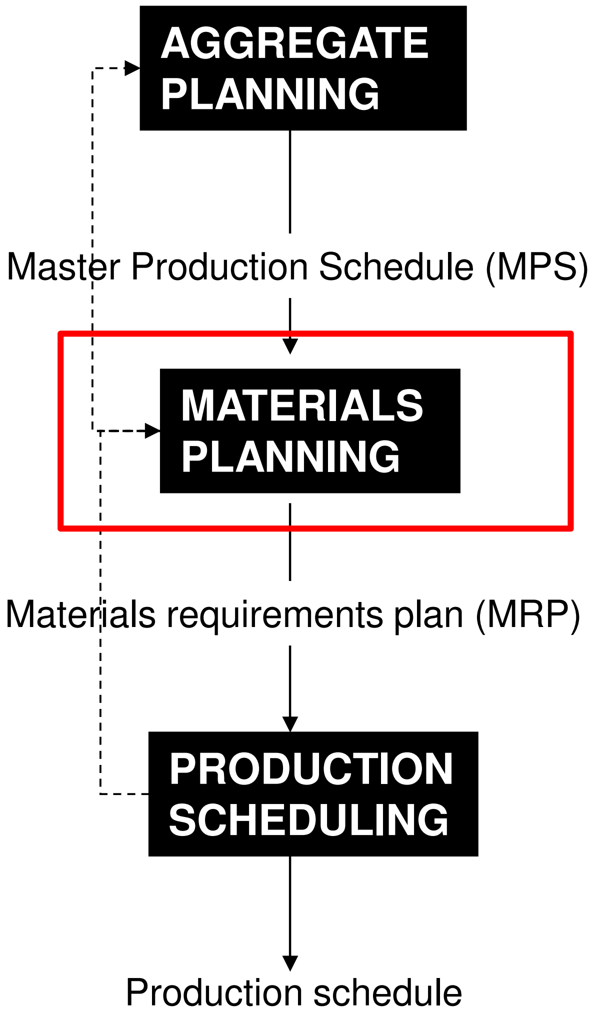
Inventory Management

Materials planning



The planning of materials requirements consists of the determination of:

- What
- How much and
- When

to order at every stage of the production process

Materials planning approaches

Materials planning approaches

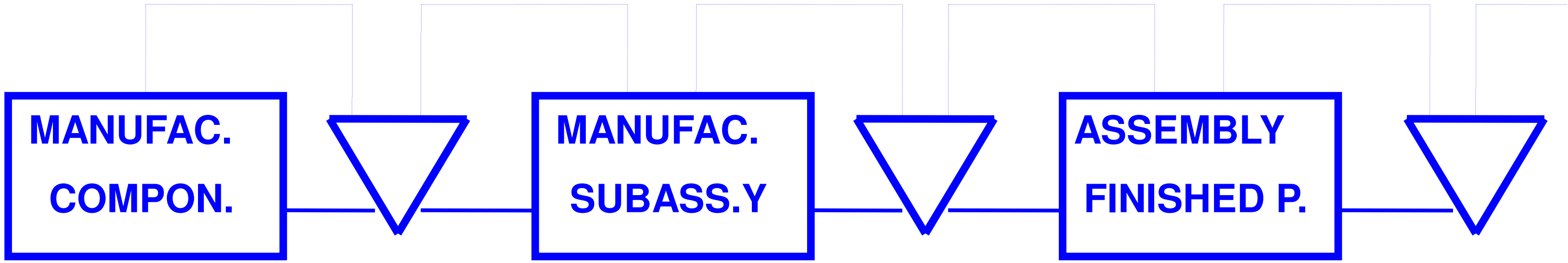
Stock based
"PULL"
also called
"INVENTORY MANAGEMENT"

Materials' production/purchase orders are issued in order to replenish a stock

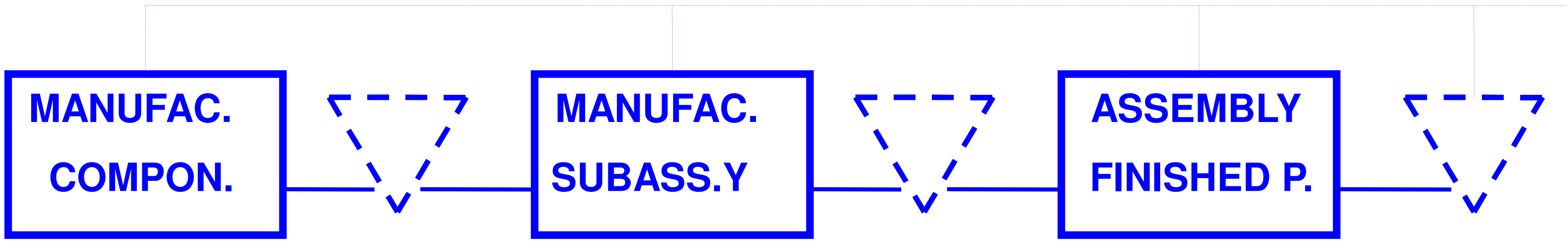
Requirements based
"PUSH"
also called
"ON DEMAND MANAGEMENT"

Materials' production/purchase orders are issued in order to satisfy a finished product requirement

Materials planning approaches



PULL SYSTEM



PUSH SYSTEM

Inventory management

- The moment and the quantity to order are decided only based on the actual level of stock.
- Stock management approach is basically to simply react to demand.
- For this reason this technique is called "Pull", i.e. pulled by downstream consumption.

Inventory management

Stock management approach should therefore be used when:

- Demand is stationary, or constant, over time, or
- It is impossible to foresee demand with enough accuracy, or
- Upstream phase is highly flexible

Why to hold inventory ?

- To absorb demand uncertainty
  - Safety stock (stock of flexibility)
  - Seasonal stock (stock of capacity)
- To decouple different operations phases
  - To keep stock is a way to allow source, make and delivery systems to work with different rate, as it allows the different phases to be “asynchronous” and it absorbs upstream variations
- Speculative stocks
- Total cost minimization
  - Setup costs reduction
  - Order emission cost reduction

Stock “performance”

- Effectiveness: service level guaranteed to downstream phases:
  - Quantity and duration of stock-outs in case of safety stocks and decoupling stocks;
- Efficiency
  - Amount of fixed costs and/or of workforce costs saved in case of seasonal stocks;
  - Amount of setup costs saved due to the application of lot policies;
  - Amount of purchasing costs saved in case of speculative stocks.

Inventory management decisions

- Replenishment policy, i.e. how much and when to order.
  - Impact on:
    - Stock holding costs;
    - Setup/order emission costs;
    - Stock out / downstream service costs;
  - Constraints:
    - Production capacity and flexibility of upstream productive stage;
    - Physical space capacity of warehouses.
- Control level, i.e. the detail and the frequency of control of the stock level of each item.
  - Impact on:
    - Cost of control
  - Constraints :
    - Availability of resources to perform the controls (people, computers, ...).

Inventory management objectives

- The main objective of inventory management is to minimize the related costs and to maximize the service level for the downstream phases.
- This objective can be expressed as the minimization of a total cost function composed by:
  - Stock holding costs;
  - Execution costs (order / setup);
  - Costs of control;
  - Costs of low service level at downstream phases (stock-out costs).

Classification profiles

According to the:

- Type of control
  - Continuous control;
  - Discrete control (time intervals)

- Order issue interval
  - Fixed-time period;
  - Variable-time period.

- Ordered quantity
  - Fixed quantity;
  - Variable quantity.

- Type of re-order
  - Independent entries: each item is re-ordered independently from the others;
  - Joint entries: orders for the different items are coordinated (combined).

Classification profiles

Discrete control
Combined entries reorder

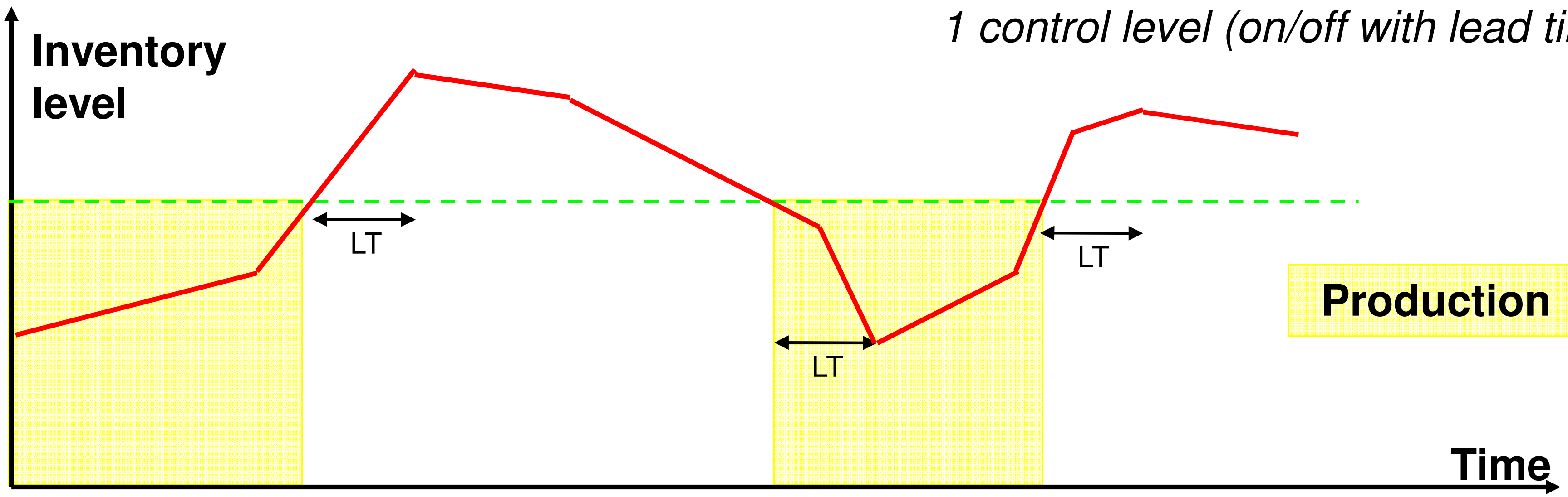
Continuous control
Independent entries reorder

Quantity ordered		Interval of order issue: Fixed	Interval of order issue: Variable
	Fixed		EOQ-ROP Model
	Variable	Fixed-Time period Model	Switch Models

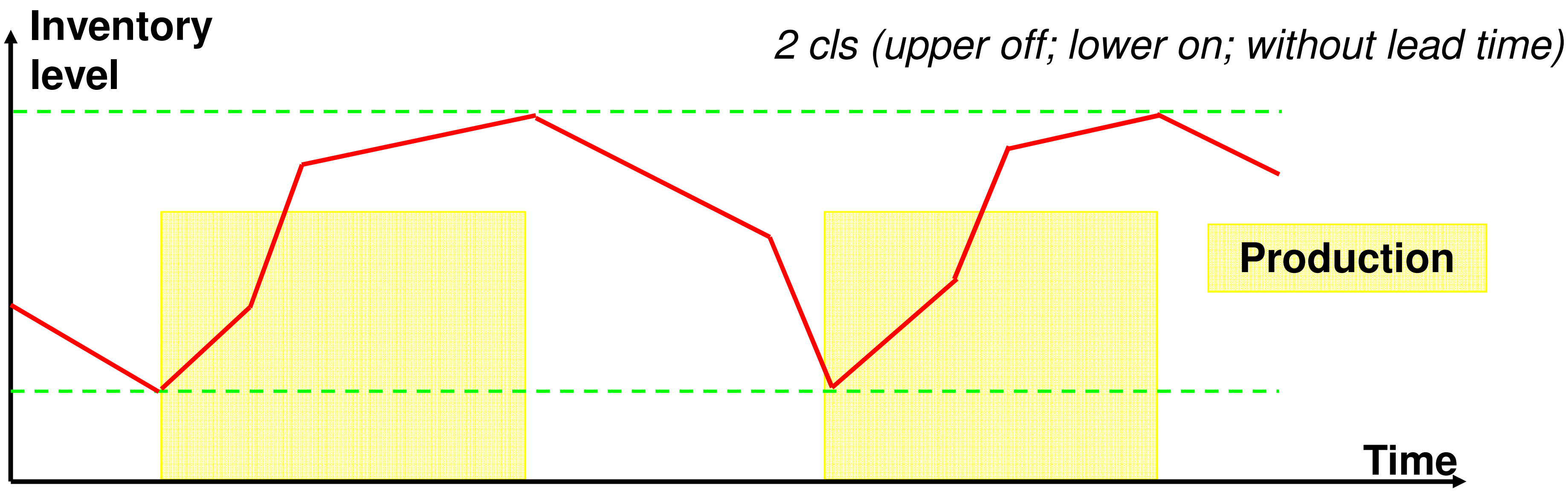
Switch models

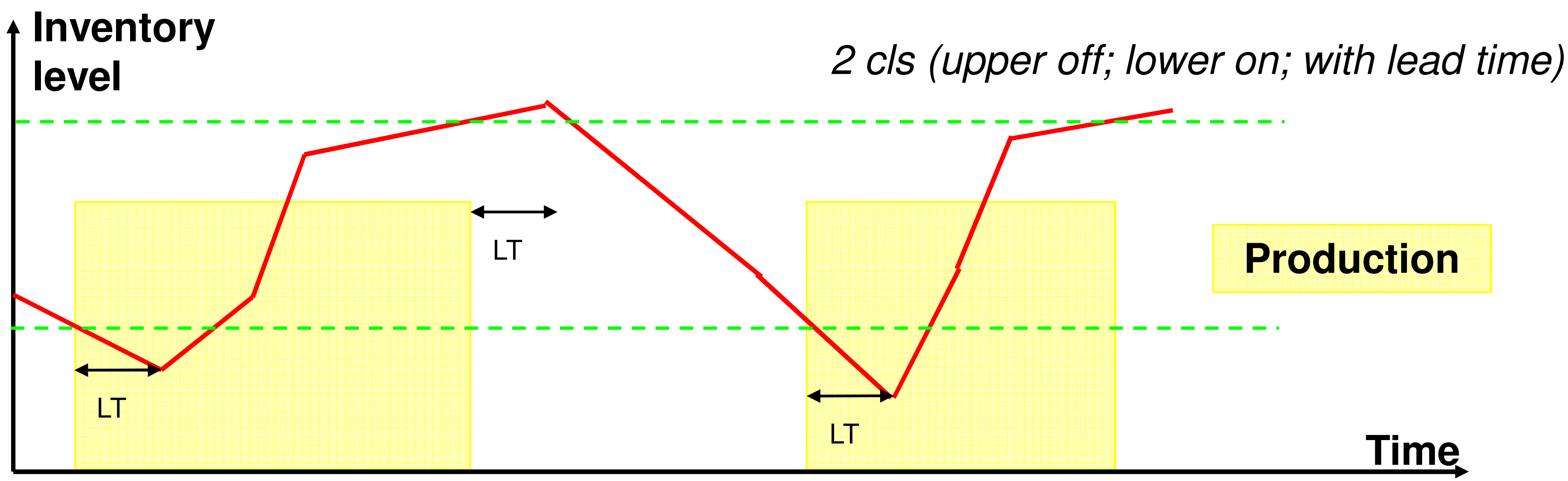
- The continuous control of inventory level allows to define the time («when») and the quantity («how much») to reorder
- They are calibrated by simulation to minimize the total costs
- Models exist with 1 (on/off with lead time), 2 (upper off; lower on; with or without lead time), and more (with slowing down/speeding up) control levels

1 control level (on/off with lead time)



Switch models





The Economic Order Quantity – Re-Order Point (EOQ-ROP) model

- Characteristics
  - Variable interval of orders issuing
  - Fixed quantity ordered
  - Continuous control
  - Independent entries reorder
- Objectives
  1. Identifying the quantity Q [pieces] to re-order that minimizes the total cost, sum of the ordering cost, purchasing/production cost and stock holding cost; and
  2. the conditions that determine the orders issuing

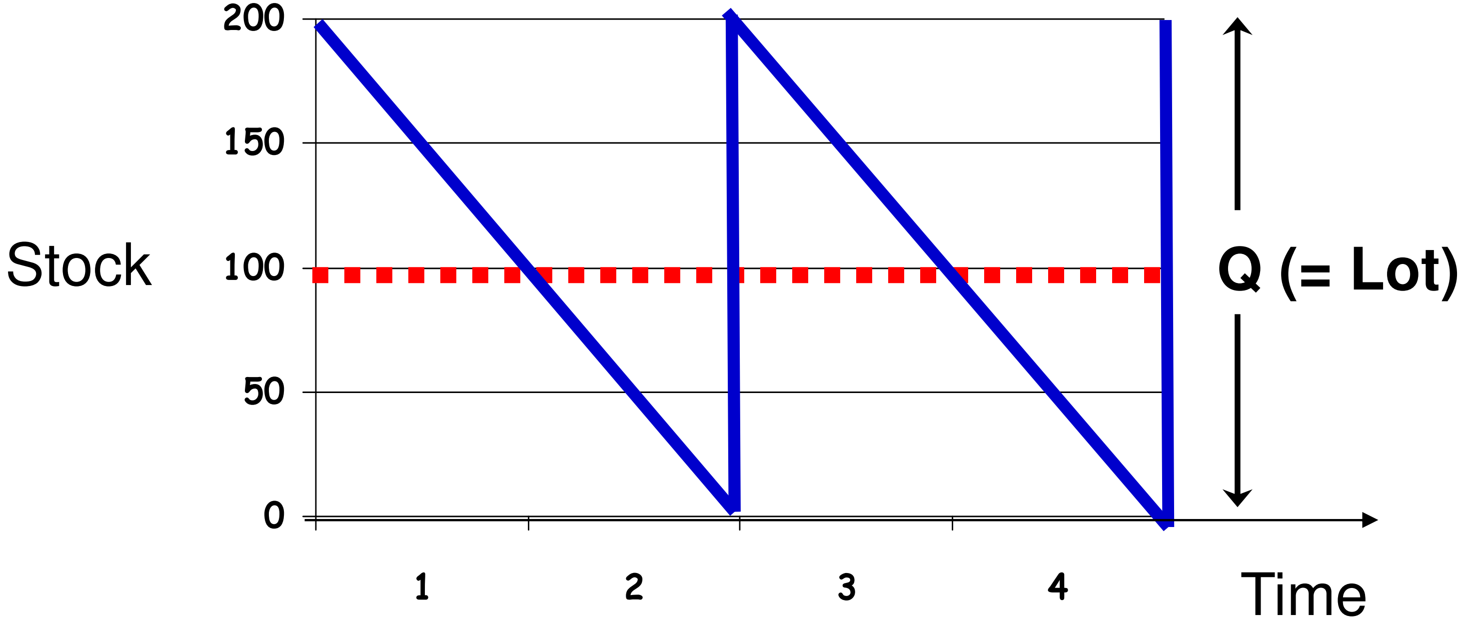
The EOQ-ROP model

Hypotheses

- Constant consumption over time — D [units/year]
- Constant setup/ordering cost — o [€]
- Constant variable cost/price per unit — p [€/unit]
- Constant ownership rate — cm [€/€*year]
- Constant lead time — LT [days]
- Pre-defined working days — H [days/year]
- Average daily consumption — d = D / H [units/day]
- Infinite stock replenishment rate — r [units/day]
- Infinite capacity of warehouse
- Shipment costs negligible (or included in the re-ordering cost)

The EOQ-ROP model

The saw-tooth diagram



- Average stock = Lot (Q) / 2
- Number of orders issued = Annual Demand (D) / Lot (Q)

The EOQ-ROP model

- Cost of stock holding Csh

$$Csh = cm \cdot p \cdot \frac{q}{2} \quad [€/year]$$

  - Under the stated hypotheses, the average inventory level is equal to q/2. So, the annual stock holding cost is linear in q.

- Cost of order issuing Co (or setup cost)

$$Co = o \cdot \frac{D}{q} \quad [€/year]$$

  - The number of orders issued in a year is equal to D/q. So the cost of order issuing Co and q are inversely proportional.

- Cost of purchasing Cp (or cost of production)

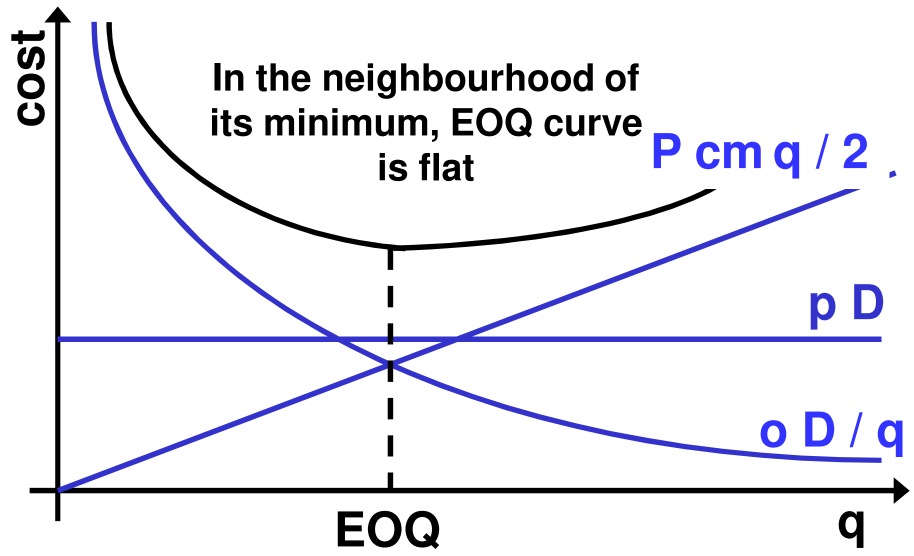
$$Cp = D \cdot p \quad [€/year]$$

  - The total cost of purchasing (or cost of production) Cp is independent from q.

The EOQ-ROP model

- If the overall cost function is derived by q and made equal to 0 ($\partial TC/\partial q = 0$), the Economic Order Quantity (EOQ) is:

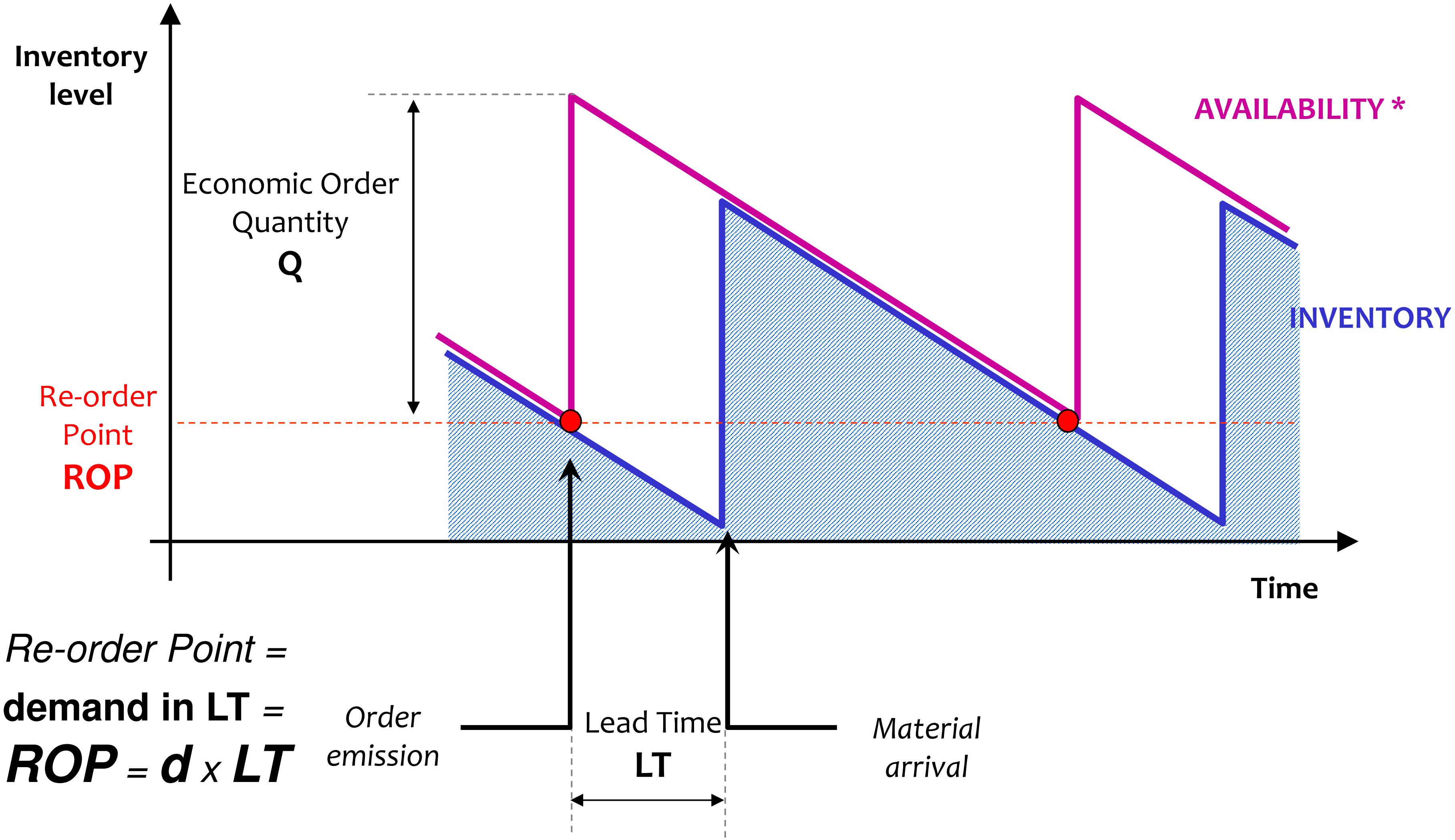
$$EOQ = \sqrt{\frac{2 \cdot o \cdot D}{p \cdot cm}}$$



- The re-order point (ROP):

$$ROP = LT \cdot D / H = LT \cdot d$$

The EOQ-ROP model



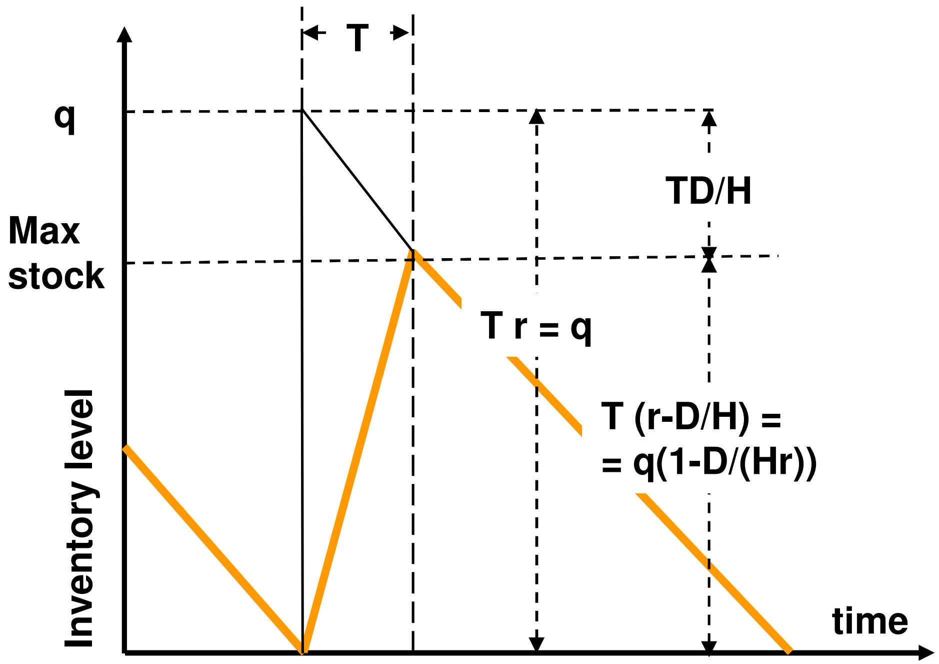
Re-order Point =

demand in LT =

$$ROP = d \times LT$$

Relaxing the hypothesis: "infinite production rate"

- If the time needed to produce a lot is high (days or weeks), the production flows into inventory not all at once. In the meantime demand is consuming the stock.
- Under finite production rate (r, units/period), EOQ can be calculated by setting reference to the "geometry" of saw-tooth diagram
- Since the consumption rate is D/H, during the replenishment period T, the inventory increases at a rate of (r – D/H)
- Since T = q/r, the following formula holds:



$$EOQ^* = \sqrt{\frac{2 \cdot o \cdot D}{p \cdot cm \cdot \left(1 - \frac{D}{H \cdot r}\right)}}$$

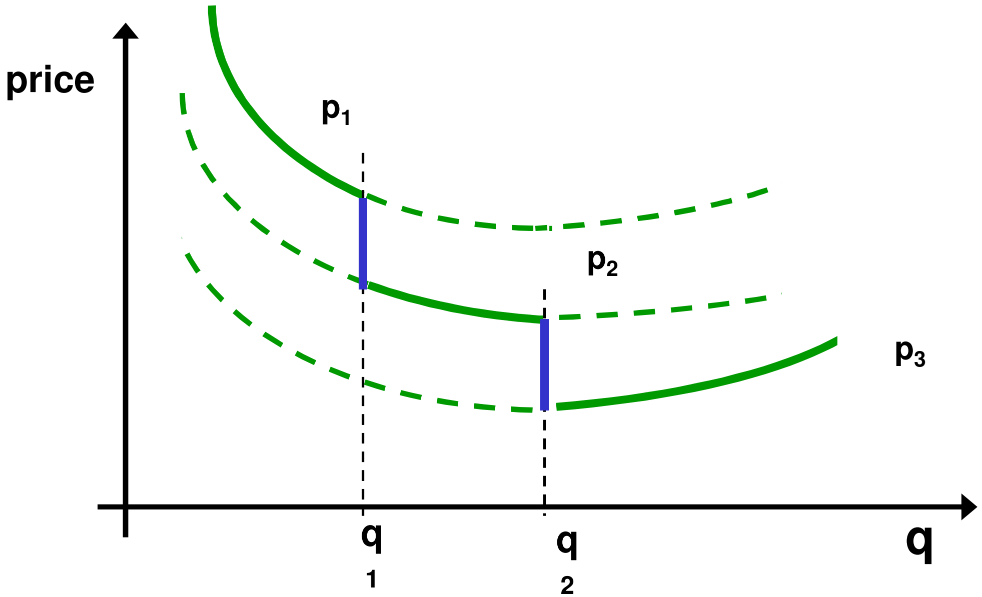
As D/H=d<r, then EOQ* > EOQ

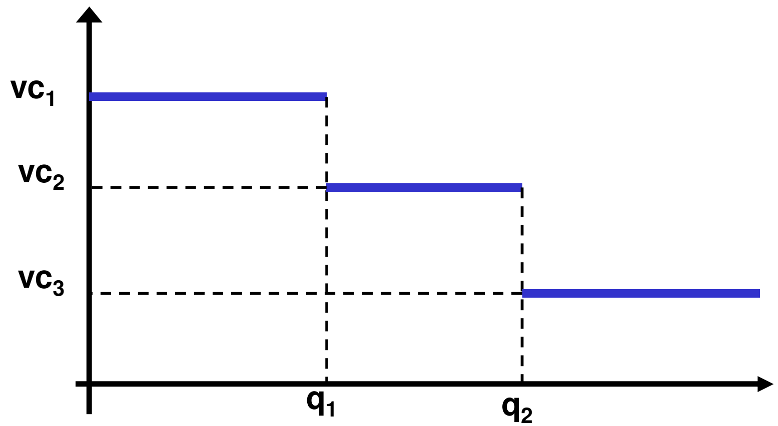
Production rate = r Consumption rate = D/H = d

Relaxing the hypothesis: "constant cost of purchasing / production"

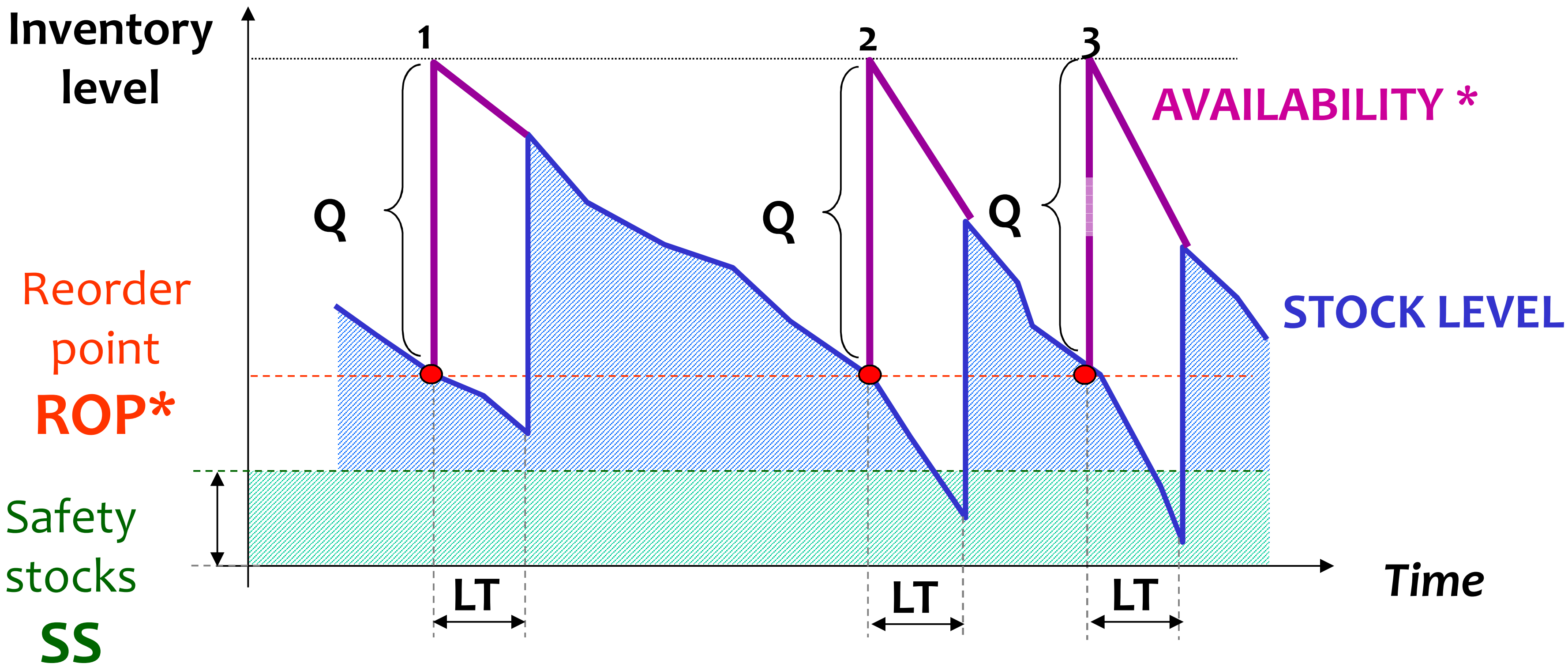
- It allows to consider quantity discounts
- It leads to consider a more complex cost function to derive, since "p" parameter changes (i.e. decreases) when q increases:

$$p = \begin{cases} p_1 & for \quad q < q_1 \\ p_2 < p_1 & for \quad q_1 < q < q_2 \\ p_3 < p_2 & for \quad q > q_2 \end{cases}$$





Relaxing the hypothesis: "constant consumption over time and constant lead time" - Safety Stock

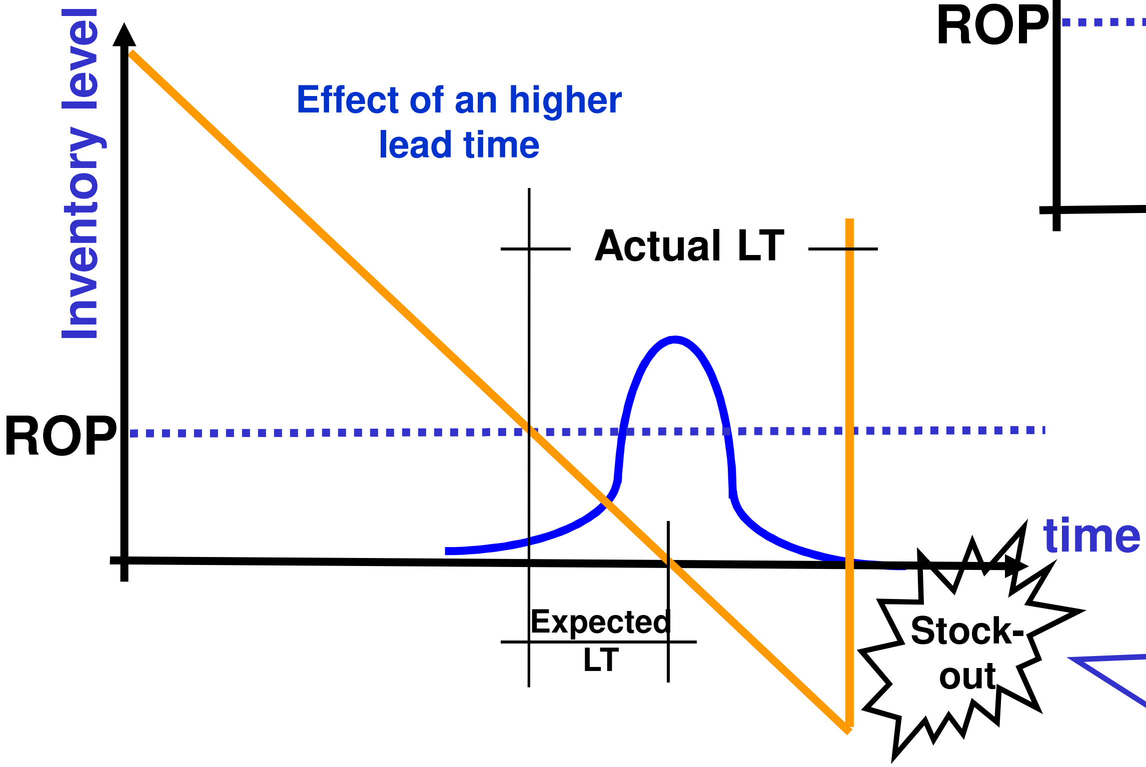


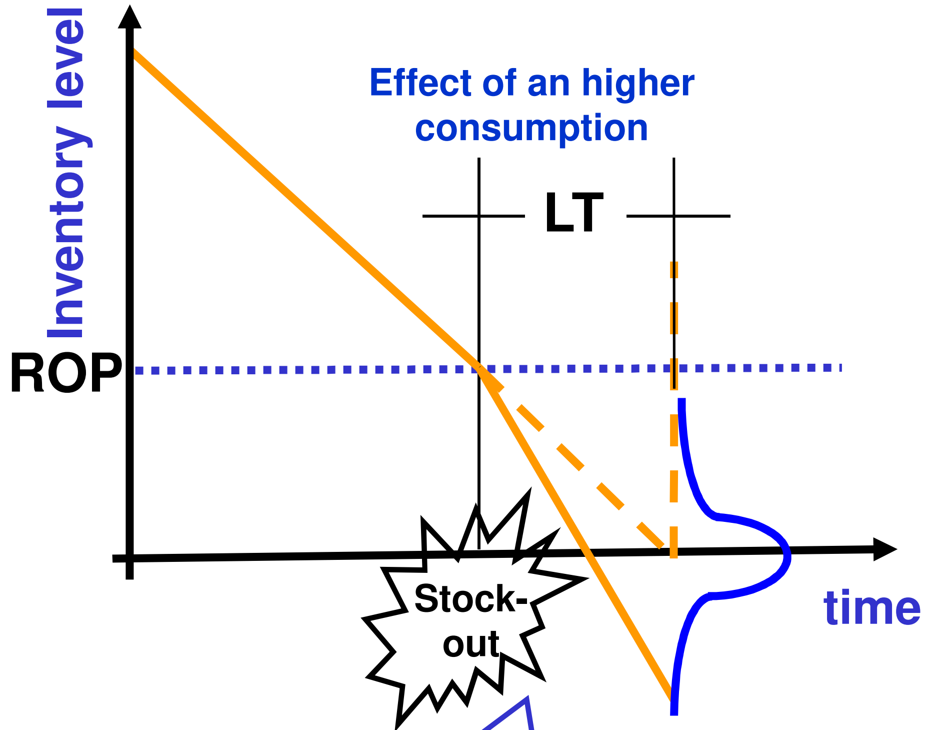
In reality: Reorder point = **demand during LT** **+ SS** = ***ROP**** = ***d*** x ***LT+ SS***

Safety Stock (SS)

- Safety stocks are used to prevent from stock-out
- The level of safety stocks depends on:
  - variation of downstream consumption (demand) with respect to forecast
  - desired service level
  - lead time duration
  - lead time reliability

The SS determination





Safety stocks are useful to prevent from stock-out when an order has been already issued, but the corresponding quantity (pieces) has not been reached the warehouse yet

The SS determination

- Basic principles
  - Safety stocks protect the inventory management system against high variability of downstream consumption (demand) and of upstream lead time
  - Safety stocks are “untouchable” and so they are “subtracted” from the the “real” availability
    - Furthermore – to ensure an adequate service level over time – they have also to be re-stored as soon as they are consumed

- Objective
  - Properly dimensioning safety stocks, by considering both consumption and lead time as random variables, distributed according the Normal distribution
    - they are provided with their own average and standard deviation

$$d, \sigma_d; \quad LT, \sigma_{LT}$$

Daily demand

The SS determination

- Safety stock is determined so that a minimum service level SL is reached
- The probability of stock coverage is equal or higher than SL
- SL: percentage of times in which company has not incurred in stockout (product was available when the market asked for it)

$$\text{P(demand during LT} \leq \text{on-hand)} \geq SL$$

$$P\left(\frac{demand \text{ in LT} - d_{LT}}{\sigma_{d,LT}} \leq \frac{on-hand-d_{LT}}{\sigma_{d,LT}}\right) \geq SL$$

$$P\left(Z \leq \frac{on-hand-d_{LT}}{\sigma_{d,LT}}\right) \geq SL$$

Normal standard variable

$$on-hand = d_{LT} + k\sigma_{d,LT} \equiv ROP^*$$

Service level factor

$$SS = k \cdot \sigma_{d,LT}$$

"Combined" standard deviation of demand during LT

Where:

d_{LT} = average demand in LT periods

$\sigma_{d,LT}$ = standard deviation of demand in LT periods

The SS determination

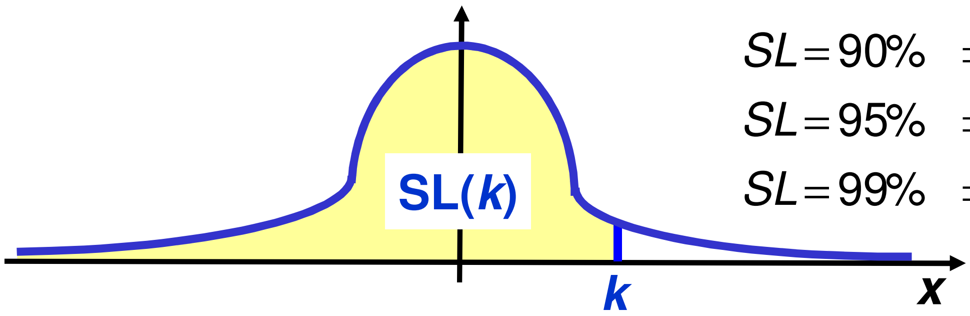
The general calculation formula

- The service level factor (k) is calculated by using the **cumulative Normal standard distribution** for random variables (Φ, provided by tables) that corresponds to the required service level (SL):

Service level factor, i.e., number of standard deviations

"Combined" standard deviation of demand during LT

$$SS = k \cdot \sigma_{d,LT}$$



$$SL = 90\% \Rightarrow k \approx 1.281$$
$$SL = 95\% \Rightarrow k \approx 1.645$$
$$SL = 99\% \Rightarrow k \approx 2.326$$

The SS determination

- The most common case is where demand and lead time are completely not-correlated

$$SS = k \cdot \sigma_{d,LT} = k\sqrt{\sigma^2_{d\ in\ LT} + \sigma^2_{LT\ in\ d}}$$

$$SS = k\sqrt{\left(\sigma_d \cdot \sqrt{LT}\right)^2 + \left(\sigma_{LT} \cdot d\right)^2}$$

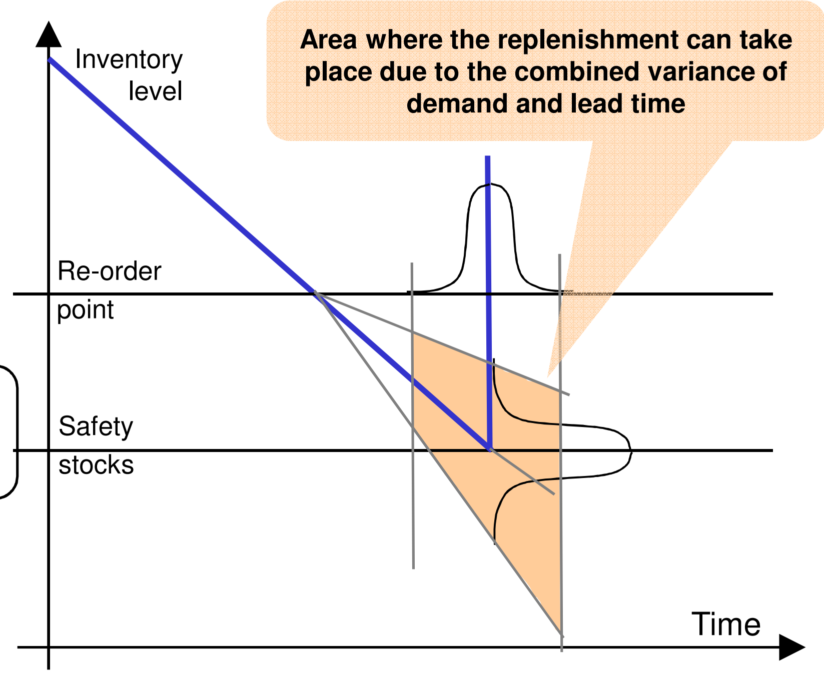
N.B.: unit of measure

- SS are [units]
- "$\sigma_d^2 \cdot LT$" is right (see later)
- Always express LT in units of measure compatible with σ_d (not vice versa!)

Service level factor

$$SS = k\sqrt{\sigma_d^2 \cdot LT + \sigma_{LT}^2 \cdot d^2}$$

- Service level factor: k
- Variance of demand: σ_d^2
- Average lead time: LT
- Variance of lead time: σ_{LT}^2
- Average demand empowered at 2: d^2



The SS determination

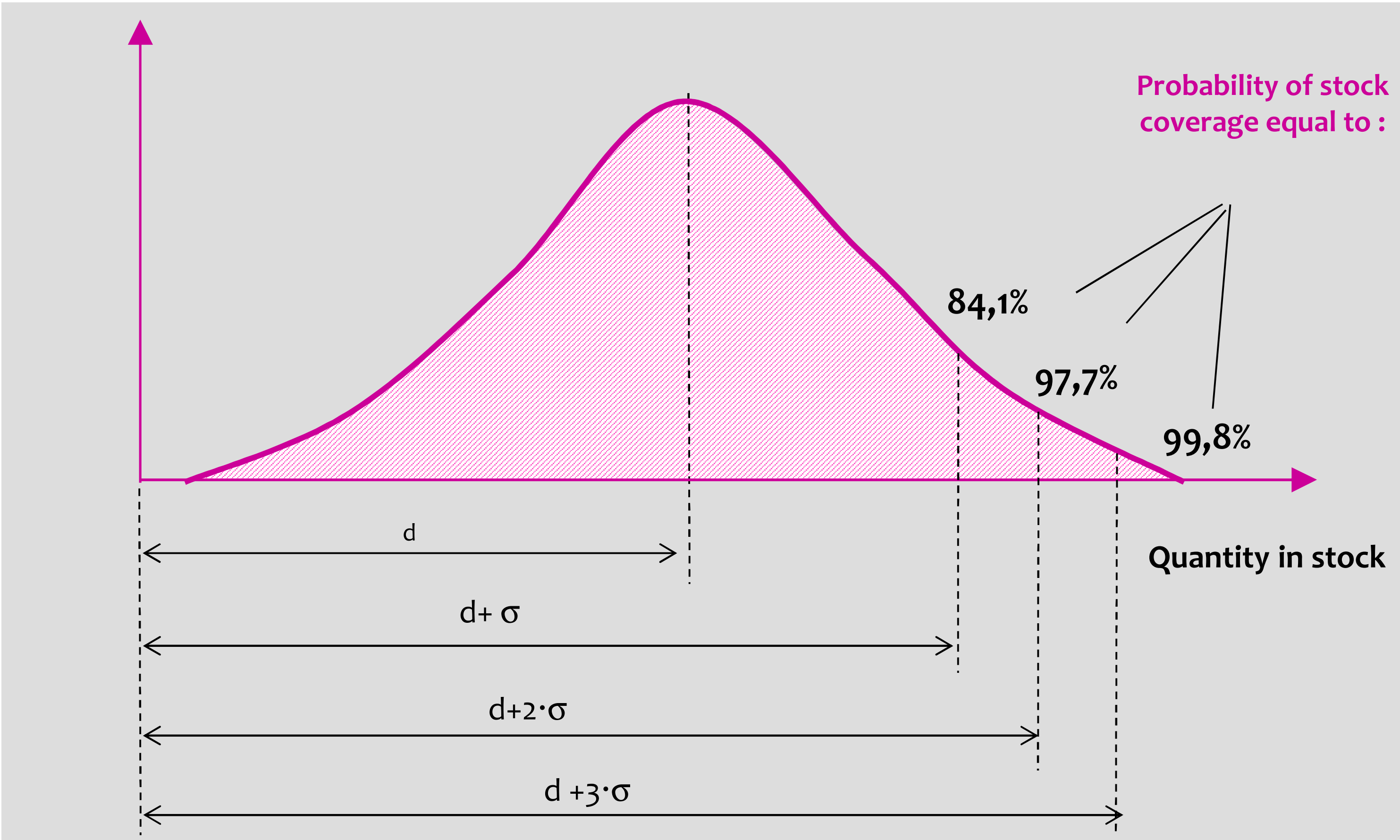
- The standard deviation of a series of independent events is the square root of the sum of the variances
- Given the average daily demand and the standard deviation of daily demand σ_d, the standard deviation during lead time LT can be calculated as follows:

$$\sigma_{d\ in\ LT} = \sqrt{\sum_{i=1}^{LT} \sigma_i^2}$$

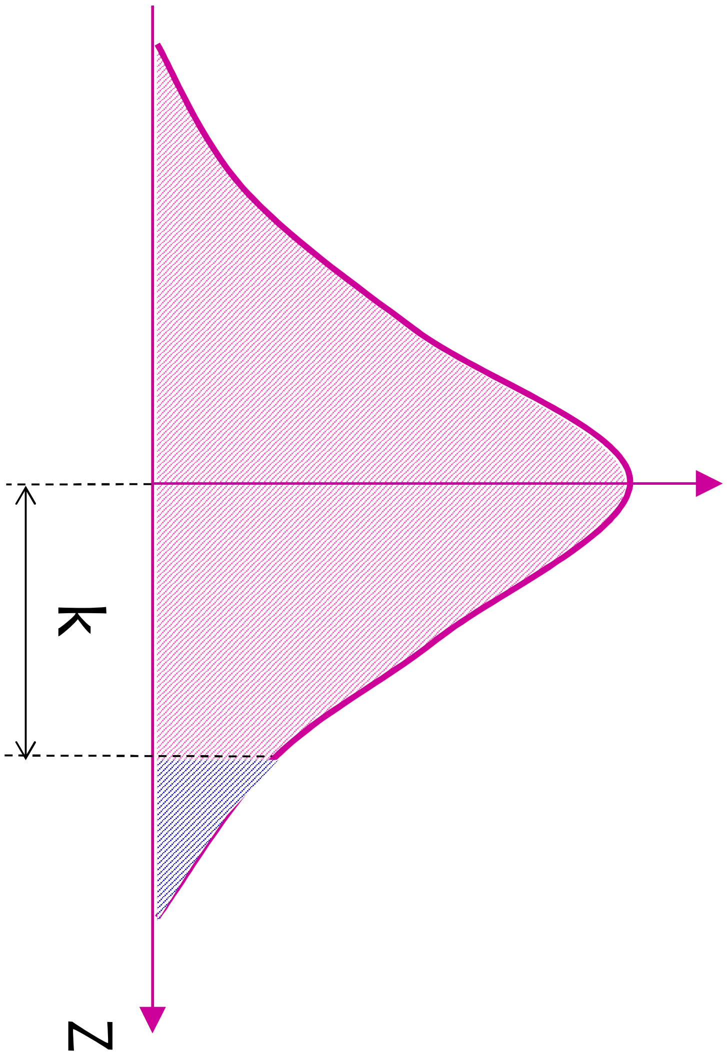
$$\text{if } \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_i^2 = \ldots = \sigma_d^2$$

$$then\ \sigma_{d\ in\ LT} = \sqrt{LT} \cdot \sigma_d$$

Safety stock



Safety stock



k	Area at the right side of k	Area at the left side: Service level
0	50%	50%
0.45	32.64	67.36
0.50	30.85	69.15
1.20	11.51	88.49
1.30	9.68	90.32
1.57	5.82	94.18
1.65	4.95	95.05
1.70	4.47	95.54
1.80	3.59	96.41
1.96	2.50	97.50
2.57	0.5	99.5

The Fixed-time period model

- Characteristics
  - Fixed-time between reviews T
  - Variable ordered quantity
  - Discontinuous control
  - Combined (or independent) entries reorder
- Reorders are placed at the (fixed) time of review T
- Objective
  - Identifying the quantity to re-order that allows the availability of each product to achieve a pre-defined level, called objective level (OL)

Availability = Physical inventory level + Orders in progress − Reserved stocks

Physical inventory level: How many pieces you can see and touch in the warehouse

Orders in progress: How many pieces have just been ordered upstream, but have not reached the warehouse yet

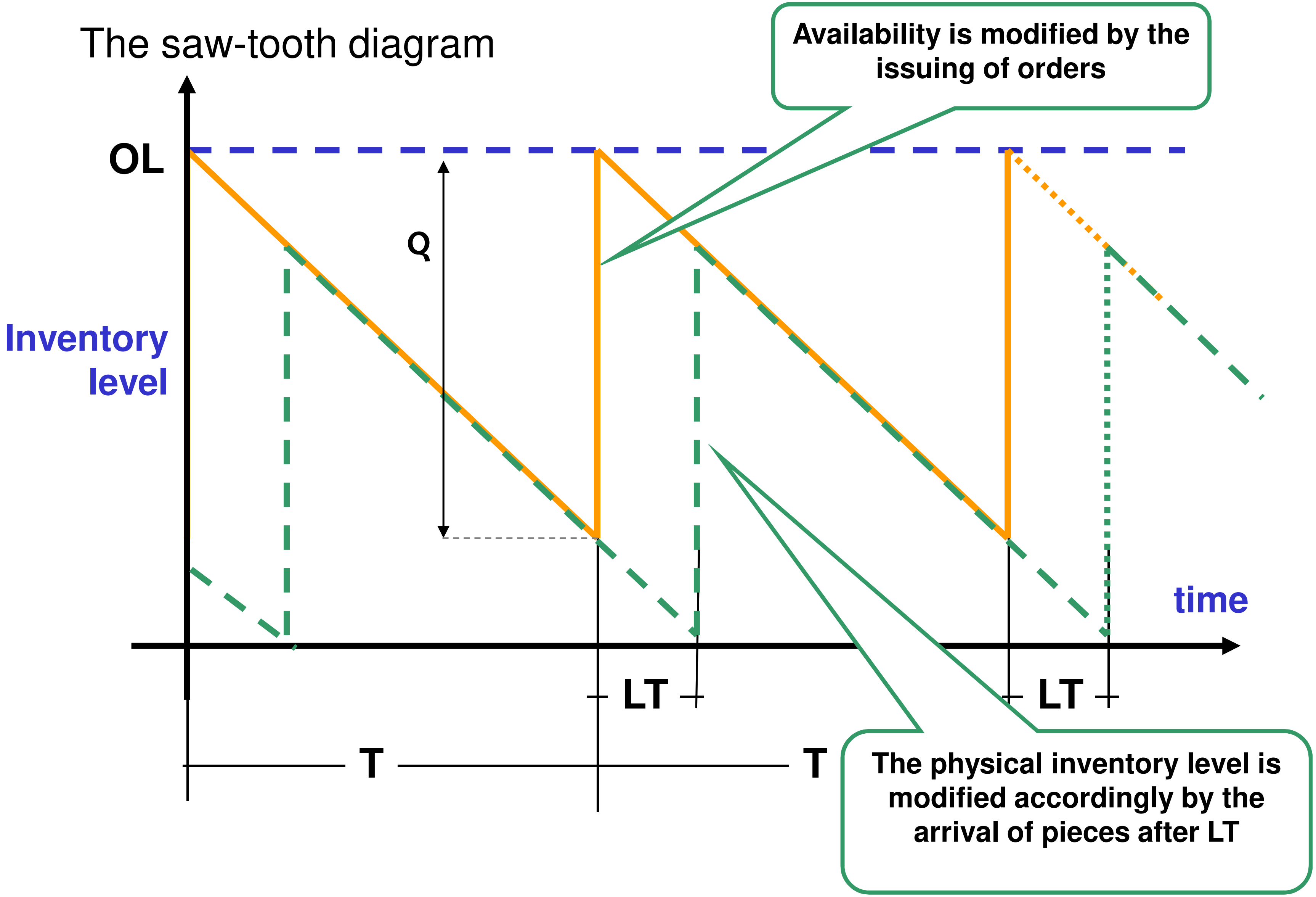
Reserved stocks: How many pieces have just been promised to a customer downstream, but that are still in the warehouse

Safety stocks: Pieces "untouchable", stored to protect the system against unpredictable events ("real" availability should not consider them)

- Hypotheses
  - The same as EOQ-ROP model

The Fixed-time period model

The saw-tooth diagram



The Fixed-time period model

The objective level

- It must cover a time fence as long as T + LT periods

Overall period to cover

Daily demand

Safety stocks (if any)

$$OL = (T + LT) \cdot D / H + SS$$

- Order quantity:

$$Q = OL - Availability$$

- In case of demand or lead time variability, safety stocks are:

$$SS = k\ \sigma_{d,T+LT}$$

The Fixed-time period model

- Safety stocks can be determined similarly to EOQ-ROP model. Nonetheless, the component derived from the demand variability has to cover a time fence as long as T + LT periods.

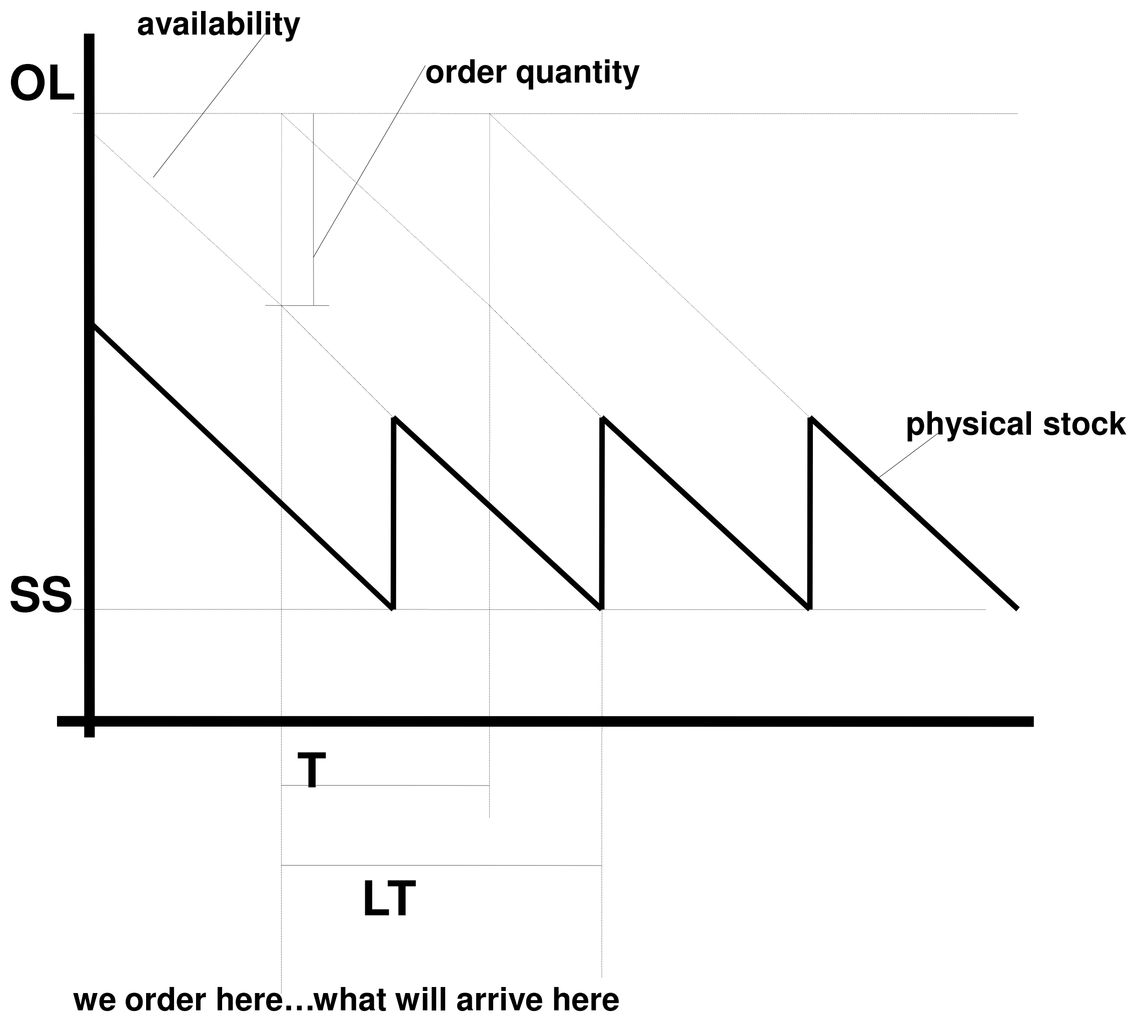
$$SS = k\sqrt{\sigma_d^2 \cdot (T + LT) + \sigma_{(T+LT)}^2 \cdot d^2}$$

with expected (if fixed-time period)

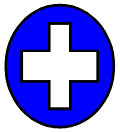
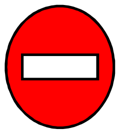
$$\sigma_{(T+LT)}^2 = \sigma_{LT}^2$$

- The average stock level is D/H* (T/2) + SS [units]

The Fixed-time period model



Comparison between the two models

	fixed quantity	fixed time
	- low average stock level (continuous control) - Minimization of relevant costs (OPTIMIZATION)	- easy joint (combined entries) re-orders planning - easy control of availiability level (periodic control)
	- difficult joint (combined entries) re-orders planning - many replenishment orders (also to the same supplier) - burdensome control of availiability level (continuous control)	- higher average stock level (periodic control)

Practice 1

Company Delta needs to purchase from a component supplier 3.500 units/year on average (with standard deviation of 65 units/year). Delivery Lead Time follows a Normal distribution with a mean of 10 days and standard deviation of 2.

If the cost of issuing an order is 3,5 €, the stock holding ownership rate is 10%/year, the unit price 4 €/unit, the Company (220 working days/year) has to determine the optimal lot (fixed quantity) to be purchased and the reorder point to get a service level of 97,5%.

Solution

$$EOQ = \sqrt{\frac{2 * o * D}{p * cm}} = \sqrt{\frac{2 * 3{,}5 * 3500}{4 * 0{,}1}} = 247{,}5 \text{ units} \quad (248)$$

The reorder point is:

ROP = d*LT +SS

$$SS = k * \sqrt{\sigma_d^{\,2} * LT + \sigma_{LT}^{\,2} * d^2}$$

N.B. the units of measure have to be consistent!

Solution

$$d * L = \frac{3500}{220} * 10 = 159 \text{ units}$$

$$SS = k * \sqrt{\sigma_d^{2} * LT + \sigma_{LT}^{2} * d^{2}} =$$

$$= 1{,}96 * \sqrt{(65)^2 * \left(\frac{10}{220}\right) + 2^2 * \frac{(3500)^2}{(220)^2}} = 68 \; \textit{units}$$

Solution

EOQ = 247,5 units, i.e., 247-248 units

ROP = d*L + SS = 159 + 68 = 227 units

Number of orders in a year = 3.500/247,5 = 14,1

Order issuing cost = 14,1 * 3,5 = 49,5 €/year

Solution

The average stock level is:

SS + EOQ/2 = 68 + 124 = 192 units

Stock holding cost = 192 * 4 * 0,1 = 76,8 €/year

N.B.:

Stock holding cost (without SS) =

= 124 * 4 * 0,1 = 49,5 €/year

(i.e., equal to order issuing cost!)

Practice 2

Company Alpha requires a calculation of the Economic Order Quantity (EOQ) and of the Re-Order Point (ROP) for product Beta so as to achieve a 95% service level, given the time series of demand during the last 10 weeks (see the table, where demand is expressed in thousands).

Week	1	2	3	4	5	6	7	8	9	10
Demand	20	30	25	35	30	25	30	20	35	25

In addition, the stock replenishment lead time accounts for 4 weeks, the ownership rate accounts for 12%/year and the set-up cost accounts for 300 € per set-up.

Finally, the accounting system provides the following data: labor cost equals to 1 €/unit; raw materials cost equals to 2 €/unit; energy cost equals to 1 €/unit; depreciation of machinery equals to 1.000.000 €/year; overhead costs equal to 250.000 €/year. The company works on a basis of 52 weeks per year (5 days per week).

Solution

Average demand equals to 27.500 pieces per week (1.430.000 per year; under stationary demand), while the standard deviation of demand equals to 5.401 pieces per week.

Parameter p (actually, it's a unit variable cost!) equals to 4 € per piece (1 + 2 + 1) (hp: all relevant – labor included -, but depreciation and overhead).

Stock holding ownership rate (c_m) equals to 0,12%/year
(N.B.: if we had accounted for units/week for demand, not always compounding effect can be overlooked for the sake of simplicity!)

As a consequence, EOQ accounts for (around) 42.279 pieces.

Safety stocks accounts for (around) 17.822 pieces (i.e. 1,65 x 5.401 x 2, the latter is namely the square root of 4)

ROP equals to 127.822 pieces.

Practice 3

Company Alpha operates on a basis of 52 weeks per year (5 days per week) and it produces product Gamma whose demand data are reported in the following table.

Week	1	2	3	4	5	6	7	8	9	10
Demand (pieces)	40	50	60	35	50	45	55	60	65	40

Gamma is replenished (review) every 10 days according to the Fixed-time model. Replenishment lead time accounts for 20 days, while the (yearly) ownership rate and the service level equal to 15% and 98% respectively.

In addition, the variable production cost accounts for 5 Euros per piece and set-up cost accounts for 10 Euros per set-up.

You are required to calculate:

- The target (objective) level of inventory
- The safety stocks level
- The average inventory level

Solution

Consider firstly safety stocks.

To this purpose:

- average demand equals to 50 pieces per week;
- standard deviation of demand equals to 10 pieces per week;
- service level factor (corresponding to 98%) accounts for (around) 2,06;
- overall lead time to cover accounts for 10 + 20 = 30 days (i.e. 6 weeks).

The safety stocks equal to 2,06 x 10 x sqrt(6) = 50,46 pieces.

The average inventory level equals to d x T / 2 + SS = 50 x 2/2 + 50,46 = 100,46 pieces.

As a consequence the target level equals to 300 pieces (i.e. 50 x 6) + SS = 350,46 pieces.