PROVA PARZIALE - A

$\boxed{\text{Ex 1}}$

$$\int_0^1 \frac{\ln(x+2)}{(x+1)^2}\,dx = \int_0^1 \underbrace{\ln(x+2)}_{g(x)} \underbrace{\frac{1}{(x+1)^2}}_{f'(x)}\,dx$$

$$f'(x) = \frac{1}{(x+1)^2} = (x+1)^{-2}$$

$$f(x) = \frac{1}{1+(-2)}(x+1)^{1+(-2)} = \frac{-1}{(x+1)}$$

$$g(x) = \ln(x+2)$$

$$g'(x) = \frac{1}{(x+2)}$$

$$\int_0^1 \frac{\ln(x+2)}{(x+1)^2}\,dx = -\frac{\ln(x+2)}{(x+1)}\Bigg|_0^1 - \int_0^1 -\frac{1}{(x+1)}\cdot\frac{1}{(x+2)}\,dx$$

$$= -\frac{\ln(x+2)}{(x+1)}\Bigg|_0^1 + \int_0^1 \boxed{\frac{1}{(x+1)}\cdot\frac{1}{(x+2)}}\,dx$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} =$$

$$= \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{(A+B)x+(2A+B)}{(x+1)(x+2)}$$

$$\begin{cases} A+B=0 \\ 2A+B=1 \end{cases} \quad \begin{cases} A=-B \\ -2B+B=1 \end{cases} \quad \begin{cases} A=-B \\ -B=1 \end{cases}$$

$$\begin{cases} A=1 \\ B=-1 \end{cases}$$

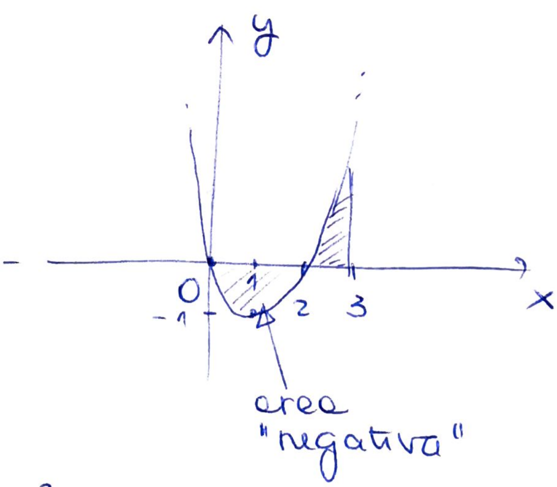
$$= -\frac{\ln(x+2)}{(x+1)}\Big|_0^1 + \int_0^1 \frac{1}{(x+1)}\,dx - \int_0^1 \frac{1}{(x+2)}\,dx =$$

$$= -\frac{\ln(x+2)}{(x+1)}\Big|_0^1 + \ln(x+1)\Big|_0^1 - \ln(x+2)\Big|_0^1 =$$

$$= -\frac{\ln(3)}{2} + \ln(2) + \ln(2) - \ln(3) + \ln(2) =$$

$$= \boxed{-\frac{3}{2}\ln(3) + 3\ln(2)}$$

Ex2 $f(x) = x^2 - 2x$



$f(x) \geq 0$ $\quad x^2 - 2x \geq 0$

$x(x-2) \geq 0$

$V_x = \frac{-b}{2a} = \frac{2}{2} = 1$

$\begin{cases} x \geq 0 \\ x \geq 2 \end{cases} \Rightarrow$

	0		2	
−		+		+
−		−		+
+		−		+

$$\Rightarrow -\int_0^2 (x^2-2x)\,dx + \int_2^3 (x^2-2x)\,dx = -\left(\frac{1}{3}x^3 - x^2\right)\Big|_0^2 + \left(\frac{1}{3}x^3 - x^2\right)\Big|_2^3 =$$

riporto la parte negativa in positivo

$$= -\left(\frac{8}{3}-4\right)+\left(\cancel{\frac{27}{3}}^{9}-\cancel{9}-\frac{8}{3}+4\right)=$$

$$= -\frac{8}{3}+4-\frac{8}{3}+4 = -\frac{16}{3}+8 = \frac{-16+24}{3} = \boxed{\frac{8}{3}}$$

$\boxed{\text{Ex 3}}$

$$\int_1^{\infty} \frac{e^{-4x^2}}{x}\,dx = \int_1^{\infty} \frac{1}{e^{4x^2}\cdot x}\,dx$$

CE: $x \neq 0 \Rightarrow$ non nel dominio di integrazione

$$\boxed{\frac{1}{e^{4x^2}} < \frac{1}{x}}$$

$\frac{1}{e^{4x^2}} > 0 \quad \forall\, x > 1$

$\frac{1}{x} > 0 \quad \forall\, x > 1$

$\Downarrow$

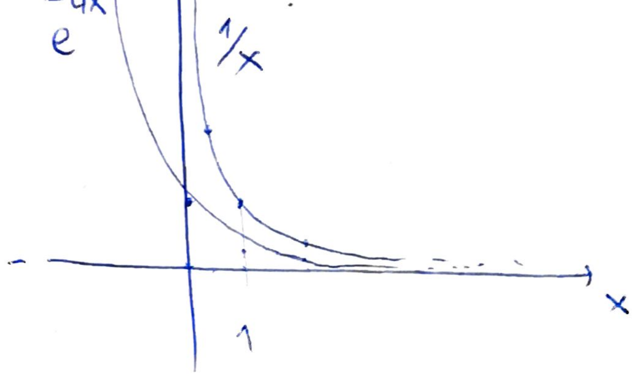
per CONFRONTO

$$\boxed{\frac{1}{e^{4x^2}\cdot x}} < \frac{1}{x\cdot x} = \frac{1}{x^2}$$

ma $\int_1^{+\infty} \frac{1}{x^2}\,dx$ converge perchè $\alpha > 1$

$\Downarrow$

CONVERGE per il CRITERIO del CONFRONTO



$\boxed{\text{Ex } 4}$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad A = \begin{bmatrix} a & 1 & 0 \\ a^2 & -2 & 1 \\ 0 & 1 & a \end{bmatrix}$$

$$\vec{y} = A\vec{x} = \begin{bmatrix} a & 1 & 0 \\ a^2 & -2 & 1 \\ 0 & 1 & a \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} a \\ a^2-1 \\ -a \end{bmatrix}$$

$$\vec{y}^T \cdot \vec{x} = \begin{bmatrix} a & a^2-1 & -a \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = [a+a] = 2a$$

$2a \neq 0 \Rightarrow a \neq 0$ i 2 ~~p~~ vettori sono perpendicolari

$$\det A = -1 \begin{vmatrix} a & 0 \\ a^2 & 1 \end{vmatrix} + a \begin{vmatrix} a & 1 \\ a^2 & -2 \end{vmatrix} =$$

$$= -(a) + a(-2a - a^2) =$$

$$= -a - 2a^2 - a^3$$

$$\det A \neq 0 \Rightarrow -a(1 + 2a + a^2) \neq 0$$

$$-a(a+1)^2 \neq 0 \quad \begin{cases} \rightarrow a \neq 0 \\ \rightarrow a \neq -1 \end{cases}$$

per $a = 1 \Rightarrow \det A = -1(1+1)^2 = -1(4) = -4$

$$\Rightarrow \det(A^{-1}) = \frac{1}{\det A} = -\frac{1}{4}$$

$$\Rightarrow \det(A^3) = \det(A) \cdot \det(A) \cdot \det(A) = -64$$

Ex 5

$$A = \begin{bmatrix} 2 & -\beta^2 & -6 \\ -1 & 2 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ \beta \end{bmatrix}$$

rank(A) $\rightarrow$ [2] det $\neq 0$

$$\begin{bmatrix} 2 & -\beta^2 \\ -1 & 2 \end{bmatrix} \rightarrow \det = 4 \cancel{\ast} \beta^2 = (2-\beta)(2+\beta)$$

$$\det \neq 0 \Leftrightarrow \beta \neq 2, \ \beta \neq -2$$

$$\begin{bmatrix} 2 & -6 \\ -1 & 3 \end{bmatrix} \rightarrow \det = 6 - 6 = 0$$

$$\Rightarrow \text{rank}(A) = 2 \Leftrightarrow \beta \neq 2 \ \vee \ \beta \neq -2$$

$$\text{rank}(A|b) = \begin{bmatrix} 2 & 2 \\ -1 & \beta \end{bmatrix} \Rightarrow \det = 2\beta + 2$$

$$\det = 2(\beta + 1) \neq 0 \Leftrightarrow \beta \neq -1$$

$\Rightarrow$ se $\beta = 2 \vee \cancel{\beta} \ \beta = -2 \Rightarrow$ rank $(A) = 1$
rank $(A|b) = 2$

$\Rightarrow$ il sistema e' impossibile

$\Rightarrow$ se $\beta \neq 2 \vee \beta \neq -2 \Rightarrow$ rank$(A) =$ rank$(A|b) = 2$

SISTEMA possibile e INDET.

$\infty^{3-2} = \infty^1$ soluzioni

$\beta = 1$

$$A = \begin{bmatrix} 2 & -1 & -6 \\ -1 & 2 & 3 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -6 \\ 3 \end{bmatrix} x_3$$

$$x_1 = \frac{\begin{vmatrix} 2+6x_3 & -1 \\ 1-3x_3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{2(2+6x_3)+1-3x_3}{4-1} =$$

$$= \frac{4+12x_3+1-3x_3}{3} =$$

$$= \frac{5+9x_3}{3} = \frac{5}{3} + 3x_3$$

$$x_2 = \frac{\begin{vmatrix} 2 & 2+6x_3 \\ -1 & 1-3x_3 \end{vmatrix}}{3} = \cancel{\frac{-1+3x_3-2(2+6x_3)}{3}} =$$

$$= \frac{2(1-3x_3)+2+6x_3}{3} =$$

$$= \frac{2-6x_3+2+6x_3}{3} = \frac{4}{3}$$

$$\vec{x} = \left[\frac{5}{3} + 3x_3 \,;\, \frac{4}{3} \,;\, x_3 \right]$$

$\boxed{Ex6}$

$$A = [\vec{u}, \vec{v}, \vec{w}] = \begin{bmatrix} 1 & 1 & 3 \\ -k & k^2 & k \\ -1 & 2 & 2 \end{bmatrix}$$

$$\det A = \cancel{-1\begin{vmatrix} 1 & 1 \\ -k & k^2 \end{vmatrix}} - 2\begin{vmatrix} 1 & 3 \\ -k & k \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ -k & k^2 \end{vmatrix}$$

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$$-1\begin{vmatrix} 1 & 3 \\ k^2 & k \end{vmatrix}$$

$$= -1(k - 3k^2) - 2(k + 3k) + 2(k^2 + k) =$$

$$= -k + 3k^2 - \cancel{2k} - 6k + 2k^2 + \cancel{2k} =$$

$$= 5k^2 - 7k$$

$$\det A \neq 0 \Rightarrow 5k^2 - 7k \neq 0$$

$$k(5k - 7) \neq 0 \to k \neq 0$$

$$\to k \neq \frac{7}{5}$$