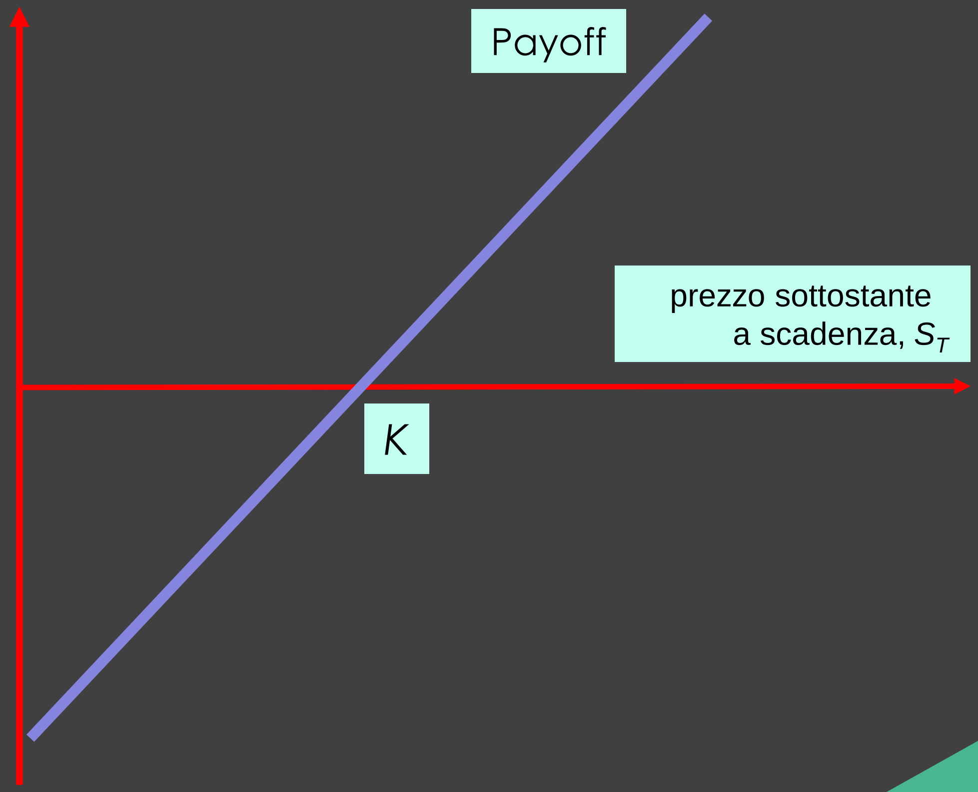
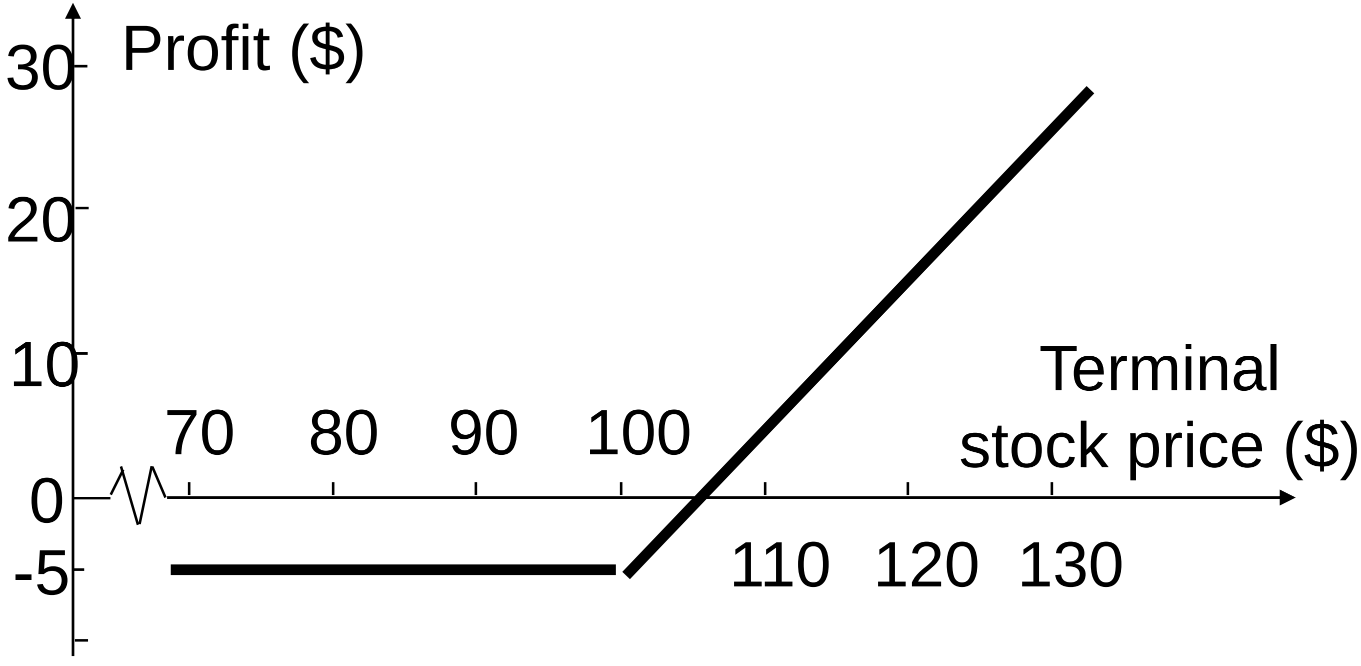
FORWARD



Options

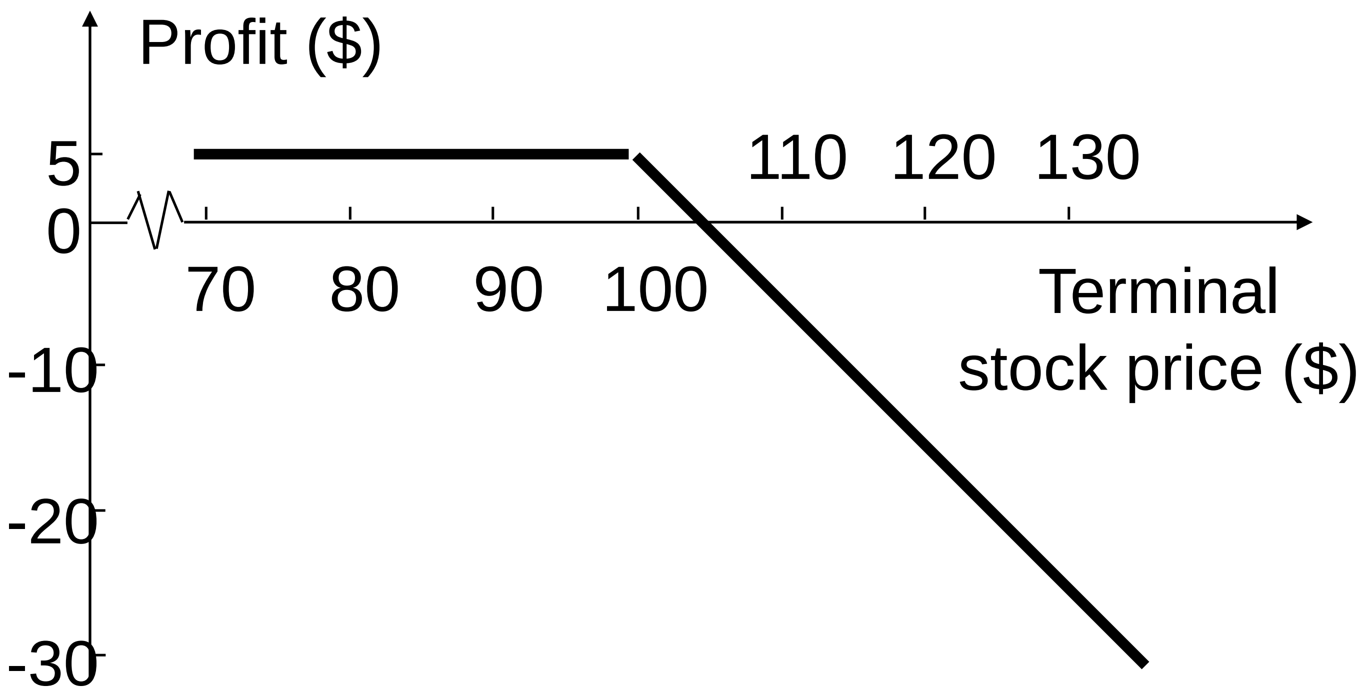
Long Call on IBM

Profit from buying an IBM European call option: option price = $5, strike price = $100, option life = 2 months



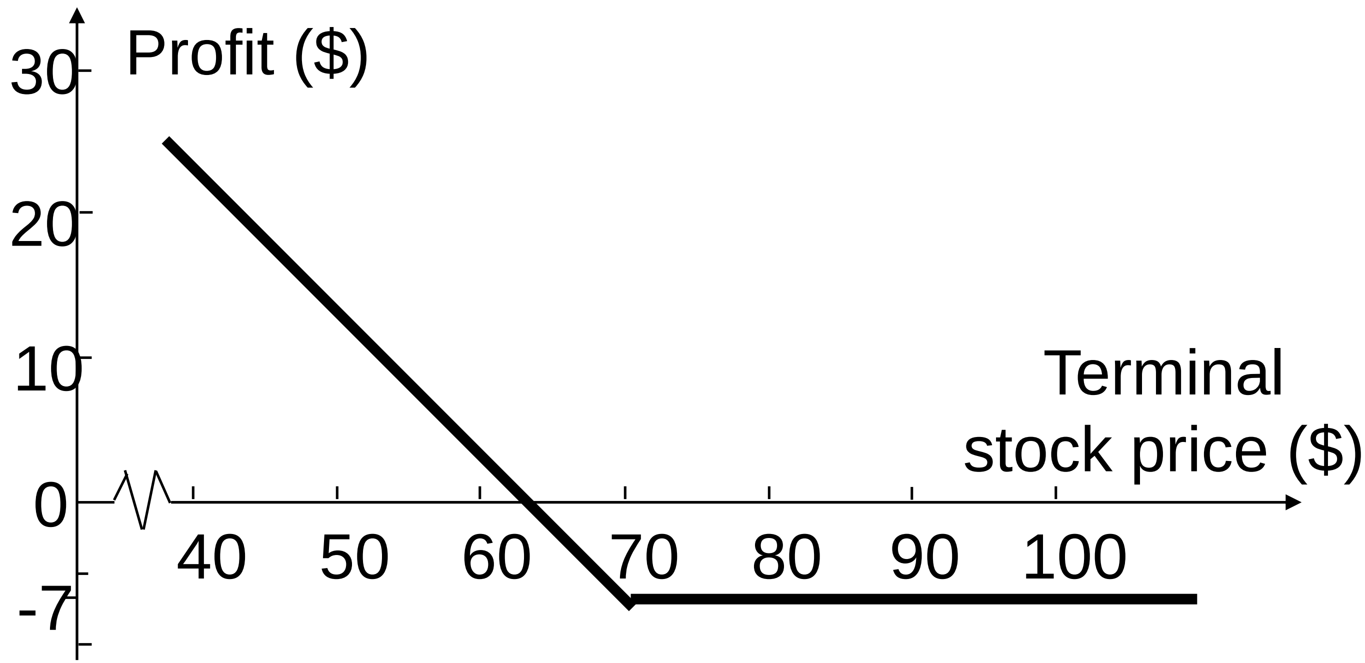
Short Call on IBM

Profit from writing an IBM European call option: option price = $5, strike price = $100, option life = 2 months



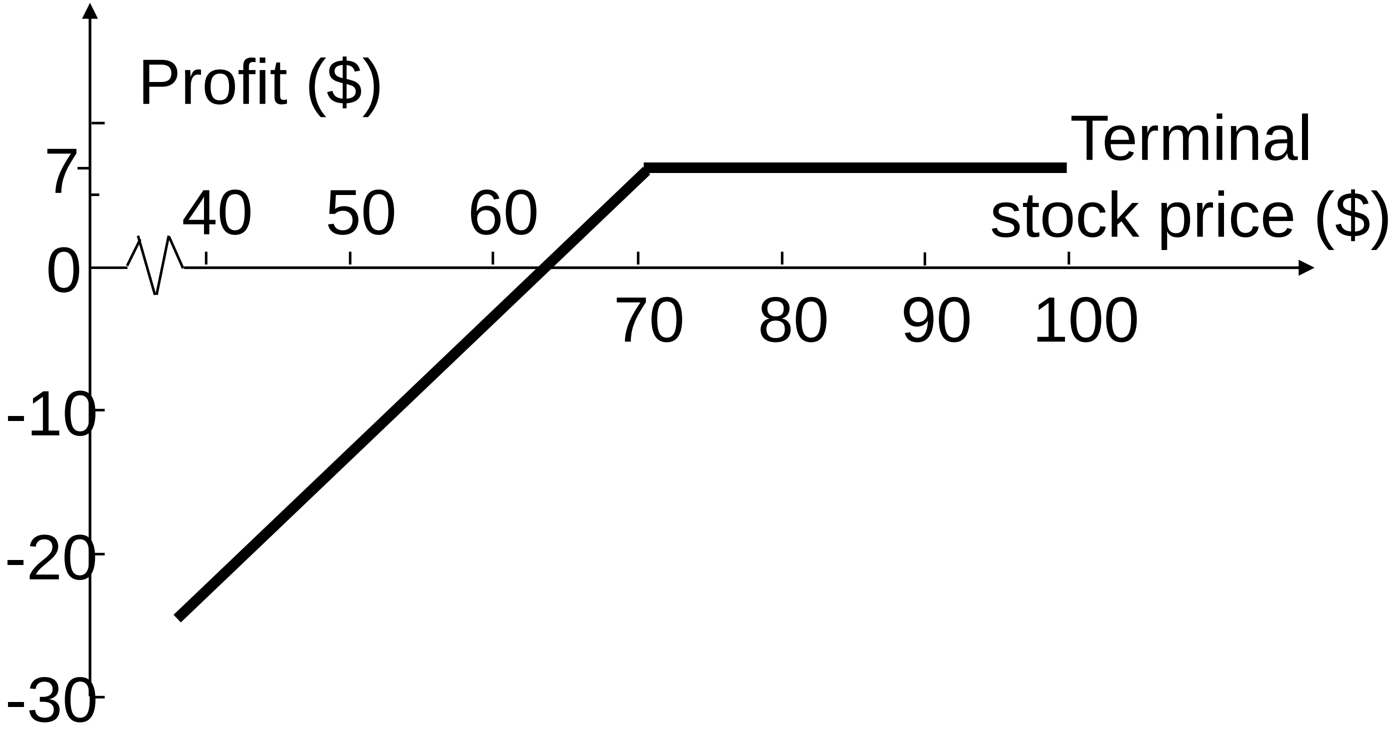
Long Put on Exxon

Profit from buying an Exxon European put option: option price = $7, strike price = $70, option life = 3 mths



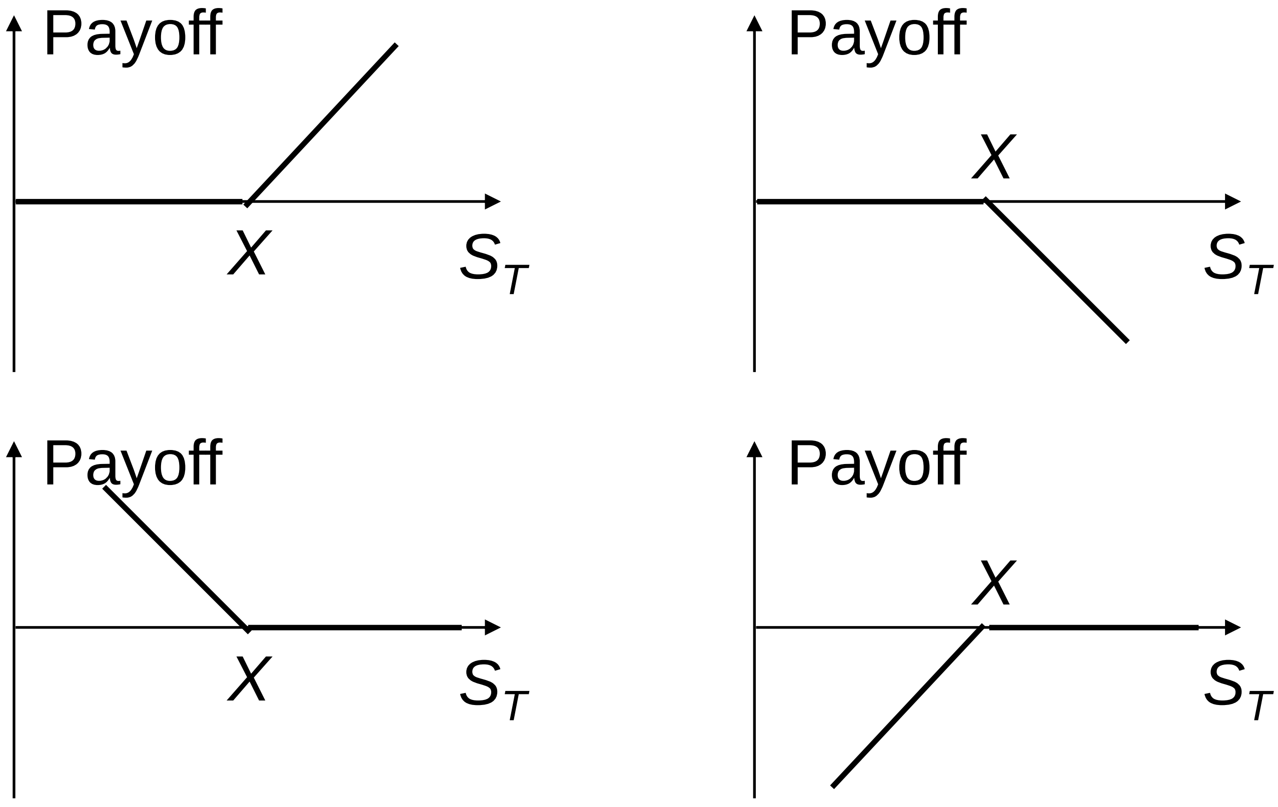
Short Put on Exxon

Profit from writing an Exxon European put option:
option price = $7, strike price = $70, option life = 3 mths



Payoffs from Options

X = Strike price, S_T = Price of asset at maturity



Terminology

Moneyness :

- At-the-money option
- In-the-money option
- Out-of-the-money option

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

Types of Options

- Exchange-traded options
  - Stocks
  - Foreign Currency
  - Stock Indices
  - Futures
- Warrants
- Convertible bonds
- swapoptions
- ....

Warrants

- Warrants are options that are issued (or written) by a corporation or a financial institution
- The number of warrants outstanding is determined by the size of the original issue & changes only when they are exercised or when they expire

Warrants
(continued)

- Warrants are traded in the same way as stocks

- When call warrants are issued by a corporation on its own stock, exercise will lead to new treasury stock being issued

Executive Stock Options

- Option issued by a company to executives
- When the option is exercised the company issues more stock
- Usually at-the-money when issued

Executive Stock Options
continued

- They become vested after a period ot time
- They cannot be sold
- They often last for as long as 10 or 15 years

Convertible Bonds

- Convertible bonds are regular bonds that can be exchanged for equity at certain times in the future according to a predetermined exchange ratio

Convertible Bonds
(continued)

- Very often a convertible is callable
- The call provision is a way in which the issuer can force conversion at a time earlier than the holder might otherwise choose

Exchangeable Bonds

- An exchangeable bond is a sort of convertible bond that provides the conversion into the shares of a company different from the issuer
- Usually, the underlying stock is the equity of a strategic partnership
- There can be adverse signalling problem which are reduced with “best of” structures

Trading Strategies Involving Options

Three Alternative Strategies

- Take a position in the option & the underlying
- Take a position in 2 or more options of the same type (A spread)
- Combination: Take a position in a mixture of calls & puts (A combination)

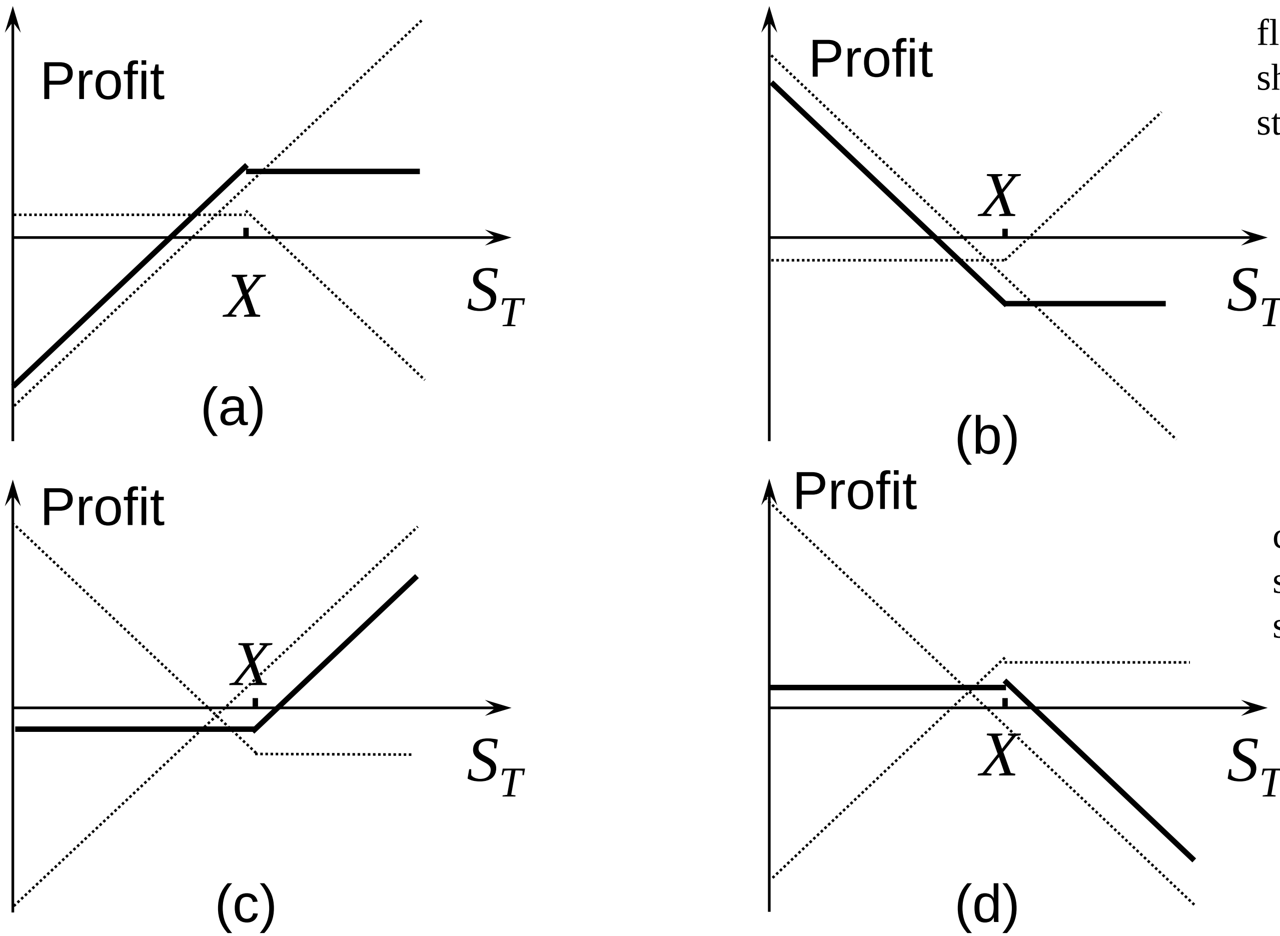
Positions in an Option & the Underlying

cap on long strategy

floor on short strategy

floor on long strategy

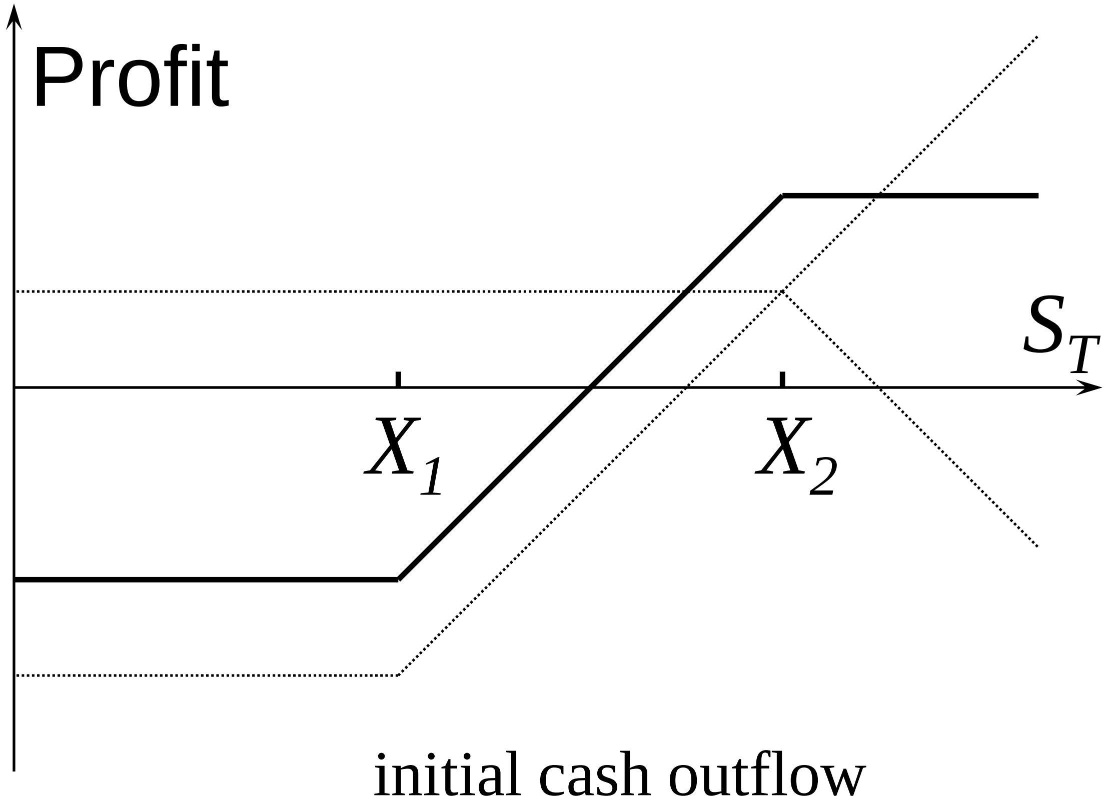
cap on short strategy



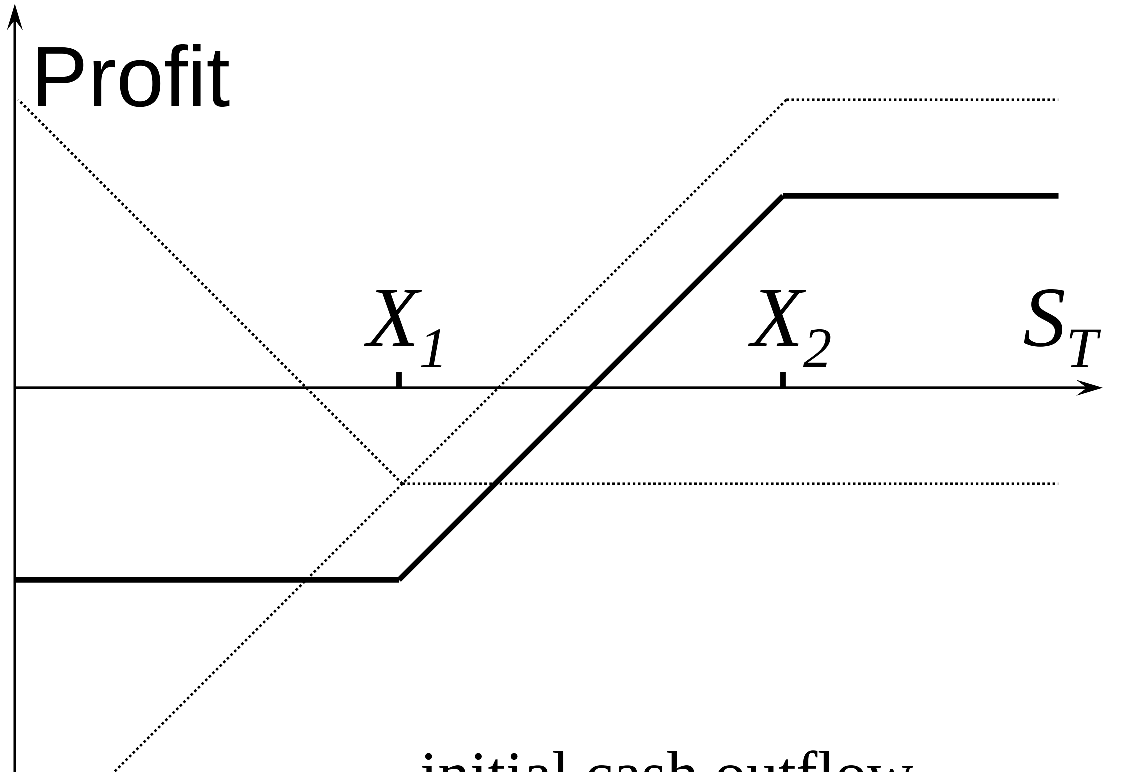
basket of options

- spread type: basket of options of the same type (call or put)
  - bull spread
  - bearish spread
  - butterfly spread
- combination type: basket of options of different types
  - straddles

Bull Spread Using Calls

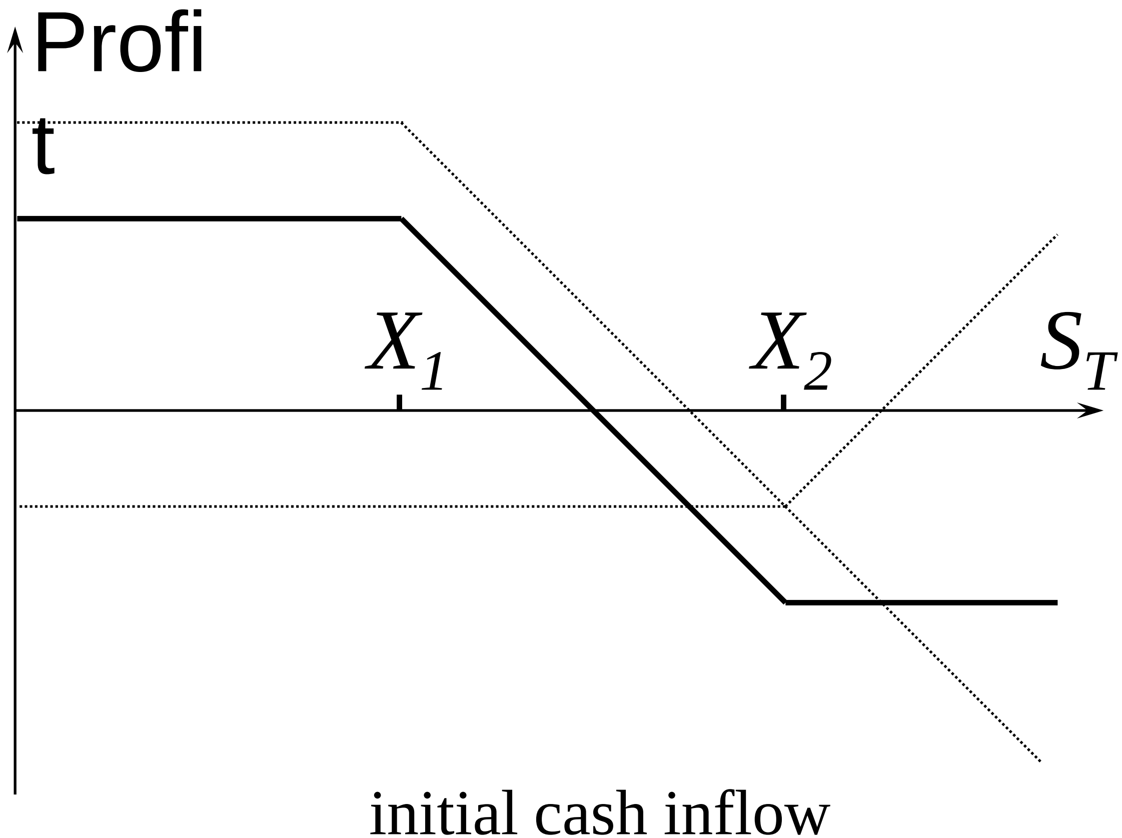


Bull Spread Using Puts

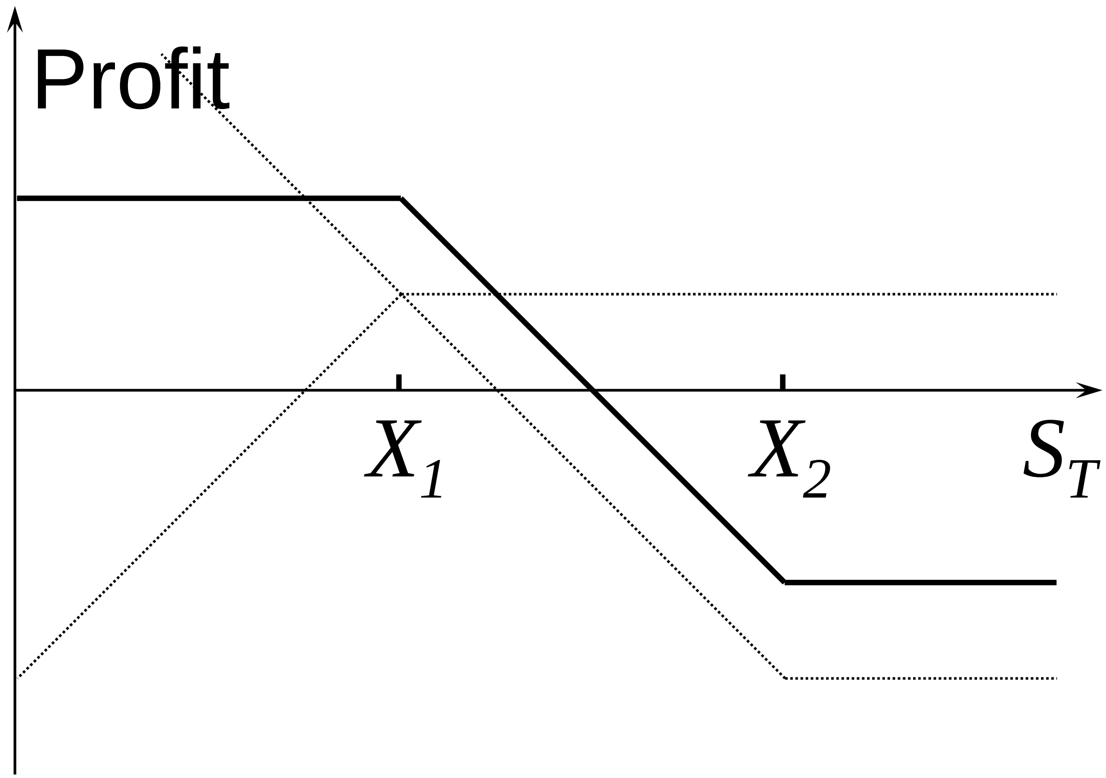


initial cash outflow

Bear Spread Using Calls



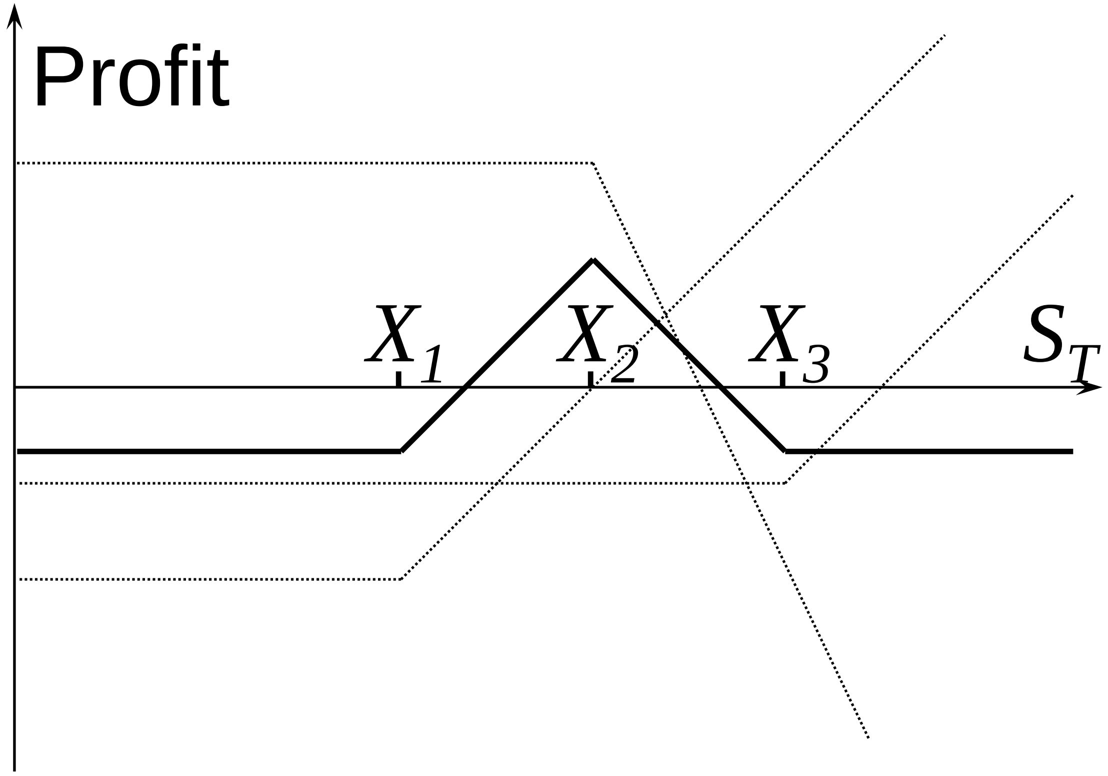
Bear Spread Using Puts



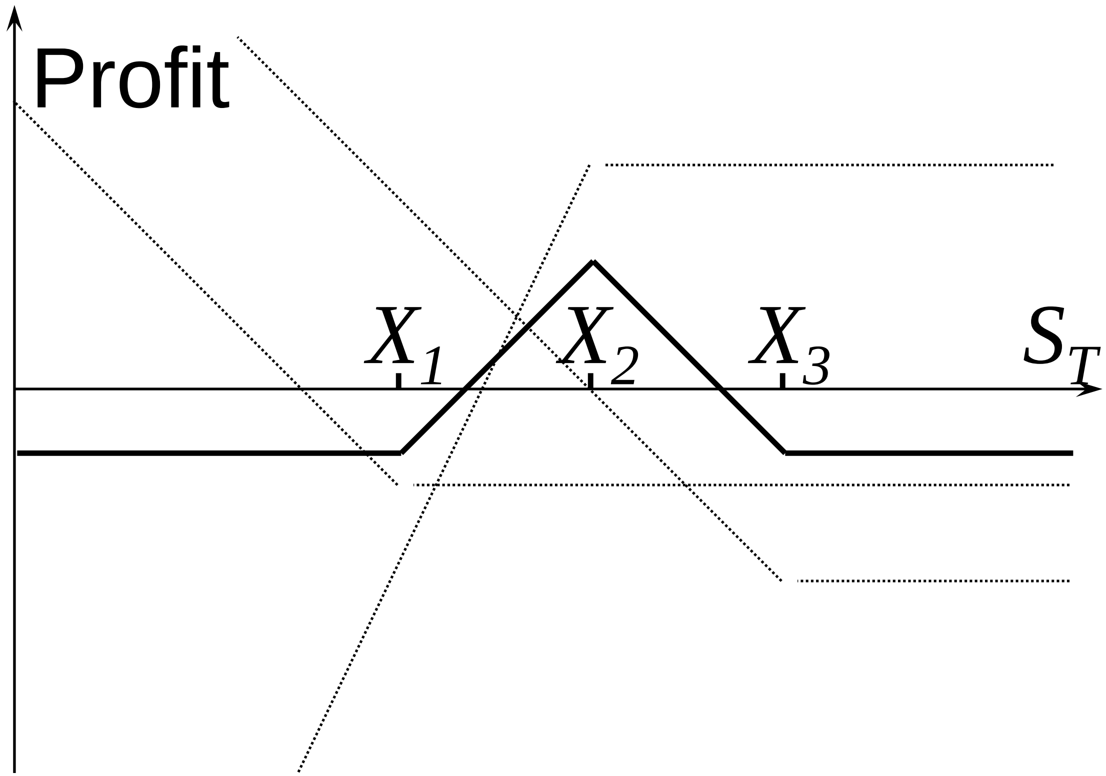
initial cash inflow

Butterfly Spread Using Calls

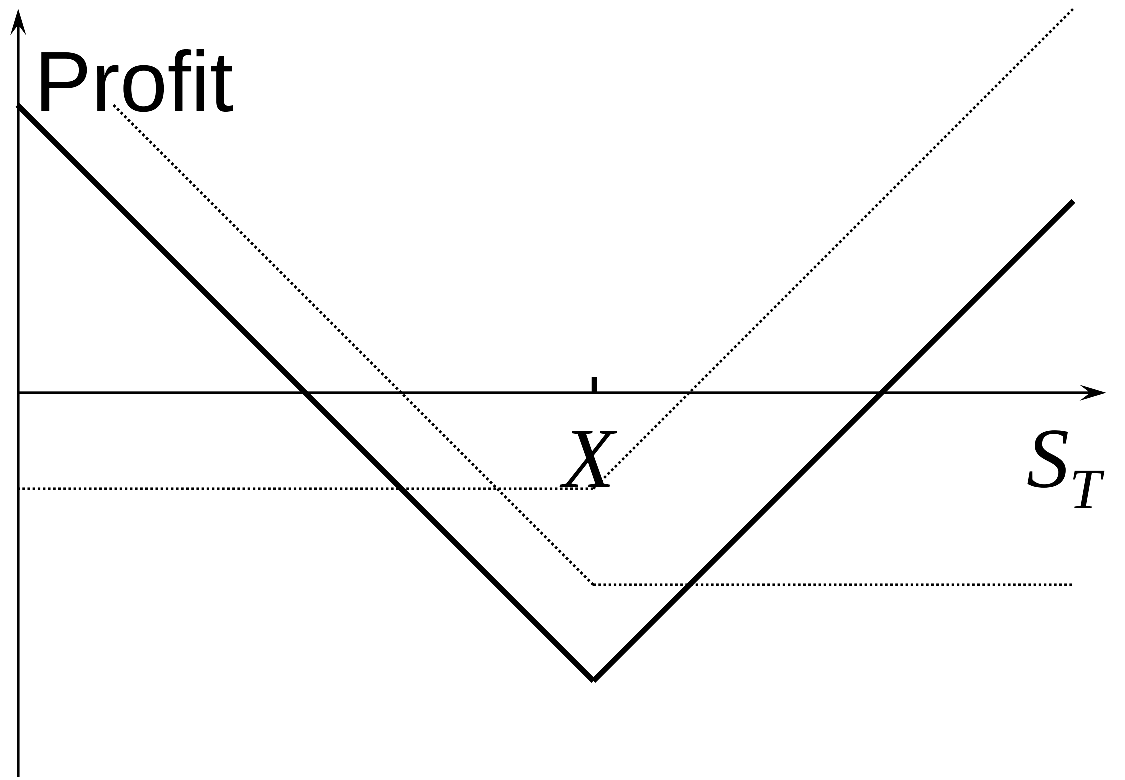
buy 2 calls and sell 2 calls



Butterfly Spread Using Puts

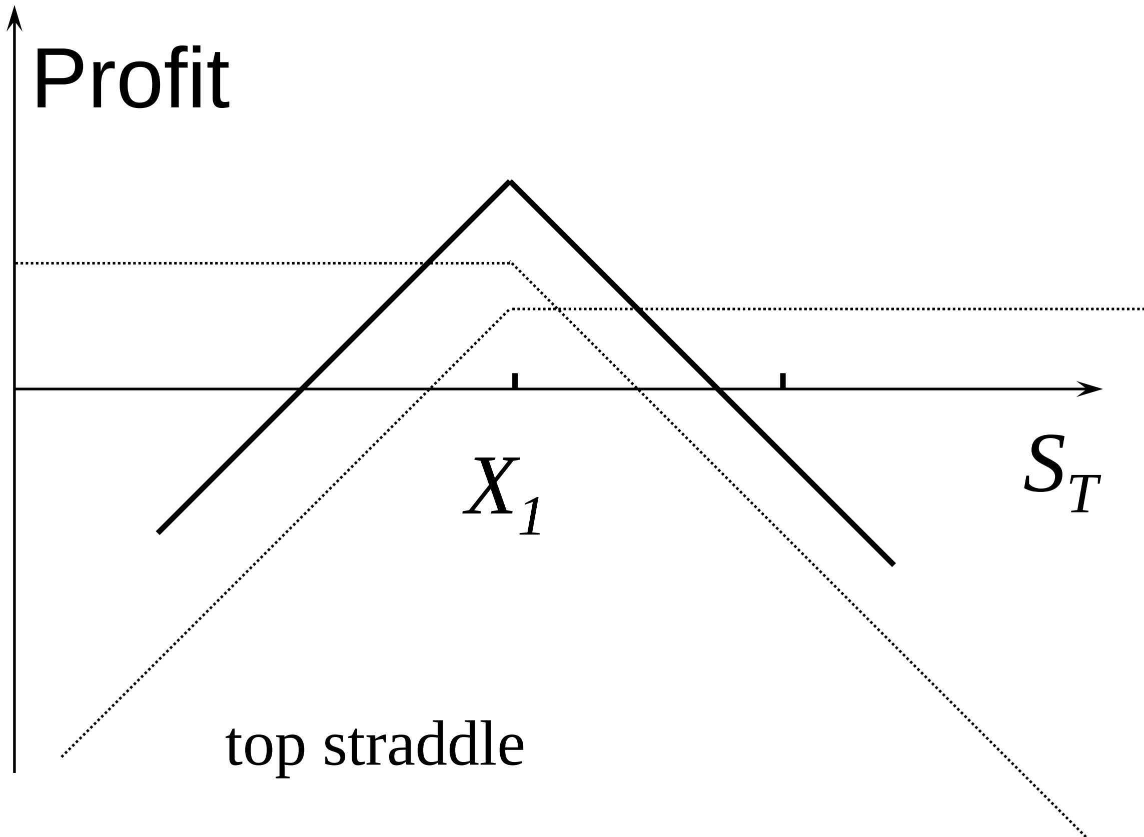


A Straddle Combination

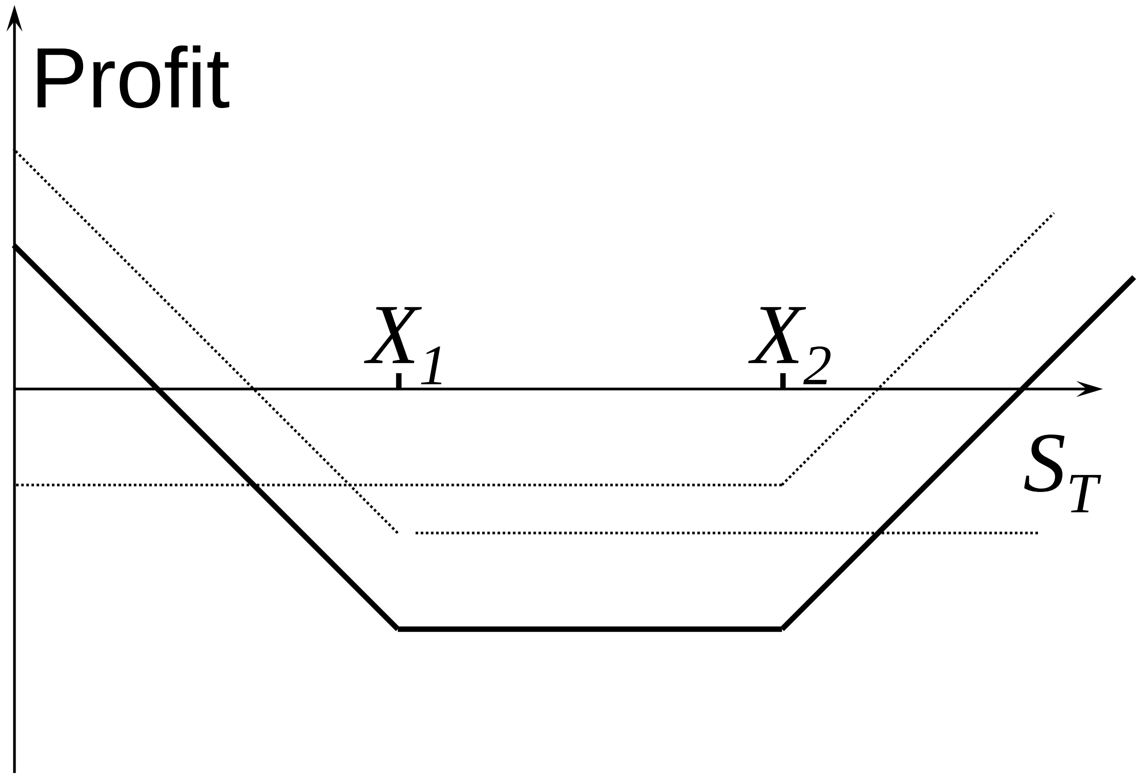


bottom straddle

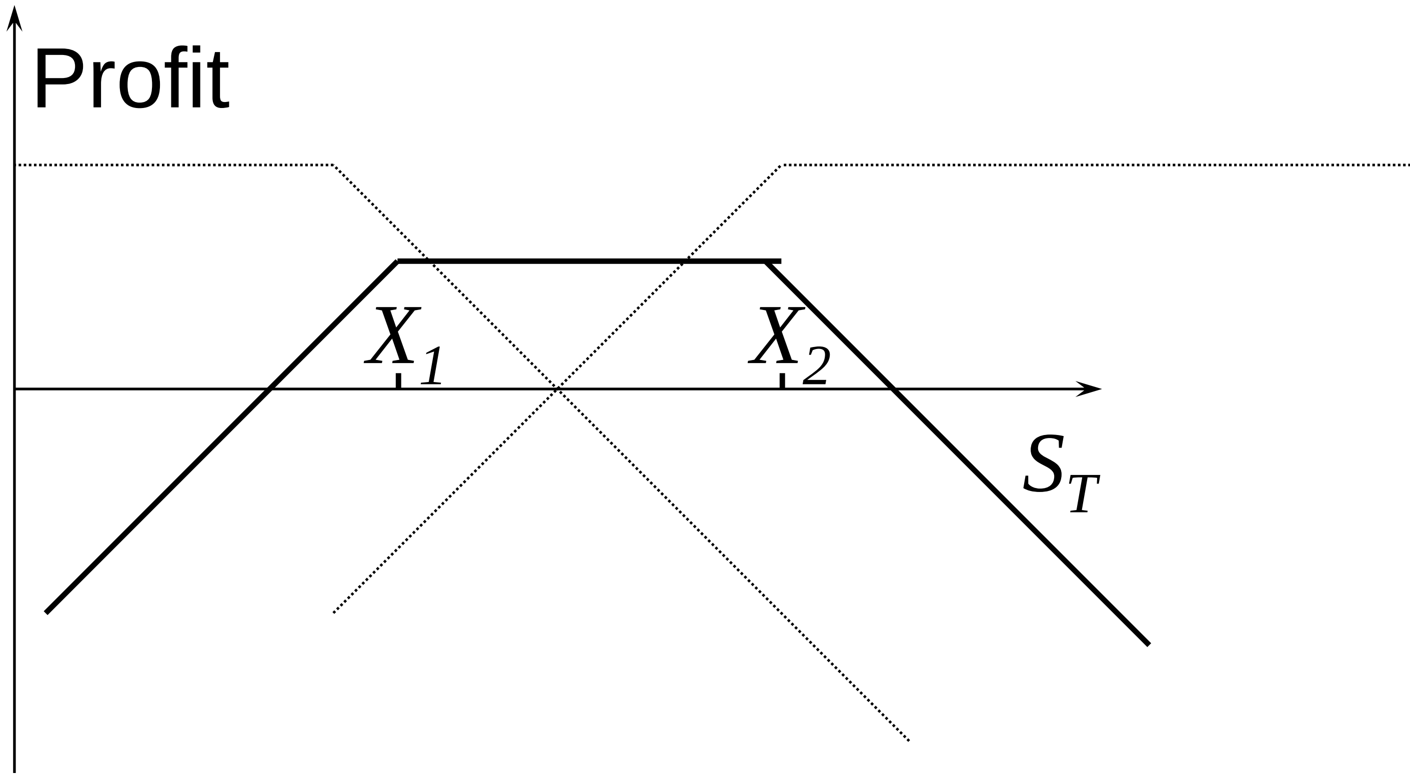
A 2nd Straddle Combination



A Strangle Combination



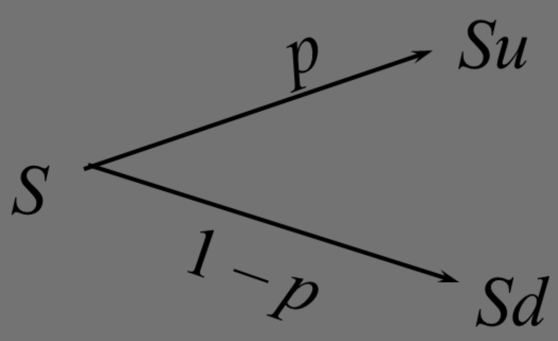
A Top Vertical Combination

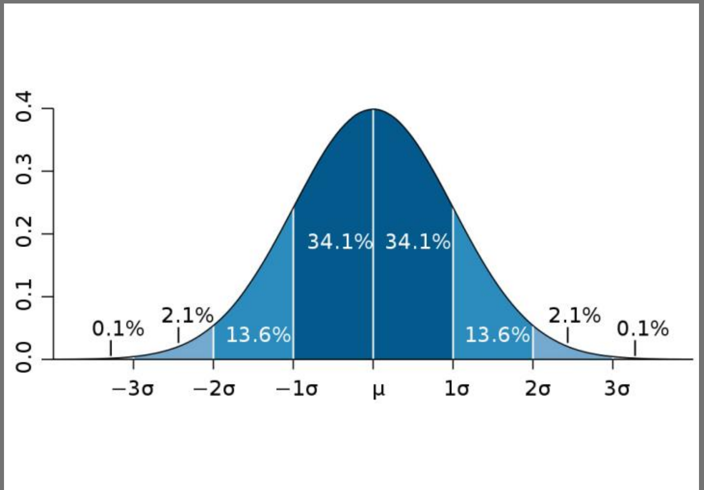


Pricing:
1. binomial tree intuition

Intuizione per capire

Binomial Tree





K = S(0)
Call = max [0 ; S(T) – S(0)]

stato «up» : Call = Su - S
stato «down» : Call = 0

Se fossimo «risk-neutral»

Premio = E (Call)
= p *(Su – S) + (1-p)*0
= p *(Su – S)

Premio = $0.5 * S * \sigma * \sqrt{T}$

altre ipotesi statistiche:

p = 50%
Su = $S(1+\sigma*\sqrt{T})$
Sd = $S(1-\sigma*\sqrt{T})$

QUANTUM

Pricing:
2. Black&Scholes

In realtà

Approssimazione binomiale: Premio = 0.5* S* $\sigma * \sqrt{T}$

Approssimazione Taylor 1 ordine: Premio = 0.4* S* $\sigma * \sqrt{T}$

esempio:

se T = 1 ; σ = 30%

*Premio = 0.12 * S*

se T = 0,25 ; σ = 30%

*Premio = 0.06 * S*

Black-Scholes (at-the-money): Premio = $S\left[2N\left(\frac{\sigma\sqrt{\tau}}{2}\right)-1\right]$

$$N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-t^2/2}dt = \frac{1}{2} + \int_0^d \frac{e^{-t^2/2}}{\sqrt{2\pi}}dt$$
$$= \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\left\{d - \frac{d^3}{6} + \frac{d^5}{40} - \cdots + \cdots\right\}.$$

QUANTUM

Pricing:
3.Monte Carlo

An Ito Process for Stock Prices

(See pages 225-6)

$$dS = \mu S dt + \sigma S dz$$

where μ is the expected return σ is the volatility.

The discrete time equivalent is

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

Monte Carlo Simulation

- We can sample random paths for the stock price by sampling values for ε
- Suppose μ= 0.14, σ= 0.20, and Δt = 0.01, then

$$\Delta S = 0.0014S + 0.02S\varepsilon$$

see simple_example.xls

Monte Carlo Simulation – One Path

(continued. See Table 10.1)

Period	Stock Price at Start of Period	Random Sample for ε	Change in Stock Price, ΔS
0	20.000	0.52	0.236
1	20.236	1.44	0.611
2	20.847	-0.86	-0.329
3	20.518	1.46	0.628
4	21.146	-0.69	-0.262

Monte Carlo Simulation

When used to value European stock options, this involves the following steps:

1. Simulate 1 path for the stock price in a risk neutral world
2. Calculate the payoff from the stock option
3. Repeat steps 1 and 2 many times to get many sample payoff
4. Calculate mean payoff
5. Discount mean payoff at risk free rate to get an estimate of the value of the option

A More Accurate Approach

Use $$d\ \ln S = \left(\hat{\mu} - \sigma^2/2\right) dt + \sigma\, dz$$

The discrete version of this is

$$\ln\left(S + \Delta S\right) - \ln(S) = \left(\hat{\mu} - \sigma^2/2\right) \Delta t + \sigma\varepsilon\sqrt{\Delta t}$$

or

$$S + \Delta S = S\, e^{\left(\hat{\mu} - \sigma^2/2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}}$$

Extensions

When a derivative depends on several underlying variables we can simulate paths for each of them in a risk-neutral world to calculate the values for the derivative