INTEGRALI DOPPI

① $\iint_D xy\,dx\,dy$ CON $D = \{(x,y) : 0 \le x \le 1,\ x^2 \le y \le 1+x\}$

↳ y-semplice

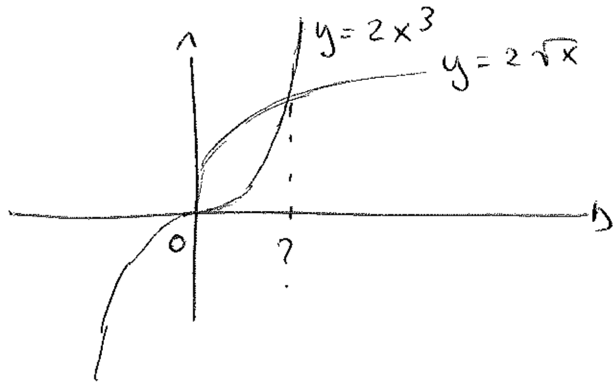
$$\iint_D xy\,dx\,dy = \int_0^1 \left[\int_{x^2}^{1+x} xy\,dy\right] dx = \int_0^1 \left[x\frac{y^2}{2}\right]_{x^2}^{1+x} dx =$$

$$= \int_0^1 \left(x\frac{(1+x)^2}{2} - x\cdot\frac{x^2}{2}\right) dx = \int_0^1 \left(\frac{x+2x^2+x^3}{2} - \frac{x^3}{2}\right) dx =$$

$$= \frac{1}{2}\int_0^1 (x+2x^2)\,dx = \frac{1}{2}\left[\frac{x^2}{2}+\frac{2}{3}x^3\right]_0^1 = \frac{7}{12}$$

② $\iint_D (x+y)\,dx\,dy$ CON $D = \{(x,y) : 2x^3 \le y \le 2\sqrt{x}\}$

DOBBIAMO STABILIRE DOVE VARIA LA X:



TROVO LE INTERSEZIONI:

$$\begin{cases} y = 2x^3 \\ y = 2\sqrt{x} \end{cases} \longrightarrow x^3 = \sqrt{x} \longrightarrow x^3 - \sqrt{x} = 0$$

$$\sqrt{x}\left(x^{\frac{5}{2}} - 1\right) = 0$$

$$\sqrt{x} = 0 \implies x = 0$$

$$x^{\frac{5}{2}} - 1 = 0 \implies \sqrt{x^5} = 1 \longrightarrow x = 1$$

quindi $D = \{(x,y) : 0 \leq x \leq 1 ,\ 2x^3 \leq y \leq 2\sqrt{x}\}$ y-semplice

$$\iint_D (x+y)\,dx\,dy = \int_0^1 \left[\int_{2x^3}^{2\sqrt{x}} (x+y)\,dy\right] dx =$$

$$= \int_0^1 \left[xy + \frac{1}{2}y^2\right]_{2x^3}^{2\sqrt{x}} dx = \int_0^1 \left[x \cdot 2\sqrt{x} + \frac{1}{2}(2\sqrt{x})^2 - x \cdot 2x^3 - \frac{1}{2}(2x^3)^2\right] dx =$$

$$= \int_0^1 \left[2x^{\frac{3}{2}} + 2x - 2x^4 - 2x^6\right] dx =$$

$$= \left[\frac{4}{5}x^{\frac{5}{2}} + x^2 - \frac{2}{5}x^5 - \frac{2}{7}x^7\right]_0^1 = \frac{4}{5} + 1 - \frac{2}{5} - \frac{2}{7} - 0 = \frac{39}{35}$$

③ $\displaystyle\iint_D \frac{x}{y}\,dx\,dy$ con $D=\{(x,y): 1\le y\le \pi \text{ e } y\le x\le \pi\}$

↳ x-semplice

$$\cancel{\iint_D \frac{x}{y}\,dx\,dy}\ \iint_D \frac{x}{y}\,dx\,dy = \int_1^{\pi}\left[\int_y^{\pi}\frac{x}{y}\,dx\right]dy =$$

$$= \int_1^{\pi}\left[\frac{1}{y}\cdot\frac{x^2}{2}\right]_y^{\pi} dy = \int_1^{\pi}\left[\frac{1}{y}\cdot\frac{\pi^2}{2} - \frac{1}{y}\cdot\frac{y^2}{2}\right]dy =$$

$$= \int_1^{\pi}\left(\frac{\pi^2}{2}\cdot\frac{1}{y} - \frac{y}{2}\right)dy = \left[\frac{\pi^2}{2}\ln y - \frac{y^2}{4}\right]_1^{\pi} =$$

$$= \frac{\pi^2}{2}\ln\pi - \frac{\pi^2}{4} - \frac{\pi^2}{2}\ln 1 + \frac{1}{4} = \frac{\pi^2}{2}\ln\pi - \frac{\pi^2}{4} + \frac{1}{4}$$