# INTEREST RATE RISK

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- Interest Rate Risk
- Duration
- Duration and Risk



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| Years to<br>Maturity | Yearly<br>Coupon | Price @2% | Price @5% | Price @8% | Price @12% |
|----------------------|------------------|-----------|-----------|-----------|------------|
| 1                    | 0                | 98.0392   | 95.2381   | 92.5925   | 89.2857    |
| 2                    | 7.5              | 110.6786  | 104.6485  | 99.1083   | 92.3947    |
| 5                    | 7.5              | 125.9240  | 110.8237  | 98.0036   | 83.7785    |
| 10                   | 7.5              | 149.4042  | 119.3043  | 96.6449   | 74.574     |
| 20                   | 7.5              | 189.9329  | 131.1555  | 95.0909   | 66.387     |
| 30                   | 7.5              | 223.1805  | 138.4311  | 94.3711   | 63.7516    |

Face Value = 100



- A rise in interest rate is associated with a fall in bond prices.
- On the contrary, a fall in interest rates is associated with a rise in bond prices.
- The more distant a bond's maturity, the greater the size of price change associated with an interest rates change.
- The more distant a bond's maturity, the lower the rate of return that occurs as a result of the increase in interest rates.
- Even though a bond has a substantial interest rate its return can turn out to be negative if interest rates rise.



| Years to Maturity | Yearly Coupon | delta -3% | delta +3% | delta +7 |
|-------------------|---------------|-----------|-----------|----------|
| 1                 | 0             | 2.94%     | -2.78%    | -6.25%   |
| 2                 | 7.5           | 5.76%     | -5.29%    | -11.71%  |
| 5                 | 7.5           | 13.63%    | -11.57%   | -24.40%  |
| 10                | 7.5           | 25.23%    | -18.99%   | -37.49%  |
| 20                | 7.5           | 44.82%    | -27.50%   | -49.38%  |
| 30                | 7.5           | 61.22%    | -31.83%   | -53.95%  |



- Prices and returns for long-term bonds are more volatile tan those for shorter-term bonds.
- □ Price variation of +20% and -20% are common for bonds with more than 20 years away from maturity.
- The riskiness of an asset's return resulting from interest rates changes is so important that it has been given a special name, interest rate risk.
- Short term bonds have low interest rate risk.
- On the contrary, long term bonds have substantial interest rate risk, as their prices change radically when interest rates vary.



- In order to measure interest rate risk, financial managers need more precise information on the actual capital gain or loss that occurs when interest rate changes by a certain amount.
- That is to say, the price variation due to a certain variation in interest rates level.
- Moreover, the only maturity does not give too much information on the interest rate risk: two bonds with same maturity can have extremely different sensitivity to interest rates.
- To do this, managers need to make use of the concept of duration.

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- The Duration is the weighted average of the maturities of the cash payments.
- In other words, it is the average lifetime of a debt security's stream of payments.

$$DUR = \sum_{t=1}^{n} \frac{t * \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}}$$



Given that:

$$\sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} = P$$

□ The formula for the duration can be rewrited as:

$$DUR = \sum_{t=1}^{n} \frac{t * \frac{CF_t}{(1+r)^t}}{P}$$



#### **Exercise**

- Compute the duration of the following bonds:
  - ZCB with constant r=5%, Face Value=100, and maturity 3 years;
  - □ Coupon bond with constant r=5%, C=3 (coupon frequency = annual), Face Value=100, 3 years to maturity.
  - □ Coupon bond with constant r=5%, C=15 (coupon frequency = annual), Face Value=100, 3 years to maturity.



- The formula for the duration is not so intuitive.
- However it can be easily programmed into a calculator or computer.
- All else being equa:
  - the longer is the term to maturity of a bond, the longer is its duration.
  - when interest rates rise, the duration of a coupon bond falls.
  - the higher the coupon rate on the bond, the shorter the bond's duration.
- The duration of a portfolio of securities is the weighted average of the durations of the individual securities, with the weights reflecting the proportion of the portfolio invested in each.



| Years to<br>Maturity | Yearly<br>Coupon | DUR @2% | DUR @5% | DUR @8% | DUR @12% |
|----------------------|------------------|---------|---------|---------|----------|
| 1                    | 0                | 1.00    | 1.00    | 1.00    | 1.00     |
| 2                    | 7.5              | 1.93    | 1.93    | 1.93    | 1.93     |
| 5                    | 7.5              | 4.43    | 4.39    | 4.34    | 4.28     |
| 10                   | 7.5              | 7.90    | 7.62    | 7.33    | 6.92     |
| 20                   | 7.5              | 13.44   | 12.09   | 10.74   | 9.05     |
| 30                   | 7.5              | 17.98   | 14.98   | 12.28   | 9.43     |

Face Value = 100

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### Duration and Interest Rate Risk



- Con be used to measure the interest rate risk.
- Duration is a particularly useful concept as it provides a good approximation, especially when interest rate changes are small, for how much the security price changes for a given change in interest rates.
- More precisely:

$$\%\Delta P \approx -DUR * \frac{\Delta i}{1+i}$$

### Duration and Interest Rate Risk



- The greater the duration of a security, the greater the percentage change in its market value for a given change in interest rates.
- The greater the duration of a security, the greater its interest rate risk.
- This reasoning applies equally to portfolio of securities.
- Duration of ZCB equals the time to maturity.
- Being equal the maturities of two bonds, the higher the coupon rate, the lower the duration.

 $0 \leq DUR \leq Time to maturity$ 

#### Duration and Interest Rate Risk



Consider the following coupon bond:

- □ Time to maturity: 3 years
- Annual coupon rate: 5%
- Constant interest rate: 3.5%
- □ Face Value: 100

Suppose that the interest rate rise to 4.1%. Which is the correspondent price variation?

Show it, by using the duration and by discounting the CFs at the new interest rate.