PROVA PARZIALE - B

$\boxed{\text{Ex 1}}$

$$\int_0^1 \frac{\ln(x+1)}{(x+2)^2}dx = \int_0^1 \underbrace{\ln(x+1)}_{g(x)} \underbrace{\frac{1}{(x+2)^2}}_{f'(x)} dx$$

$$g(x) = \ln(x+1) \qquad f'(x) = (x+2)^{-2}$$

$$g'(x) = \frac{1}{x+1} \qquad f(x) = -\frac{1}{(x+2)}$$

$$\Rightarrow = -\frac{\ln(x+1)}{(x+2)}\Bigg|_0^1 - \int_0^1 \frac{1}{(x+1)}\left(-\frac{1}{(x+2)}\right)dx =$$

$$= -\frac{\ln(x+1)}{(x+2)}\Bigg|_0^1 + \int_0^1 \frac{1}{(x+1)}\frac{1}{(x+2)}dx$$

$$\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)} =$$

$$= \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{(A+B)x+(B+2A)}{(x+1)(x+2)}$$

$$\begin{cases} A+B=0 \\ B+2A=1 \end{cases} \quad \begin{cases} A=-B \\ -B=1 \end{cases} \quad \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$\Rightarrow \int_0^1 \frac{1}{(x+1)}dx - \int_0^1 \frac{1}{(x+2)}dx$$

$$= -\frac{\ln(x+1)}{(x+2)}\Big|_0^1 + \ln(x+1)\Big|_0^1 - \ln(x+2)\Big|_0^1 =$$

$$= -\frac{\ln(2)}{3} + \ln(2) - \ln(3) + \ln(2) =$$

$$= \frac{5}{3}\ln(2) - \ln(3)$$

$\boxed{Ex\ 2} \Rightarrow$ come prova A

$\boxed{Ex\ 3} \Rightarrow$ come prova A

$\boxed{Ex\ 4}$

- $$\vec{y} = A\vec{x} = \begin{bmatrix} a^{\cancel{2}} & a^2 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & a \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ -a \end{bmatrix}$$

- $$y \cdot x = a + 0 + a = 2a$$

  $$\vec{y} \cdot \vec{x} \neq 0 \Leftrightarrow 2a \neq 0 \Rightarrow a \neq 0$$

- $\det A \neq 0$

$$A = \begin{bmatrix} a & a^2 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & a \end{bmatrix}$$

$$\det A = a \begin{vmatrix} -2 & 1 \\ 1 & a \end{vmatrix} - 1 \begin{vmatrix} a^2 & 0 \\ 1 & a \end{vmatrix} =$$

$$= a(-2a-1) - a^3 =$$

$$= -2a^2 - a - a^3 = -a(a^2+2a+1)$$

$$-a \neq 0 \rightarrow a \neq 0$$

$\det A \neq 0$

$$a^2 + 2a + 1 \neq 0 \rightarrow (a+1)^2 \neq 0$$

$$a \neq -1$$

• $\det A = -a(a+1)^2$

$a = 1 \Rightarrow \det A = -1\,(2)^2 = -4$

$\det(A^{-1}) = \frac{1}{\det A} = -\frac{1}{4}$

$\det(A^3) = (\det A)^3 = (-4)^3 = -64$

$\boxed{Ex\ 5}$

$A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & -\beta^2 & -6 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} \beta \\ 2 \end{bmatrix}$

$A \in (2,3) \qquad \text{rank}(A) \leq \min\{2,3\} = 2$

$M_1 \to [-1] = -1 \neq 0$

$M_2 \to \begin{bmatrix} -1 & 2 \\ 2 & -\beta^2 \end{bmatrix} \to \det = \beta^2 - 4$

$\begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix} \to \det = 6 - 6 = 0$

$\Rightarrow \text{rank}(A) = 2 \iff \beta^2 - 4 \neq 0$

$(\beta + 2)(\beta - 2) \neq 0$

$\beta \neq -2 \vee \beta \neq 2$

$\text{rank}(A|b)$?

$A|b = \begin{bmatrix} -1 & 2 & 3 & \beta \\ 2 & -\beta^2 & -6 & 2 \end{bmatrix}$

Considero il nuovo minore di ordine 2

$$\begin{bmatrix} -1 & \beta \\ 2 & 2 \end{bmatrix} \rightarrow \det = -2-2\beta$$
$$= -2(1+\beta)$$

$$\det \neq 0 \Rightarrow \beta \neq -1$$

Quindi:

- Se $\beta = 2 \lor \beta = -2$

$\mathrm{rank}(A) = 1$
$\mathrm{rank}(A|b) = 2$ $\Rightarrow$ sistema impossibile

- Se $\beta \neq 2 \lor \beta \neq -2$

$\mathrm{rank}(A) = \mathrm{rank}(A|b) = 2$

$\Rightarrow$ sistema possibile con $\infty^{3-2} = \infty^1$ soluzioni

- per $\beta = 1$

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & -1 & -6 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\downarrow \bar{A}$

$$\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ -6 \end{bmatrix} x_3$$

$$\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1-3x_3 \\ 2+6x_3 \end{bmatrix}$$

$$x_1 = \frac{\det(B_1)}{\det \breve{A}} = -\frac{(5+12x_3)}{-3} = \frac{5+12x_3}{3} = \frac{5}{3} + 4x_3$$

$$\breve{A} = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix} \rightarrow \det \breve{A} = 1 - 4 = -3$$

$$B_1 = \begin{bmatrix} 1-3x_3 & 2 \\ 2+6x_3 & -1 \end{bmatrix} \qquad \det B_1 = -(1-3x_3) - 2(2+6x_3) =$$

$$= -1 + 3x_3 - 4 - 12x_3 =$$

$$= -5 - 12x_3$$

$$x_2 = \frac{\det(B_2)}{\det \breve{A}} = \frac{-4}{-3} = \frac{4}{3}$$

$$B_2 = \begin{bmatrix} -1 & 1-3x_3 \\ 2 & 2+6x_3 \end{bmatrix} =$$

$$= -2 - 6x_3 - 2(1-3x_3) =$$

$$= -2 - 6x_3 - 2 + 6x_3 = -4$$

$$\vec{x} = \left[\frac{5}{3} + 4x_3 ; \frac{4}{3} ; x_3 \right] \qquad x_3 \in \mathbb{R}$$

- $A\vec{x} = 0$

se $\beta = 2 \vee \beta = -2$

$\text{rank}(A) = \text{rank}(A|b) = 1$

sistema possibile con $\infty^{3-1} = \infty^2$ soluzioni

se $\beta \neq 2 \vee \beta \neq -2$

$\text{rank}(A) = \text{rank}(A|b) = 2$

sistema possibile con $\infty^{3-2} = \infty^1$ soluzioni

$\boxed{\text{Ex } 6}$

$$A = \begin{bmatrix} 3 & 1 & 1 \\ k & -k & k^2 \\ 2 & -1 & 2 \end{bmatrix}$$

Vettori lin. INDIP.
SSE
$\det A \neq 0$

$$\det A = 3 \begin{vmatrix} -k & k^2 \\ -1 & 2 \end{vmatrix} - 1 \begin{vmatrix} k & k^2 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} k & -k \\ 2 & -1 \end{vmatrix} =$$

$$= 3(-2k + k^2) - (2k - 2k^2) + (-k + 2k) =$$

$$= -6k + 3k^2 - 2k + 2k^2 + k =$$

$$= 5k^2 - 7k$$

$\det A \neq 0 \iff 5k^2 - 7k \neq 0$

$k(5k - 7) \neq 0 \to k \neq 0$

$\to k \neq \frac{7}{5}$