SOLUZIONI ESERCIZI - INTEGRALI DEFINITI

1.

a) $\displaystyle\int_1^2 \left(\frac{3}{x^2} - \frac{3x+1}{x^3-9x}\right) dx$ FUNZIONE CONTINUA SU $[1,2]$

$\Rightarrow$ INTEGRABILE

$$\int_1^2 \frac{3}{x^2}\,dx - \int_1^2 \frac{3x+1}{x(x+3)(x-3)}\,dx = *$$

SCOMPONGO

$$\frac{3x+1}{x(x+3)(x-3)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3} = \frac{(A+B+C)x^2 + (3C-3B)x - 9A}{x(x+3)(x-3)}$$

$$\begin{cases} A+B+C=0 \\ 3C-3B=3 \\ -9A=1 \end{cases} \qquad \begin{cases} A = -\frac{1}{9} \\ B = -\frac{4}{9} \\ C = \frac{5}{9} \end{cases}$$

$$* = \int_1^2 \frac{3}{x^2}\,dx - \frac{1}{9}\int_1^2 \frac{1}{x}\,dx - \frac{4}{9}\int_1^2 \frac{1}{x+3}\,dx + \frac{5}{9}\int_1^2 \frac{1}{x-3}\,dx =$$

$$= \left[-\frac{3}{x}\right]_1^2 - \frac{1}{9}\left[\ln|x|\right]_1^2 - \frac{4}{9}\left[\ln|x+3|\right]_1^2 + \frac{5}{9}\left[\ln|x-3|\right]_1^2 =$$

$$= -\frac{3}{2} + 3 - \frac{1}{9}\ln 2 + 0 - \frac{4}{9}\ln 5 + \frac{4}{9}\ln 4 + 0 - \frac{5}{9}\ln 2 =$$

$$= \frac{3}{2} - \frac{4}{9}\ln 5 + \frac{2}{9}\ln 2$$

b) $\displaystyle\int_1^2 \frac{2t+1}{(t-4)(t-3)^2}\,dt$ FUNZ. CONT. SU $[1,2]$

$\Rightarrow$ INTEGR. SU $[1,2]$

$$\frac{2t+1}{(t-4)(t-3)^2} \overset{\text{SCOMPONGO}}{=} \frac{A}{t-4}+\frac{B}{t-3}+\frac{C}{(t-3)^2} = \frac{(A+B)t^2+(-7B+C-6A)+9A-4C}{(t-4)\,(t-3)^2}$$

$$\begin{cases} A+B=0 \\ -7B+C-6A=2 \\ 9A-4C=1 \end{cases} \qquad \begin{cases} A=\frac{9}{13} \\ B=-\frac{9}{13} \\ C=\frac{17}{13} \end{cases}$$

SEGUE COME IL PRECEDENTE ...

c) $\displaystyle\int_0^1 \frac{e^{1-2\sqrt{x+3}}}{\sqrt{x+3}}\,dx = \int_0^1 e^{\overset{f(x)}{1-2\sqrt{x+3}}}\cdot \overset{f'(x)}{\frac{1}{\sqrt{x+3}}}\,dx =$

$$\int e^{f(x)}\cdot f'(x) = e^{f(x)}$$

$$=\left[e^{1-2\sqrt{x+3}}\right]_0^1 = e^{-3}-e^{1-2\sqrt{3}}$$

d) $\displaystyle\int_2^3 \frac{2x+3}{x(x^2-1)}\,dx = \circledast$

$$\frac{2x+3}{x(x^2-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} = \frac{(A+B+C)x^2+(C-B)x-A}{x(x+1)(x-1)}$$

$$\begin{cases} A+B+C=0 \\ C-B=2 \\ -A=3 \end{cases} \qquad \begin{cases} A=-3 \\ B=\frac{1}{2} \\ C=\frac{5}{2} \end{cases}$$

SI PROSEGUE COME NEI PUNTI a) e b)

$$-3\int_2^3 \frac{1}{x}\,dx + \frac{1}{2}\int_2^3 \frac{1}{x+1}\,dx + \frac{5}{2}\int_2^3 \frac{1}{x-1}\,dx = \dots$$

e) $\displaystyle\int_0^1 (x^2+x)\, e^{3x}\,dx = \circledast$

PER PARTI CERCO UNA PRIMITIVA

$$\int (x^2+x)\, e^{3x}\,dx = \frac{1}{3}e^{3x}\cdot(x^2+x) - \int \frac{1}{3}e^{3x}(2x+1)\,dx =$$

INTEGRO PRIMA e^{3x}

ANCORA PER PARTI

$$= \frac{1}{3}e^{3x}(x^2+x) - \frac{1}{9}e^{3x}(2x+1) + \int \frac{1}{9}e^{3x}\cdot 2\,dx =$$

$$= \frac{1}{3}e^{3x}(x^2+x) - \frac{1}{9}e^{3}(2x+1) + \frac{2}{27}e^{3x}$$

2.

$$f(x) = \begin{cases} 1 & x \leq 0 \\ x \, \mathrm{sen}(x^2) & 0 < x < 1 \\ \dfrac{\ln x}{x} & x \geq 1 \end{cases}$$

$$\int_0^1 f(x)\,dx = \int_0^1 x \, \mathrm{sen}(x^2)\,dx = \frac{1}{2}\int_0^1 \overset{f'(x)}{\overbrace{2x}} \, \mathrm{sen}(\overset{f(x)}{\overbrace{x^2}}) = \left[-\frac{1}{2}\cos(x^2)\right]_0^1 =$$

$$= -\frac{1}{2}\cos(1) + \frac{1}{2}\cos(0) = -\frac{1}{2}\cos(1) + \frac{1}{2}$$

$$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{0} 1\,dx + \int_0^1 x \, \mathrm{sen}(x^2)\,dx + \int_1^3 \frac{\ln x}{x}\,dx =$$

$$= \left[x\right]_{-2}^{0} + \left[-\frac{1}{2}\cos(x^2)\right]_0^1 + \left[\frac{\ln^2 x}{2}\right]_1^3 =$$

$$= 2 - \frac{1}{2}\cos(1) + \frac{1}{2} + \frac{\ln^2 3}{2}$$

$$(*) = \left[\frac{e^{3x}}{3}\left(x^2 + \frac{1}{3}x - \frac{1}{9}\right)\right]_0^1 =$$

$$= \frac{e}{3}\left(\frac{11}{9}\right) - \frac{1}{3}\left(-\frac{1}{9}\right) = \frac{11e+1}{27}$$

f) $$\int_0^1 \frac{2x+1}{9x^2-1}\,dx + \int_0^1 \frac{2}{4x^2+1}\,dx$$

OSSERVO CHE IL PRIMO INTEGRALE È ILLIMITATO PER $x \to \frac{1}{3}$ E $x \to -\frac{2}{3}$. HO DUNQUE UN PROBLEMA SU $[0,1]$.

per $x \to \frac{1}{3}$ $\quad \frac{2x+1}{9x^2-1} = \frac{2x+1}{(3x+1)(3x-1)} \sim \frac{\frac{5}{3}}{2(3x-1)} = \frac{5}{6(3x-1)}$ INFINITA DI ORDINE 1

$\Rightarrow$ NON INTEGRABILE

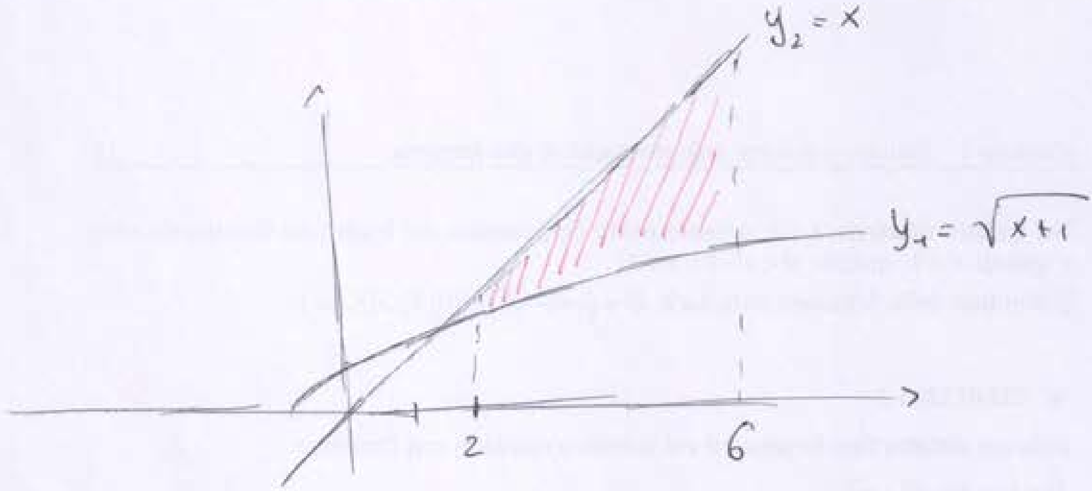
INVECE

$$\int_0^1 \frac{2}{4x^2+1}\,dx = \int_0^1 \frac{\overset{f'(x)}{2}}{1+(\underset{f(x)}{\boxed{2x}})^2}\,dx = \left[\operatorname{artg}(2x)\right]_0^1 = \operatorname{artg} 2$$

$$\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{artg}[f(x)]$$

~~PERÒ LA SOM~~

3.



Su $[2,6]$ la y_2 è maggiore di y_1. Quindi:

$$A = \int_2^6 (y_2 - y_1)\,dx = \int_2^6 \left(x - \sqrt{x+1}\right)dx =$$

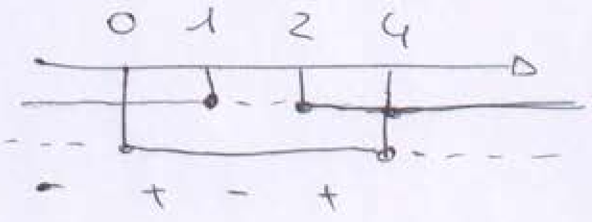
$$= \left[\frac{x^2}{2} - \frac{2}{3}\sqrt{(x+1)^3}\right]_2^6 = \frac{36}{2} - \frac{2}{3}\sqrt{343} - \frac{4}{2} + \frac{2}{3}\sqrt{27}$$

4. $$A = \int_1^3 |f(x)|\,dx$$

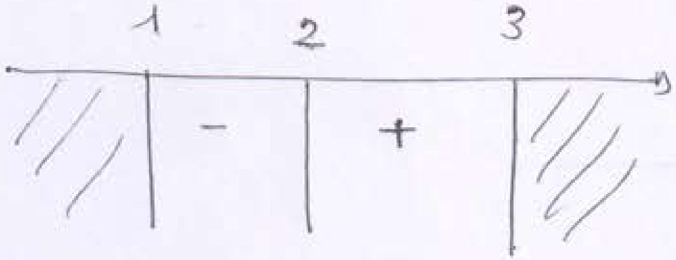
Come si comporta $f(x)$ su $[1,3]$?

$$\frac{x^2 - 3x + 2}{4x - x^2} \geq 0 \qquad x^2 - 3x + 2 \geq 0 \qquad x \leq 1 \vee x \geq 2$$

$$4x - x^2 > 0 \qquad 0 < x < 4$$



Su $[1,3]$ si ha dunque:



è negativa su $[1,2]$ e positiva su $[2,3]$

$\Rightarrow$ "SPEZZO" l'integrale:

$$A = -\int_1^2 \frac{x^2-3x+2}{4x-x^2}\,dx + \int_2^3 \frac{x^2-3x+2}{4x-x^2}\,dx =$$

Cerco una primitiva: $\displaystyle\int \frac{x^2-3x+2}{4x-x^2}\,dx = (*)$

devo prima operare una divisione:

$$\begin{array}{ccc|l} x^2 & -3x & +2 & -x^2+4x \\ \cline{4-4} -x^2 & +4x & & -1 \\ \cline{1-3} 0 & x & +2 & \end{array}$$

$$\Rightarrow \quad x^2-3x+2 = (-x^2+4x)\cdot(-1) + (x+2)$$

$$\Rightarrow \quad \frac{x^2-3x+2}{4x-x^2} = \frac{-1}{4x-x^2} + \frac{x+2}{4x-x^2}$$

$$\circledast = \int \frac{1}{x(x-4)}\,dx + \int \frac{x+2}{x(4-x)}\,dx = \circledast$$

$$\frac{1}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4} = \frac{(A+B)x-4A}{x(x-4)} \quad \begin{cases} A+B=0 \\ -4A=1 \end{cases} \quad \begin{cases} A=-\frac{1}{4} \\ B=\frac{1}{4} \end{cases}$$

$$\frac{x+2}{x(4-x)} = \frac{C}{x} + \frac{D}{4-x} = \frac{(D-C)x+4C}{x(4-x)}$$

$$\begin{cases} D-C=1 \\ 4C=2 \end{cases} \quad \begin{cases} C=\frac{1}{2} \\ D=\frac{3}{2} \end{cases}$$

$$\circledast = -\frac{1}{4}\int \frac{1}{x}\,dx + \frac{1}{4}\int \frac{1}{x-4}\,dx + \frac{1}{2}\int \frac{1}{x}\,dx + \frac{3}{2}\int \frac{1}{4-x}\,dx =$$

$$= -\frac{1}{4}\ln|x| + \frac{1}{4}\ln|x-4| + \frac{1}{2}\ln|x| - \frac{3}{2}\ln|4-x|$$

QUINDI L'AREA RISULTA:

$$A = -\left[-\frac{1}{4}\ln|x| + \frac{1}{4}\ln|x-4| + \frac{1}{2}\ln|x| - \frac{3}{2}\ln|4-x|\right]_1^2 + \left[\ \dots\ \right]_2^3 = \dots$$

5. $g(x) = \ln(|x|+1) = \begin{cases} \ln(x+1) & x \geq 0 \\ \ln(-x+1) & x < 0 \end{cases}$

$$\int_{-1}^{1} g(x)\,dx = \int_{-1}^{0} \ln(-x+1)\,dx + \int_{0}^{1} \ln(x+1)\,dx =$$

$$= \Big[x\ln(1-x) - x - \ln(1-x)\Big]_{-1}^{0} + \Big[x\ln(x+1) - x + \ln(x+1)\Big]_{0}^{1}$$

$= \ldots\ldots$