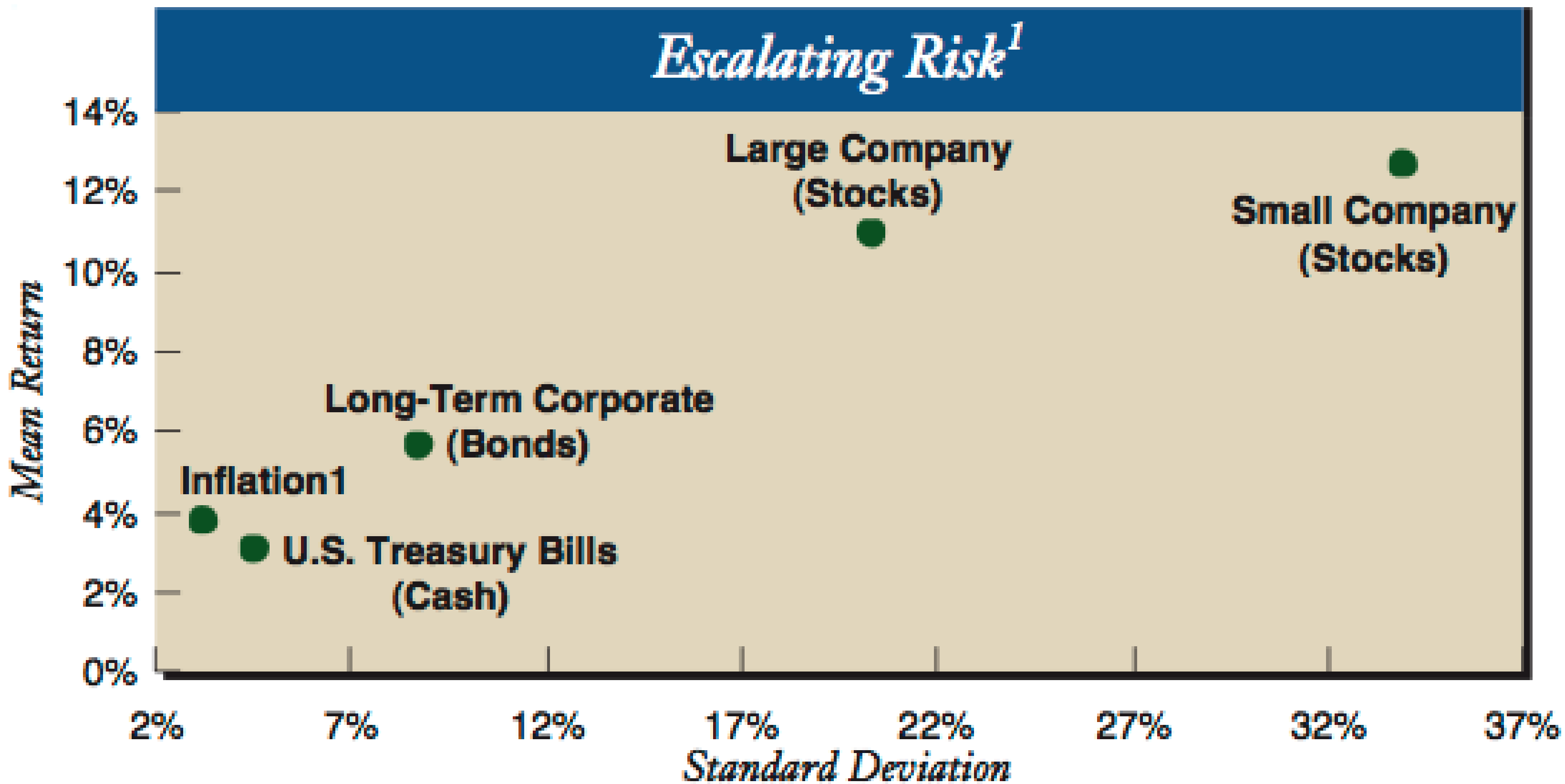
Risk

$$\sigma_A^e > \sigma_B^e \rightarrow A \text{ is riskier than } B$$

- Holding everything else constant, if an asset's risk rises relative to that of alternative assets, its quantity demanded will fall.

Determinants of Assets Demand

Risk



Determinants of Assets Demand

Liquidity

- Another factor that affects the demand for an asset is how quickly it can be converted into cash at low cost.
- That is to say, how much the asset is liquid.
- All being equal, The more liquid an asset is relative to alternative assets, the more desirable it is, and the greater will be the quantity demanded.

Determinants of Assets Demand

Variable	Change in variable	Change in quantity demanded
Wealth	↑	↑
Expected return relative to other assets	↑	↑
Risk relative to other assets	↑	↓
Liquidity relative to other assets	↑	↑

Supply and Demand in the Bond Market

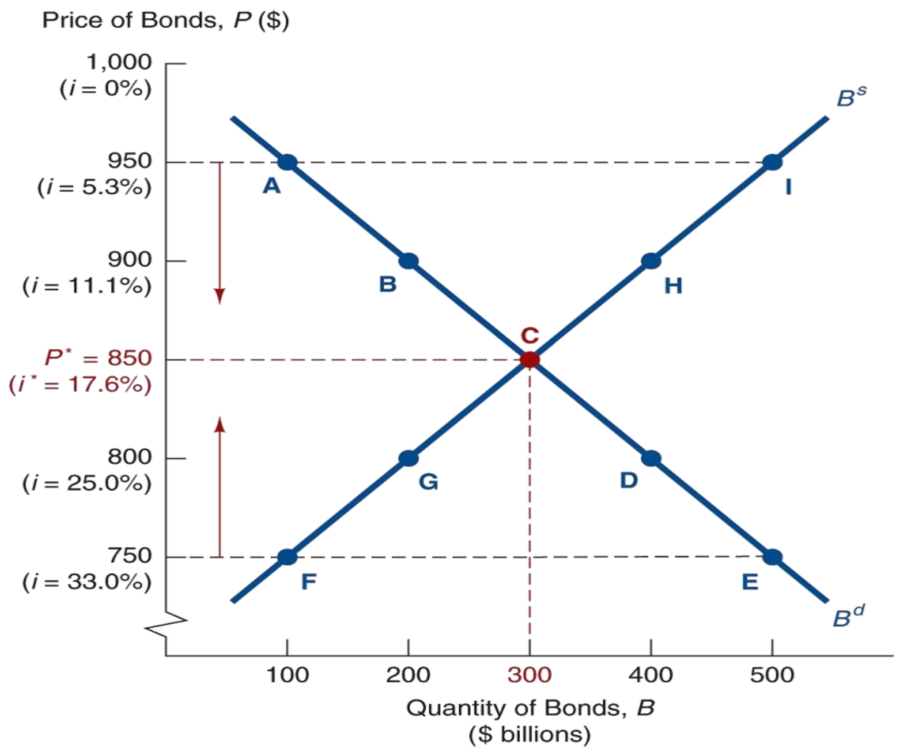
- Bond prices determine the interest rates levels
- Bond prices represent equilibrium condition in the bond market
- That is to say, each price is formed when the demand meets the supply.

Supply and Demand in the Bond Market

- As every demand curve, the one of the bonds represents the relationship between the demanded quantity and the price when all other economic variables are held constant.
- As every supply curve, the one of the bonds represents the relationship between the quantity supplied and the price when all other variables are held constant.

Supply and Demand in the Bond Market

- Market equilibrium occurs when the amount that people are willing to buy (demand) equals the amount that people are willing to sell (supply) at a given price.



Shifts in the Demand for Bonds

Wealth

- When economy is growing rapidly and wealth is increasing the demanded quantity at each price increases.
- As a consequence, the demand curve shifts to the right.
- Economy ↑, wealth ↑, Bd ↑, Bd shifts to the right.

Shifts in the Demand for Bonds

Wealth

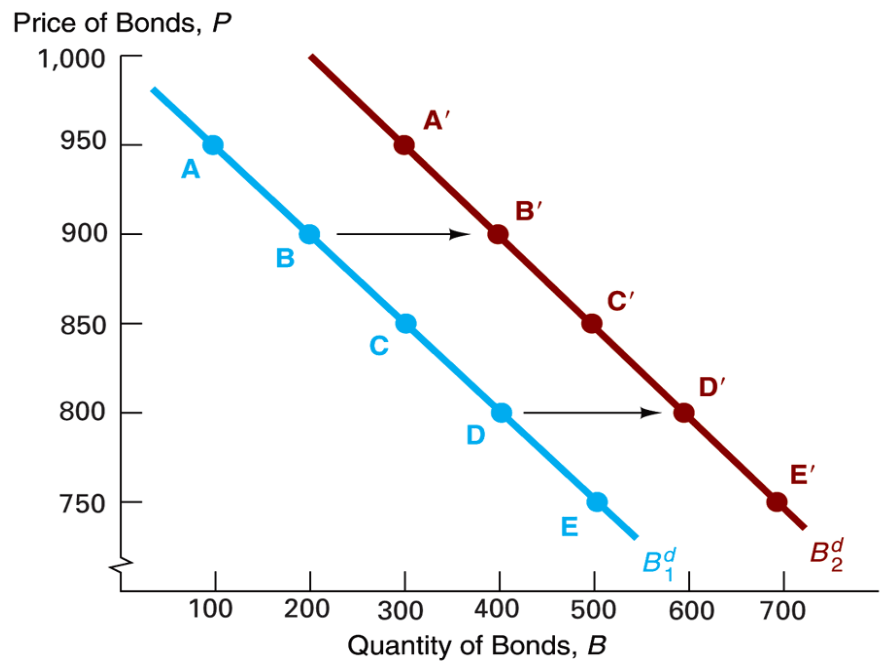


Figure 4.2 Shift in the Demand Curve for Bonds

When the demand for bonds increases, the demand curve shifts to the right as shown.

Shifts in the Demand for Bonds

Expected Returns

- If people begin to think that interest rates would be higher next year than they had originally anticipated, the expected return today on long-term bonds would fall.
- Higher expected interest rates in the future:
  - Lower the expected return for long-term bond
  - Decrease the demand
  - Shift the demand curve to the left

Shifts in the Demand for Bonds

Expected Returns

- On the contrary lower expected interest rates in the future:
- Increase the demand for long-term bonds;
- Shift the demand curve to the right.

Shifts in the Demand for Bonds

Risk

- An increase (decrease) in the riskiness of bonds causes the demand for bonds to fall (rise) and the demand curve to shift to the left (right)r.
- An increase in the riskiness of alternative assets causes the demand for bonds to rise and the demand curve to shift to the right.

Shifts in the Demand for Bonds

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Liquidity

- Increased liquidity of bonds results in an increased demand for bonds, and the demand curve shifts to the right.
- Increased liquidity of alternative assets lowers the demand for bonds and shifts the demand curve to the left.

Shifts in the Supply of Bonds

Expected Profitability of Investment Opportunities

- When economy grows, investment opportunities abound and consequently the supplied quantity of bonds.
- In a business cycle expansion (recession), the supply of bonds increases (decreases), and the supply curve shifts to the right (left).

Shifts in the Supply of Bonds

Expected Inflation

- More than looking at the nominal interest rate, investors look at the real interest rate.
- Real rates are nominal interest rates adjusted by the expected level of prices.
- The higher the inflation, the lower the real cost of borrowing money, the higher the supply of bonds.

Shifts in the Supply of Bonds

Government Budget

- When the government revenues are less than its expenditure, the treasury must issue new bonds to finance the gap.
- Higher government deficits increase the supply of bonds and shift the supply curve to the right.
- On the other hand, government surpluses, decrease the supply of bonds and shift the supply curve to the left.

Shifts in the Supply of Bonds

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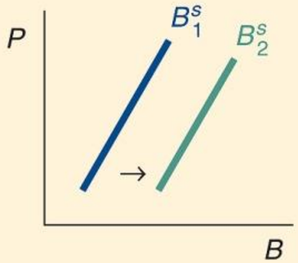
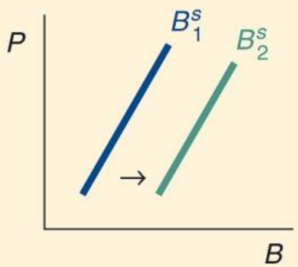
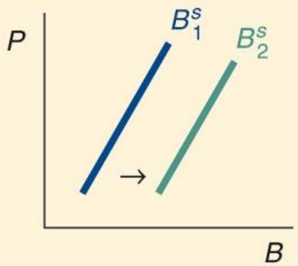
Variable	Change in Variable	Change in Quantity Supplied at Each Bond Price	Shift in Supply Curve
Profitability of investments	↑	↑	
Expected inflation	↑	↑	
Government deficit	↑	↑	

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Yield Curve

- Up to now, we have supposed a unique interest rate, that remains constant over time.
- However, it is opportune to recognize that the short term interest rates is different from the long term interest rates.
- Indeed, different zero coupon bonds have different prices, according to the various maturity.

Yield Curve

- Plotting the different yields against the maturity, we obtain the Yield Curve.
- The yield curve shows the different interest rates corresponding at each maturity.
- Therefore, it shows the relationship between the short and long term interest rates.

Spot and forward contract

- All rates we have considered up to now are computed using spot contract.
- A spot contract represents, like the US T-bill, a transaction for immediate delivery.
- Indeed, the purchaser of a T-bill is entitled to interest from the settlement date onwards.

Yield curve

- In general, the <u>spot</u> rates (on annual basis!) of a <u>spot</u> contract which has
  - maturity (in years), t
  - terminal value, x_t
  - price, $P=v(0, x_t)$

is the following one:

$$s(0,t) = [\frac{x_t}{v(0,x_t)}]^{\frac{1}{t}} - 1$$

Yield Curve

- Exercise
- Let consider the following situation:
  - 1y ZCB – Price=98 and Face Value=100
  - 2y ZCB – Price=94 and Face Value=100
- The former has a interest rate of (…) - indicated with (…) while the latter has a interest rate of (…) - indicate with (…).

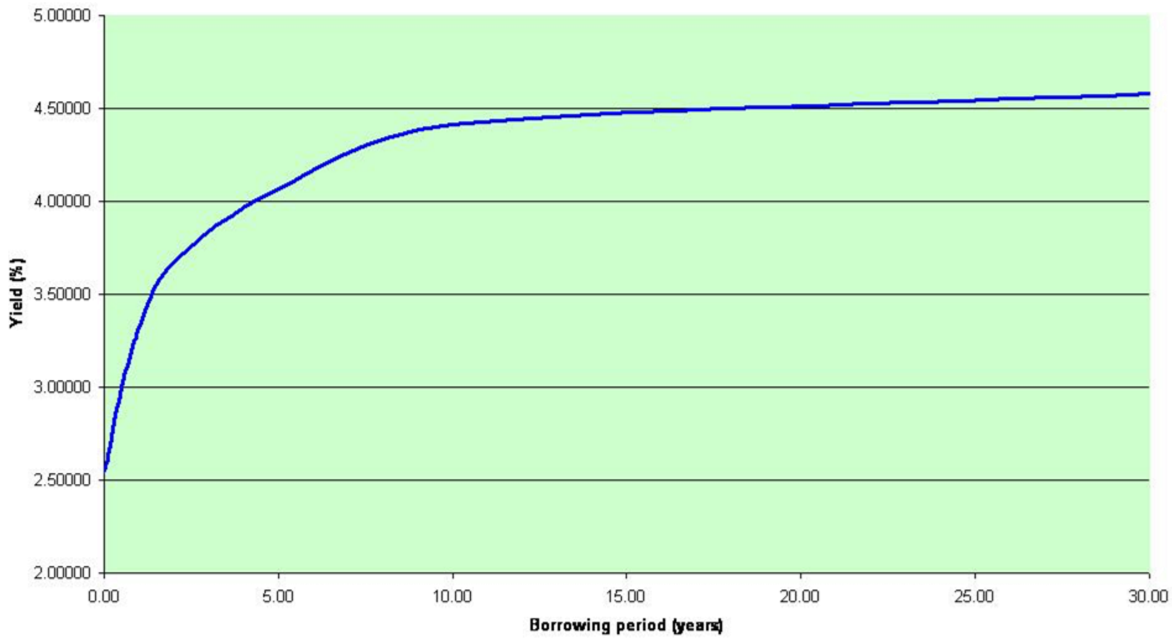
Yield Curve

Security	Time to Maturity	Face Value	Price	Interest Rate
ZCB	1m	100	99.9585	s(0,1)=…
ZCB	2m	10	9.9876	s(0,2)=…
ZCB	3m	50	49.8511	s(0,3)=…
ZCB	6m	65	64.3596	s(0,6)=…
ZCB	9m	1	0.9781	s(0,9)=…
ZCB	12m	1000	956.9378	s(0,12)=…

Using the data in the table above, graph the term structure of interest rates.

Yield Curve

- The zero coupon bond yield curve, as all the other yield curve, plots the spot rates corresponding to the different maturity.



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Spot and forward contract

- Financial markets permits also forward contract.
- Unlike the spot contract, the forward one refers to a contract in which the parties to the trade agree today to exchange a security for cash at a future date, but at a price agreed today.
- Therefore, as can be inferred from the definition, a forward rate is the interest rate set today which will be applied to a contract at a future date.

Spot and forward contracts

- From the spot contracts and the spot rates one can infer the forward rates implicit in the market.
- In order to avoid any possibility of arbitrage, the following relation must be respected:

$$[1+s(0,k)]^{k} * [1+f(k,k+i)]^{i} = [1+s(0,k+i)]^{k+i}$$

Spot and forward contracts

- Therefore $f(k, k+i)$ can be computed using the following relation.

$$[1+f(k,k+i)]^{i}=\frac{[1+s(0,k+i)]^{k+i}}{[1+s(0,k)]^{k}}$$

- Where $f(k, k+i)$ is the interest rate agreed today for a contract which will start in the year k and last for i years.

Spot rate, forward rates and arbitrage

- When the relation above mentioned is not satisfied, the market offer a possibility of arbitrage.
- An arbitrage is a strategy which ensures non-negative cash flows and, at least, one cash flow strictly positive.
- In other words, the arbitrage consists of the simultaneous sell and purchase of different security, in order to achieve a “free meal”

Spot rate, forward rates and arbitrage

Example.

- Suppose that the market offers the following investment opportunity:
  a) A spot contract with maturity k
  b) A spot contract with maturity $k+i$
  c) A forward contract with settlement date k and maturity $k+i$.

Spot rate, forward rates and arbitrage

- If the spot and forward rates satisfies the following inequality…

$$[1+f(k,k+i)]^{i} < \frac{[1+s(0,k+i)]^{k+i}}{[1+s(0,k)]^{k}}$$

- …One can
  - Invest \$1 for *k+i* periods at the spot rate s(0,k+i);
  - Borrow \$1 for *k* periods at the interest rate s(0,k);
  - Borrow $\$[\mathbf{1+s(0,k)}]^{k}$ at the forward rate f(k,k+i), for i periods, from k to k+i.

Spot rate, forward rates and arbitrage

- Strategy payouts:

contracts \ time	0	k	k+i
a)	-1	-	$+[1+s(0,k+i)]^{k+i}$
b)	1	$-[1+s(0,k)]^{k}$	-
c)	-	$+[1+s(0,k)]^{k}$	$-[1+s(0,k)]^{k}*[1+f(k,k+i)]^{i}$
TOTAL	0	0	>0

Spot rate, forward rates and arbitrage

- In word, one must be indifferent between the two following choices:
  - investing first in the 1Y ZCB and secondly, as the zcb goes to maturity, reinvest in another 1Y ZCB (whose interest rate is known in 0);
  - Investing directly in the 2Y ZCB.
- If the markets do not respect the preceding relation one can have a free meal and, in other word, can implement an arbitrage.

Spot rate, forward rates and arbitrage

- Let consider the following situation:
  - 1y ZCB – Price=98 and Face Value=100
  - 2y ZCB – Price=94 and Face Value=100
- Considering the two spot rates s(0,1)=2.0408% and s(0,2)=3.1421%, there is one and only one forward rates which does not permit any risk free arbitrage.
- The forward rate f(1,2)=….

Spot rate, forward rates and arbitrage

- Consider again the following spot rates:
  - s(0,1)=2.0408% (Spot price=98)
  - s(0,2)=3.1421% (Spot price=94)
- Suppose that the forward interest rate f(1,2)=3.0928% (Forward price=97).
- Show how one can realize a risk free arbitrage.

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Bonds pricing……was the formula correct?

When we priced the bond, we used a constant interest rate.

However, as we have just seen, the interest rate may vary according to the different maturity.

Therefore, exploiting the term structure and the relation between maturities and interest rates we can re-write the bond pricing formula

Bond pricing formula

$$P = \frac{C}{[1+s(0,1)]^1} + \frac{C}{[1+s(0,2)]^2} + \ldots \frac{C}{[1+s(0,t)]^t} + \ldots + \frac{C+FV}{[1+s(0,n)]^n}$$

where:

C is the coupon;

s(0, t) is the spot rate referred to the t^{th} maturirity;

FV is the principal (/face value);

n is the bond maturity.