LIUC

Università Cattaneo

# International financial markets

## Why do interest rates change?

Luigi Vena

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# Today's agenda

- **Debt and bonds**
- Changes in interest rates
- Supply and demand in the bond market
- Yield curve
- Spot and forward rate

Mishkin, Eakins – ch. 3, 4 and 5

# Where we are…

## Present value

- The concept of present value (or discounted value) is based on the notion that a "*dollar today is worth more than a dollar tomorrow*"
- The process of computing today's value of monetary units received in the future is called discounting
- The present value permits us to compare the value of two instruments with very different timing of their cash flows

| $PV = FV/(1+i*t)$ | $PV = FV/(1+i)^t$ | $PV = FV * e^{-i*t}$ |
|---|---|---|
| **Simple rate** | **Compound rate** | **Continuous rate** |

# Debt

If a company wants to finance its business, it can alternatively:

- Issue new equity.
- Borrow money, promising to make regular interest payments and to repay the principal. But…

… who can be the lender?

  - A financial institution.
  - A bank
  - The households.

… what type of instruments can be used?

  - Mortgages
  - Bonds

# Bonds

## What are bonds?

Bonds are debt instruments* that represent cash flows payable during a specified time period.

The cash flows they represent are:

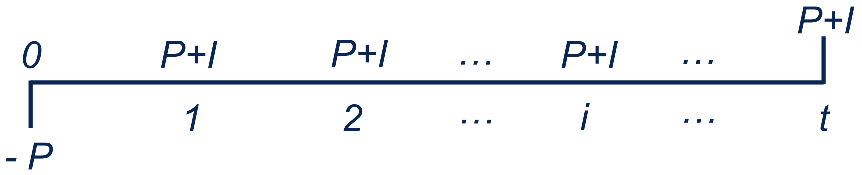
- Interest payments on the loan;
- The redemption of the loan.

*Bonds are portions of a single operation of indebtedness.

# Credit instruments

- Simple loan


- Fixed-payment loan




- Coupon bond


- Discount bond (ZCB)

# Bonds vs shares

Bonds are debt instruments, whereas shares are fraction of equity.

- By purchasing equity (shares), an investor becomes the owner of the corporation.
- By purchasing debt (bonds), an investor becomes a company's creditor.
- The primary advantage of being a creditor is that you have a higher claim on assets than shareholders do: that is, in the case of bankruptcy, bondholders will be paid before shareholders.
- However, bondholders do not share in the profits if a company does well – they are entitled only to the principal plus interest.

# Bonds key features

- Maturity:
  The maturity of a bond refers to the date that the debt will cease to exist.
- Term to maturity:
  The term to maturity of a bond is the number of periods after which the issuer will repay the obligation.
- Face value (a.k.a. Redemption value/ Par value):
  The face value is the amount that the issuer agrees to repay the bondholder on the maturity date.
- Coupon Rate:
  The coupon rate is the interest rate used to compute the coupon that the issuer agrees to pay each period. It can vary or be fixed.

# Focus: T-bill and Money Market

- The money market consists of very short term debt securities that usually are highly marketable.

- The most marketable securities of the money markets are the US Treasury-Bills (or simply T-bills).

- T-bills are issued at initial maturity of 1 month, 3 months, 6 months and 1 year.

# Bond categories

With respect to the *issuer*, bonds can be classified into:

- ✓ Sovereign governments B;
- ✓ Local government authorities B;
- ✓ Supranational bodies B (e.g. the World Bank);
- ✓ Corporate B

According to the interest payment *scheme*, bond can be classified into:

- ZCB – Zero Coupon Bond
- CB – Coupon Bond, in turn classified into:
  - ✓ **Fixed income bonds** which, in all period, pay the same coupon;
  - ✓ **Floating Rate Notes** which have, for each period, a different interest rate (and hence a different coupon) depending on a specific “underlying” interest rate.

# Zero Coupon Bond

The Zero Coupon Bond is a debt instruments which grants the lender a profit originated by the difference between:

- Issuing Price
- Face/Nominal Value

# Valuation of a ZCB

Since the interest rate of a Zero Coupon Bond is implicit in the difference between the issuing price and the par value…

EG, the interest rate of a 1Y ZCB.

$$r = \left(\frac{FV}{P}\right)^{\frac{1}{t}} - 1 \rightarrow r_{0,1} = \frac{FV}{P} - 1$$

# Price and interest rates: which relation?

- Calculate the interest rate of the following zero coupon bonds:
  – Face Value: 100; – Price: 98 – Maturity: 2 years
  – Face Value: 100; – Price: 95 – Maturity: 2 years
  – Face Value: 100; – Price: 90 – Maturity: 2 years
- Calculate the price of the following zero coupon bonds:
  – Face Value: 100; – r: 2% – Maturity: 3 years
  – Face Value: 100; – r: 5% – Maturity: 3 years
  – Face Value: 100; – r: 10% – Maturity: 3 years

# Fixed Income Bonds

- The fixed income bond grants the owner
  - the payment of a certain amount of interest each period and
  - the repayment of the principal at the maturity.
- In all periods (e.g. quarters, years…) the interest is computed on the same interest rate which remains fixed over time.

$$P = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^t} + \cdots + \frac{C+FV}{(1+r)^n}$$

- Where:
  - P is the fair price of the bond
  - C is the coupon
  - FV is the par (or face) value
  - r is the interest rate.

# Fixed Income Bond

Compute the fair price of the following three bonds:

- Interest paid on a yearly basis;
- Face value: 100$.
- Time to maturity: 3 years
- Interest rate, r=5%.
- Coupon rate:
  - 7% on annual basis, for the $1^{st}$ bond
  - 5% on annual basis, for the $2^{st}$ bond
  - 3% on annual basis, for the $3^{st}$ bond

# Bonds fair price

The fair price of a bond depends on two rate:

- Discount rate, *r*
- Coupon rate, *c*

More precisely, if

- $r > c \rightarrow P < FV$
- $r = c \rightarrow P = FV$
- $r < c \rightarrow P > FV$

# Bonds fair price

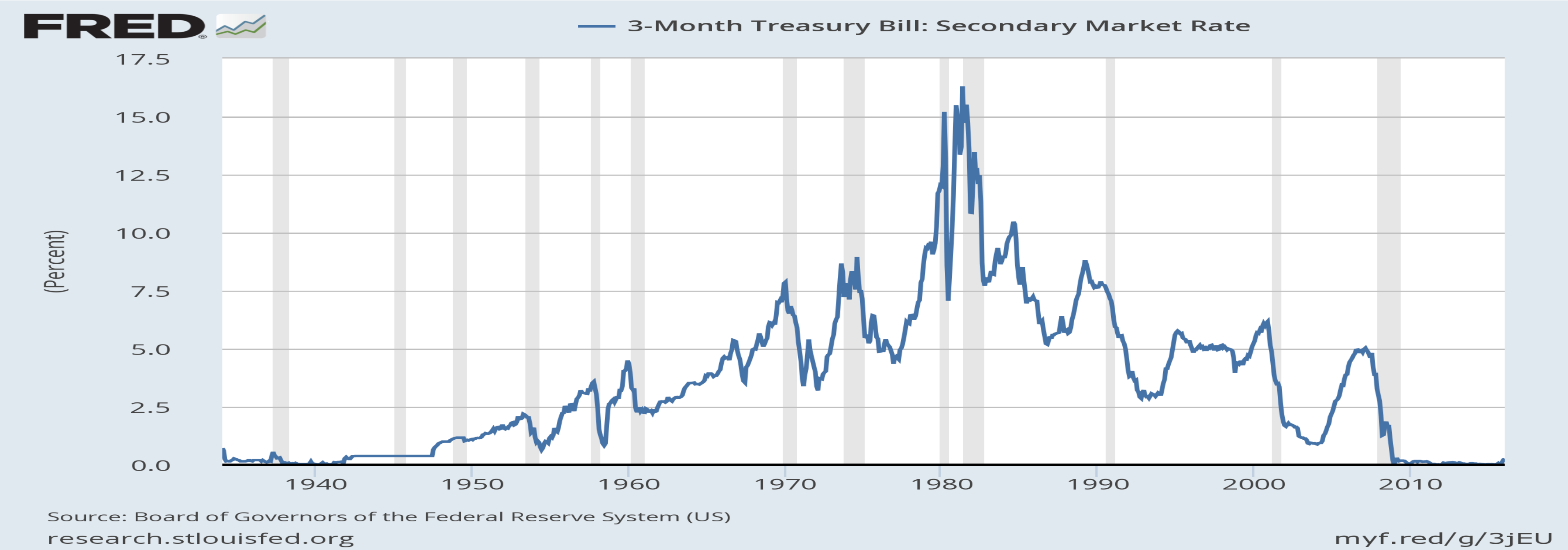
- To discount the different cash flows granted to the bondholder we used the same interest rate, r.
- However it is important to know that:
  - Interest rates may, and usually do, vary over time;
  - The 1y interest rate differs from the 2y interest rate. That is to say, each maturity has its interest rate.

# Today's agenda

- Debt and bonds
- **Changes in interest rates**
- **Supply and demand in the bond market**
- Yield curve
- Spot and forward rate

Mishkin, Eakins – ch. 3, 4 and 5

# Why do interest rates change?



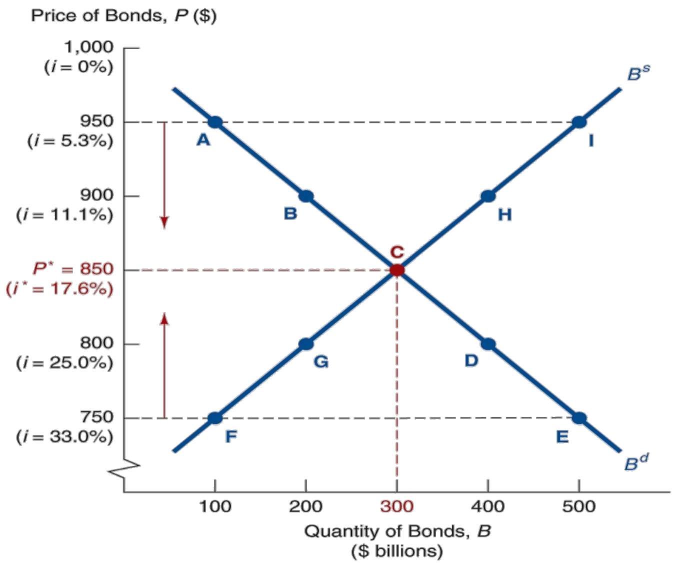

- We have seen that bond prices determine the level of the interest rates.
- Changes in bond prices determine changes in interest rates.
- If we explain why bond prices change, we can also explain why interest rates change.

# Supply and Demand in the Bond Market

- Bond prices determine the interest rates levels
- Bond prices represent equilibrium condition in the bond market
- That is to say, each price is formed when the demand meets the supply.
- As every demand curve, the one of the bonds represents the relationship between the demanded quantity and the price when all other economic variables are held constant.
- As every supply curve, the one of the bonds represents the relationship between the quantity supplied and the price when all other variables are held constant.

# Supply and Demand in the Bond Market

- Market equilibrium occurs when the amount that people are willing to buy (demand) equals the amount that people are willing to sell (supply) at a given price.



# Determinants of Asset Demand

- An asset is a piece of property that is a store of value. An individual who want to purchase an asset must take into account:
  - Wealth
  - Expected Return
  - Risk
  - Liquidity

# Determinants of Assets Demand

## Wealth

- Should the wealth increase, the demand for asset may increase, due to the more resources available with which to purchase assets.
- All else equal, the greater is the wealth the greater will be the quantity demanded for an asset.

## Expected Returns

- When we make a decision to buy an asset, we are influenced by what we expect the return on that asset to be.
- If a Walt Disney bond has a return of 15% half of the time and 5% the other half, its expected return is…

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## Expected Returns

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- If a Walt Disney bond has a return of 15% half of the time and 5% the other half, its expected return is…**10%**

# Determinants of Assets Demand

## Expected Returns

- You can imagine the expected return as the weighted average of all possible returns.
- The weights are the probabilities of occurrence

$$R^e = p_1 * r_1 + p_2 * r_2 + \cdots + p_n * r_n$$

where $R^e$ is the expected return, n the number of possible outcomes, $r_i$ the return in the i-th state of nature, and $p_i$ the probability of occurrence of the i-th return

# Determinants of Assets Demand

**Expected Returns**

- An increase in an asset’s expected return relative to that of an alternative assets, holding everything else unchanged, raises the quantity demanded of the asset. It happens if:
  - The expected return on the asset increases, while the ones of the other assets remain the same.
  - The expected return on alternative assets fall while the return on the asset remains constant.

# Determinants of Assets Demand

**Risk**

- The degree of risk (or uncertainty) of asset's returns also affects the demand for the asset.
- Consider the two alternative assets:
  - A – $r_1 = 15\%, p_1 = 50\%; r_2 = 5\%, p_2 = 50\%$
  - B – $r_1 = 10\%, p_1 = 100\%;$
- A has an expected return of 10%; B has a fixed certain return of 10%.
- However, A has uncertainty associated with its returns and, hence, a greater risk than B.
- To evaluate the asset's riskiness, we will use the standard deviation.

# Determinants of Assets Demand

## Risk

- The standard deviation (*sd* or $\sigma$) is a statistical measure that summarize how the observations are far from the mean value.
- We compute the *sd* in the following way:

$$\sigma = \sqrt{p_1 * (r_1 - R^e)^2 + p_2 * (r_2 - R^e)^2 + \cdots + p_n * (r_n - R^e)^2}$$

- The higher the standard deviation, the greater the risk of an asset.
- A risk-averse person prefers less risky assets, while a risk-lover person prefers most risky assets.
- The majority of people is risk-averse, especially when evaluating financial decisions.

# Determinants of Assets Demand

## Risk - Exercise

- Consider once again the two investments A and B:
  - A – $r_1 = 15\%, p_1 = 50\%; r_2 = 5\%, p_2 = 50\%$
  - B – $r_1 = 10\%, p_1 = 100\%;$
- Of the two assets, which is riskier?

# Determinants of Assets Demand

**Risk**

- The Expected Return of A is:

$$R_A^e = 50\% * 15\% + 50\% * 5\% = 10\%$$

- Its Standard Deviation is:

$$\sigma_A^e = \sqrt{50\%(15\% - 10\%)^2 + 50\%(5\% - 10\%)^2}$$

$$\sigma_A^e = \sqrt{0.25\%} = 5\%$$

- The Expected Return of B is:

$$R_B^e = 100\% * 10\% = 10\%$$

- Its Standard Deviation is:

$$\sigma_B^e = \sqrt{100\%(10\% - 10\%)^2}$$

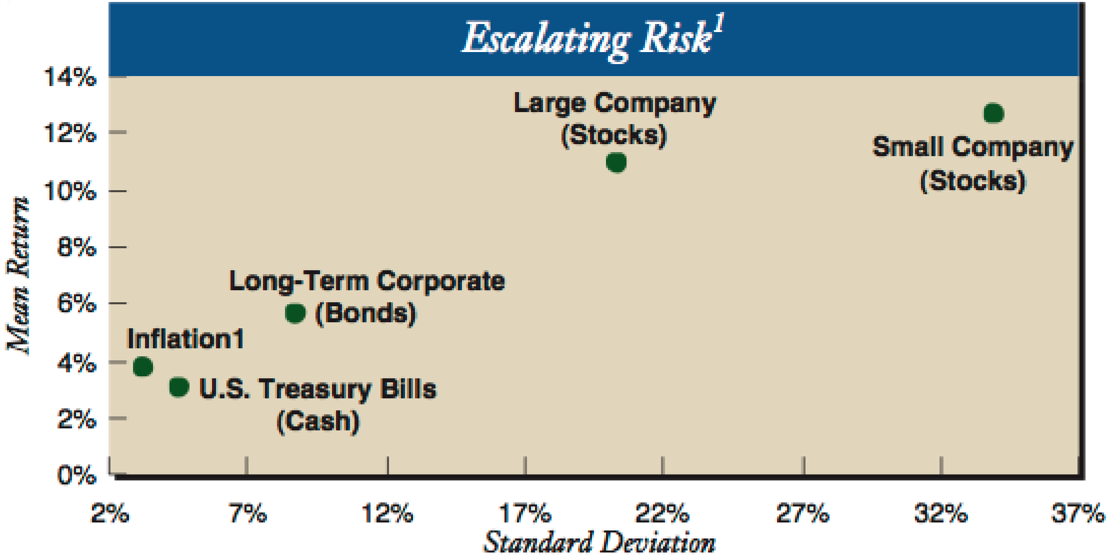
$$\sigma_B^e = \sqrt{0\%} = 0\%$$

# Determinants of Assets Demand

## Risk

$$\sigma_A^e > \sigma_B^e \rightarrow A\ is\ riskier\ than\ B$$

- Holding everything else constant, if an asset's risk rises relative to that of alternative assets, its quantity demanded will fall.



# Determinants of Assets Demand

**Liquidity**

- Another factor that affects the demand for an asset is how quickly it can be converted into cash at low cost.
- That is to say, how much the asset is liquid.
- All being equal, the more liquid an asset is relative to alternative assets, the more desirable it is, and the greater will be the quantity demanded.

# Determinants of Assets Demand

| Variable | Change in variable | Change in quantity demanded |
|---|---|---|
| Wealth | ↑ | ↑ |
| Expected return relative to other assets | ↑ | ↑ |
| Risk relative to other assets | ↑ | ↓ |
| Liquidity relative to other assets | ↑ | ↑ |

# Shifts in the Demand for Bonds

**Wealth**

- When economy is growing rapidly and wealth is increasing the demanded quantity at each price increases.
- As a consequence, the demand curve shifts to the right.

□ Economy ↑, wealth ↑, Bd ↑, Bd shifts to the right.

# Shifts in the Demand for Bonds

## Wealth

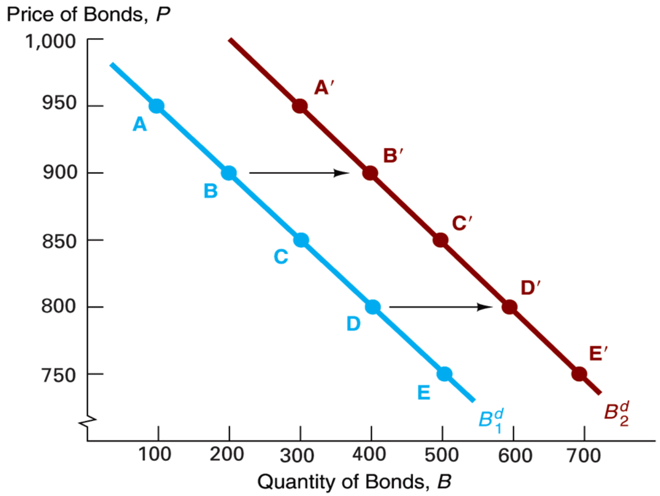



**Figure 4.2** Shift in the Demand Curve for Bonds

When the demand for bonds increases, the demand curve shifts to the right as shown.

# Shifts in the Demand for Bonds

## Expected Returns

- If people begin to think that interest rates would be higher next year than they had originally anticipated, the expected return today on long-term bonds would fall.
- Higher expected interest rates in the future:
  - Lower the expected return for long-term bond
  - Decrease the demand
  - Shift the demand curve to the left
- On the contrary lower expected interest rates in the future:
- Increase the demand for long-term bonds;
- Shift the demand curve to the right.

# Shifts in the Demand for Bonds

**Risk**

- An increase (decrease) in the riskiness of bonds causes the demand for bonds to fall (rise) and the demand curve to shift to the left (right).
- An increase in the riskiness of alternative assets causes the demand for bonds to rise and the demand curve to shift to the right.

**Liquidity**

- Increased liquidity of bonds results in an increased demand for bonds, and the demand curve shifts to the right.
- Increased liquidity of alternative assets lowers the demand for bonds and shifts the demand curve to the left.

# Shifts in the Supply of Bonds

## Expected Profitability of Investment Opportunities

- When economy grows, investment opportunities abound and consequently the supplied quantity of bonds.
- In a business cycle expansion (recession), the supply of bonds increases (decreases), and the supply curve shifts to the right (left).

## Expected Inflation

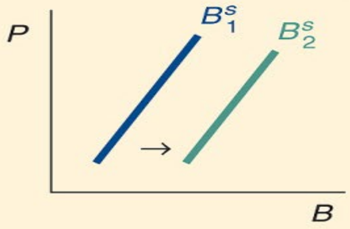
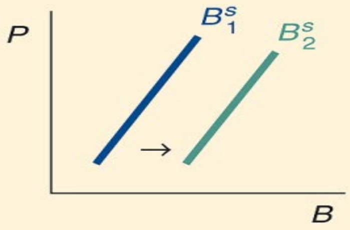
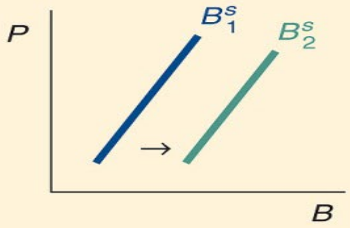
- More than looking at the nominal interest rate, investors look at the real interest rate.
- Real rates are nominal interest rates adjusted by the expected level of prices.
- The higher the inflation, the lower the real cost of borrowing money, the higher the supply of bonds.

# Shifts in the Supply of Bonds

**Government Budget**

- When the government revenues are less than its expenditure, the treasury must issue new bonds to finance the gap.
- Higher government deficits increase the supply of bonds and shift the supply curve to the right.
- On the other hand, government surpluses, decrease the supply of bonds and shift the supply curve to the left.

# Shifts in the Supply of Bonds

| Variable | Change in Variable | Change in Quantity Supplied at Each Bond Price | Shift in Supply Curve |
|---|---|---|---|
| Profitability of investments | ↑ | ↑ |   |
| Expected inflation | ↑ | ↑ |   |
| Government deficit | ↑ | ↑ |   |

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- **Yield curve**
- Spot and forward rate

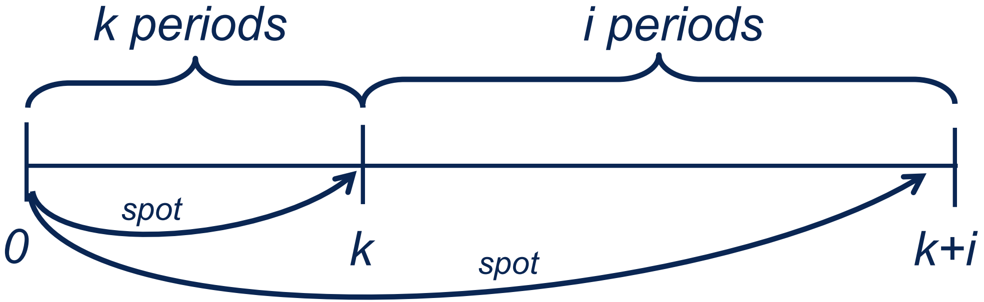
Mishkin, Eakins – ch. 3, 4 and 5

# Yield Curve

- Up to now, we have supposed a unique interest rate, that remains constant over time.
- However, it is opportune to recognize that the short term interest rates differ from the long term interest rates.
- Indeed, different bonds have different prices, according to the various maturity.
- Plotting the different yields against the maturity, we obtain the so-called *Yield Curve*.
- The yield curve shows the different interest rates corresponding to each maturity.
- Therefore, it shows the relationship between short and long term interest rates.

# Spot and forward contract

- All rates we have considered up to now are computed using spot contracts.
- A spot contract represents, like the US T-bill, a transaction for immediate delivery.
- Indeed, the purchaser of a T-bill is entitled to interest from the settlement date onwards.



# Yield curve

- In general, the spot rate (on annual basis!) of a spot contract which has
  - maturity (in years), *t*
  - terminal value or face value, $x_t$
  - price, $P = v(0, x_t)$

is the following one: $s(o,t) = \left(\frac{x_t}{v(0,x_t)}\right)^{\frac{1}{t}}$

# Yield Curve

- Exercise
- Let's consider the following situation:
  - 1y ZCB – Price=98 and Face Value=100
  - 2y ZCB – Price=94 and Face Value=100
- The former has a interest rate of (…) indicated with (…) while the latter has a interest rate of (…) indicated with (…).

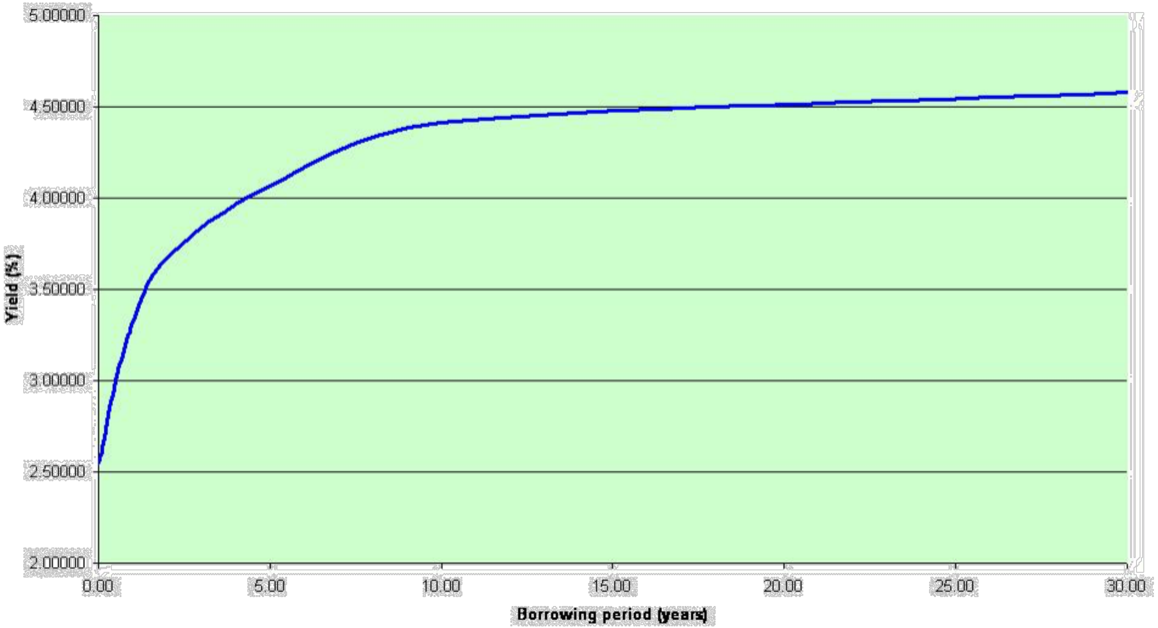
# Yield Curve

- Using the data in the table below, graph the term structure of interest rates.

| Security | Time to Maturity | Face Value | Price | Interest Rate |
|---|---|---|---|---|
| ZCB | 1m | 100 | 99.9585 | s(0,1)=… |
| ZCB | 2m | 10 | 9.9876 | s(0,2)=… |
| ZCB | 3m | 50 | 49.8511 | s(0,3)=… |
| ZCB | 6m | 65 | 64.3596 | s(0,6)=… |
| ZCB | 9m | 1 | 0.9781 | s(0,9)=… |
| ZCB | 12m | 1000 | 956.9378 | s(0,12)=… |

# Yield Curve

- The zero coupon bond yield curve, as all the other yield curve, plots the spot rates corresponding to the different maturity.



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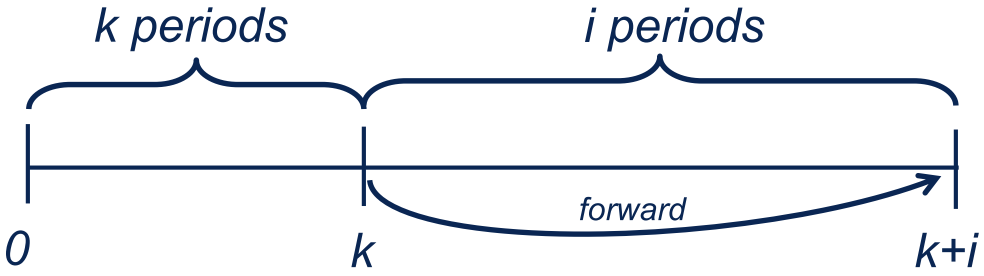
Mishkin, Eakins – ch. 3, 4 and 5

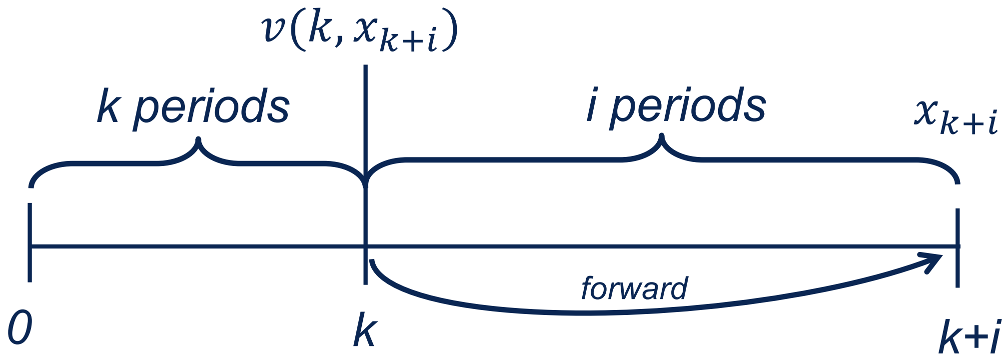
# Spot and forward contract

- Financial markets permits also forward contract.
- Unlike the spot contract, the forward one refers to a contract in which the parties to the trade agree today to exchange a security for cash at a future date, but at a price agreed today.
- Therefore, as can be inferred from the definition, a forward rate is the interest rate set today which will be applied to a contract at a future date.



# Spot and forward contract

- Suppose that today (time 0), we decide the following forward contract:



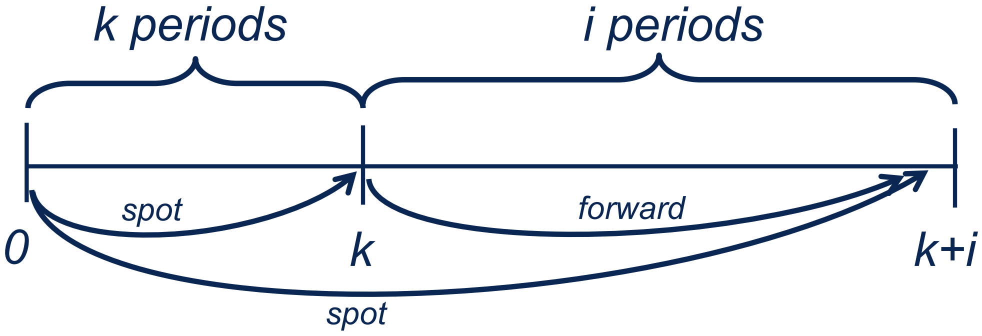

Knowing that: $r = \left(\frac{\mathrm{FV}}{\mathrm{PV}}\right)^{\frac{1}{t}} - 1 \;\rightarrow\; f(k, k+i) = \left(\frac{x_{k+i}}{v(k, x_{k+i})}\right)^{\frac{1}{i}} - 1$

# Spot and forward contracts

- From the spot contracts and the spot rates one can infer the forward rates implicit in the market.
- In order to avoid any possibility of arbitrage, the following relation must be respected:

$$[1 + s(o, k)]^k * [1 + f(k, k + i)]^i = [1 + s(0, k + i)]^{k+i}$$

# Spot and forward contracts




- Therefore *f(k, k+i)* can be computed using the following relation.

$$f(k, k+i) = \left(\frac{[1+s(0,k+1)]^{k+i}}{[1+s(0,k)]^{k}}\right)^{\frac{1}{i}} - 1$$

- Where *f(k, k+i)* is the interest rate agreed today for a contract which will last for *i* periods, from *k* to *k+i*.

# Spot rate, forward rates and arbitrage

- When the relation above mentioned is not satisfied, the market offers a possibility of arbitrage.
- An arbitrage is a strategy which ensures non-negative cash flows and, at least, one cash flow strictly positive.
- In other words, the arbitrage consists of the simultaneous sell and purchase of different securities, in order to achieve a “free meal”

Example.

- Suppose that the market offers the following investment opportunity:

a) A spot contract with maturity k

b) A spot contract with maturity k+i

c) A forward contract with settlement date k and maturity k+i.

# Spot rate, forward rates and arbitrage

- If the spot and forward rates do not satisfy the equilibrium condition:

$$[1+f(k,k+1)]^i \neq \frac{[1+s(0,k+i)]^{k+i}}{[1+s(s0,k)]^k}$$

That is, for example:

$$[1+f(k,k+1)]^i < \frac{[1+s(0,k+i)]^{k+i}}{[1+s(s0,k)]^k}$$

- …One can
  - Invest $1 for *k+i* periods at the spot rate s(0,k+i)*;*
  - Borrow $1 for *k* periods at the interest rate s(0,k)*;*
  - Borrow $**[1+s(0,k)]$^{k}$** at the forward rate f(k,k+i), for i periods, from k to k+i.

# Spot rate, forward rates and arbitrage

- Strategy payouts:

| Time / Contracts | 0 | k | k+i |
|---|---|---|---|
| a) | -1 | - | $+[1+s(0,k+i)]^{k+i}$ |
| b) | 1 | $-[1+s(0,k)]^{k}$ | - |
| c) | - | $+[1+s(0,k)]^{k}$ | $-[1+s(0,k)]^{k}*[1+f(k,k+i)]^{i}$ |
| TOTAL | 0 | 0 | >0 |

# Spot rate, forward rates and arbitrage

- In word, one must be indifferent between the two following choices:
  - invest first in a 1Y ZCB and secondly, as the ZCB goes to maturity, reinvest in another 1Y ZCB which features have been decided in 0;
  - Investing directly in a 2Y ZCB.
- If the markets do not respect the preceding relation one can have a risk-free meal and, in other word, implement an arbitrage.

# Spot rate, forward rates and arbitrage

- Consider the following situation:
  - 1y ZCB – Price=98 and Face Value=100
  - 2y ZCB – Price=94 and Face Value=100
- Considering the two spot rates s(0,1)=2.0408% and s(0,2)=3.1421%, there is one and only one forward rate which does not permit any risk free arbitrage.
- This forward rate f(1,2)=….

# Spot rate, forward rates and arbitrage

- Consider again the following spot rates:
  - s(0,1y)=2.0408% (Spot price=98)
  - s(0,2y)=3.1421% (Spot price=94)
- Suppose that there is a forward contract with the following features:
  - Forward price = 97.
  - Face value = 100
  - Settlement date/period: 1y; Maturity: 2y
- Show how one can realize a risk free arbitrage, if any.

# Bonds pricing……was the formula correct?

When we priced the bond, we used a constant interest rate.

However, as we have just seen, the interest rate may vary over time.

Therefore, exploiting the term structure and the relation between maturities and interest rates we can re-write the bond pricing formula

$$P = \frac{C_1}{[1+s(0,1)]^1} + \frac{C_2}{[1+s(0,2)]^2} + \cdots + \frac{C_t}{[1+s(0,t)]^t} + \cdots + \frac{P+C_n}{[1+s(0,n)]^n}$$

where:

*C* is the coupon;
*s(0, t)* is the spot rate referred to the $t^{th}$ maturirity;
*FV* is the principal (/face value);
*n* is the bond maturity.