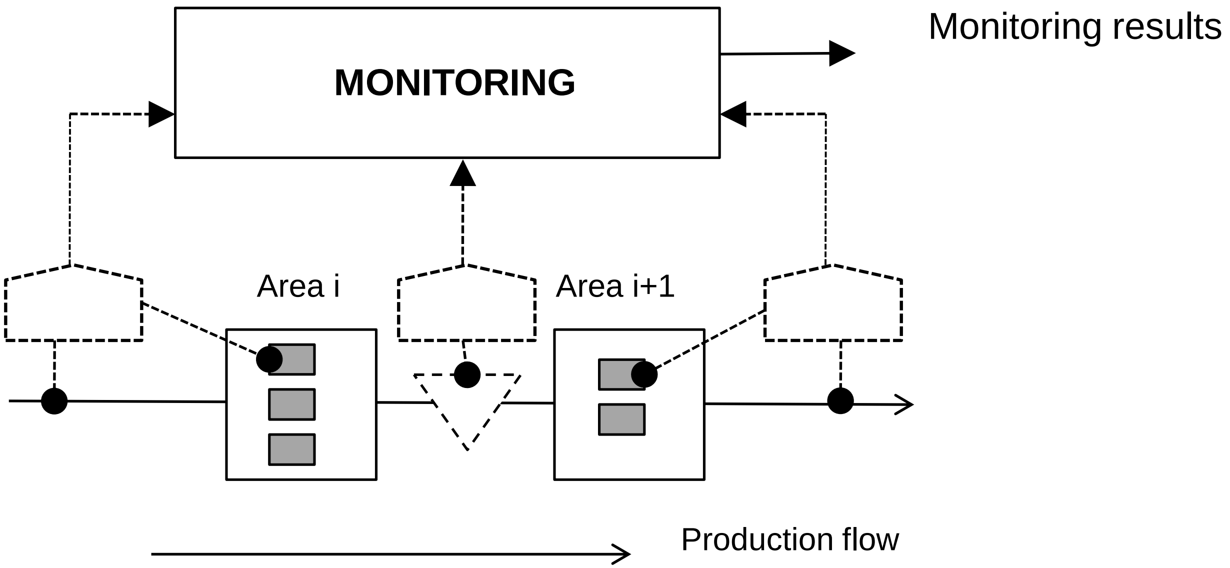
Performance Analysis in Manufacturing System– Little's Law

Performance analysis

- Performance analysis (or monitoring) related to a system composed of independent resources, (e.g. manufacturing system that produces a set of different pieces, each of them asking for system resources on the basis of its own cycle), is the analysis of the relationships among the different performance variables of the system, given the boundaries conditions.
- Performance analysis is used both for making predictions on new system behaviour, or for checking the performances for existing systems.
- There are different performance analysis methods, depending on their complexity and approximation level. For example simulation is very precise and detailed, but very expensive to apply.
- Little’s Law and Throughput Diagrams (or analysis) are two of the most simple methods.

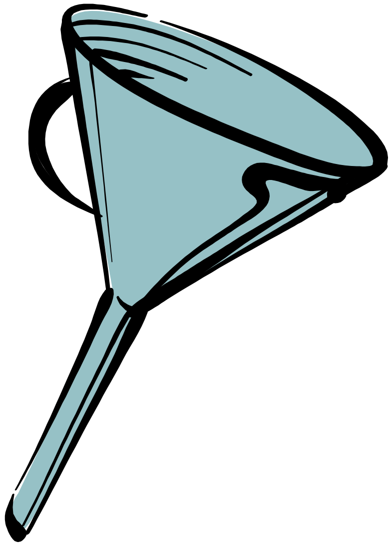
Performance analysis

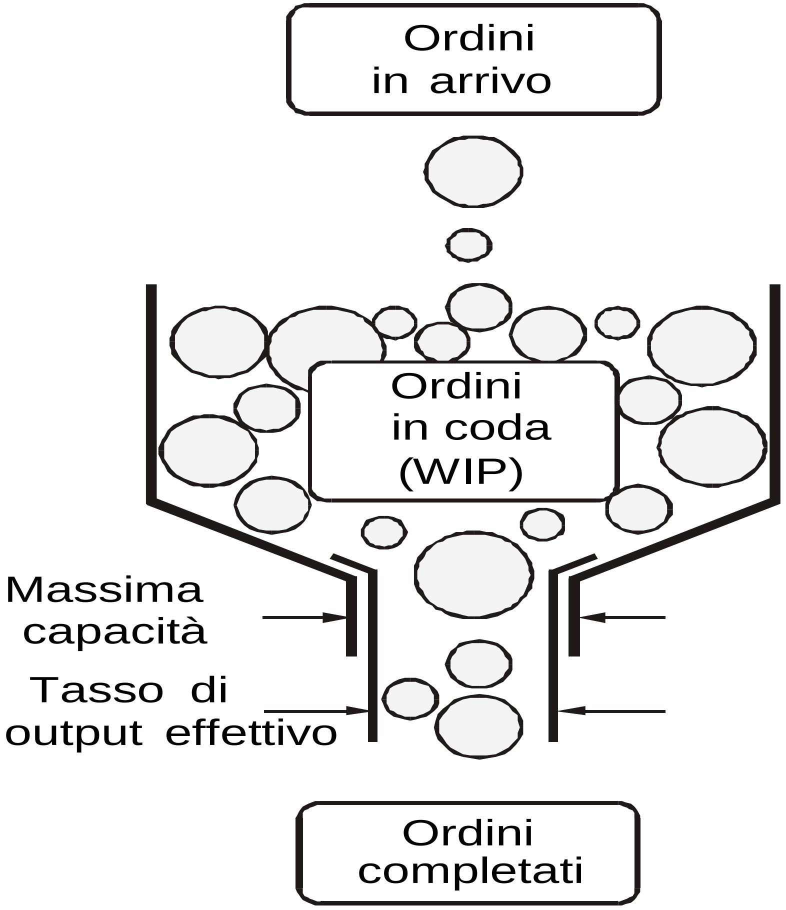
- Define the main performance of the system
- By monitoring it is possible:
  - To obtain real-time information on the system
  - To make an analysis on system behaviour history
  - To compute and analyse the trand of performance index



Performance analysis

- Funnel Model





Base relationship

- **Critical WIP (*WIP**):** it is the WIP level that allows to a manufacturing system to reach the maximum throughput (THmax) and the minimum lead time (LTmin)

$$\boldsymbol{WIP^* = \text{THmax} \times \text{LTmin}}$$

The Penny Fab

- Characteristics:
  - 4 identical machines (flow shop)
  - Manufacturing time equal to 2 h/penny for each machine
  - Fixed time
  - The pieces are loaded into the system in order to maintain a constant WIP level (CONWIP)
    - When a penny is released by the system, another one can be loaded

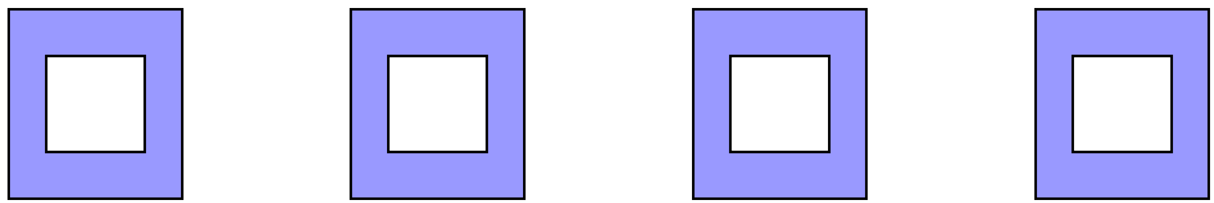
- Parameters:

  THcb = 0,5 penny/h

  LTmin = 8 h

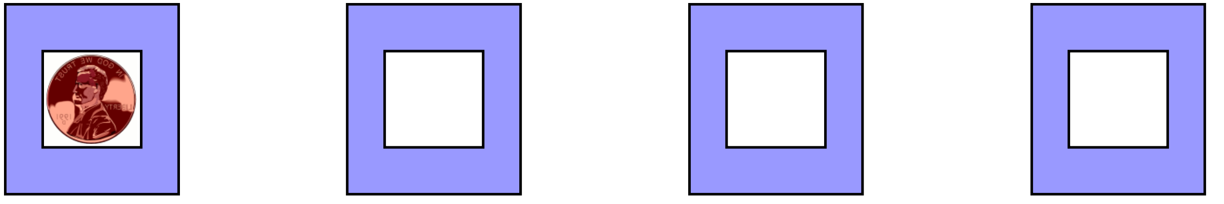
  Critical WIP - WIP*= 0,5 penny/h x 8 h = 4 penny

The Penny Fab



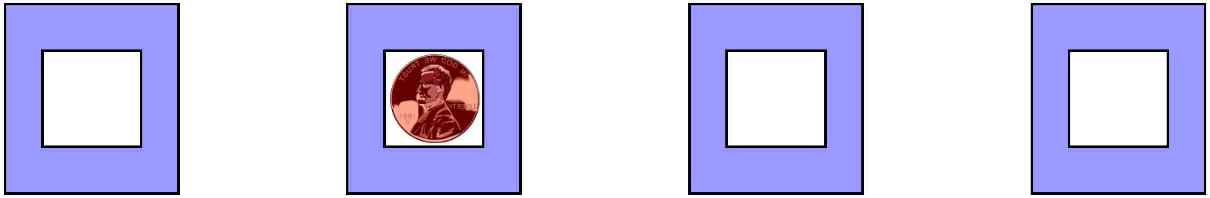
- Factory Physics, Wallace J. Hopp and Mark L. Spearman, Irwin/McGraw-Hill, 1996
  - Penny Fab represents a simple production line that makes **giant one-cent pieces** used extensively in Fourth of July parades
  - The line consists of four machines in sequence
    - The first machine is a punch press that cuts penny blanks, the second stamps Lincoln's face on one side and the Lincoln Memorial on the back, the third puts a rim on the penny, and the fourth cleans away any burrs
    - After each penny is processed, it is moved immediately to the next machine
  - The line runs 24 hours per day, with breaks and lunches covered by spare operators
  - The market for giant pennies is assumed to be unlimited, so that all product made is sold; thus, more throughput is unambiguously better for this system

The Penny Fab (WIP=1)



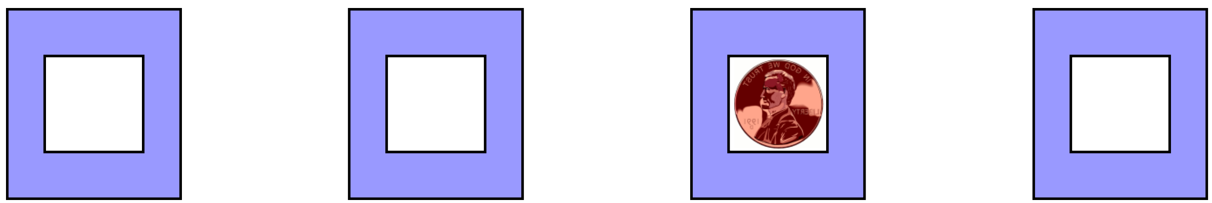
Time = 0 hours

The Penny Fab (WIP=1)



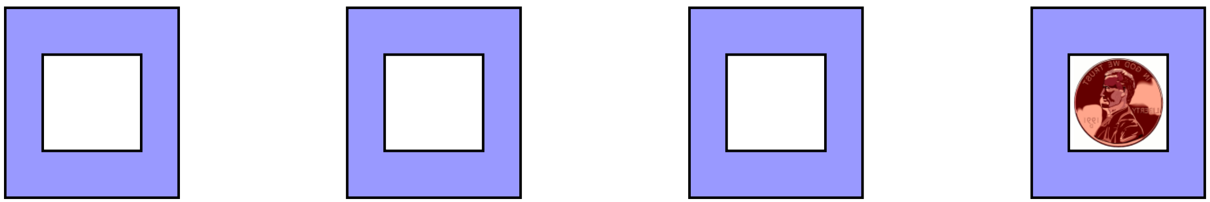
Time = 2 hours

The Penny Fab (WIP=1)



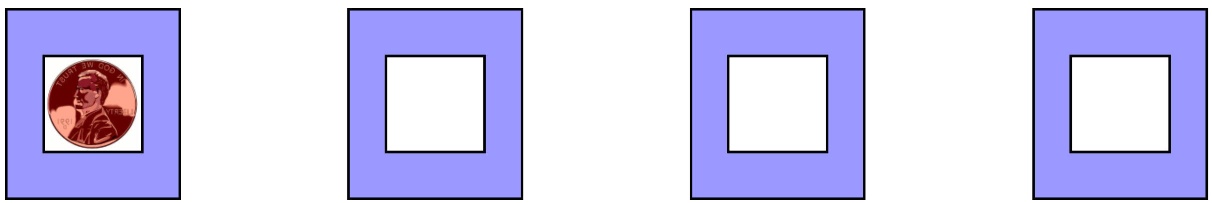
Time = 4 hours

The Penny Fab (WIP=1)



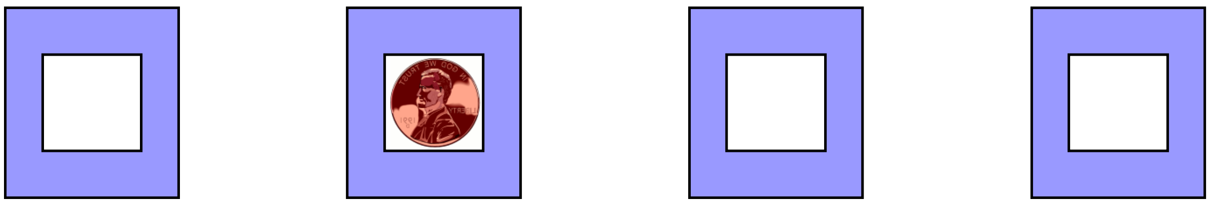
Time = 6 hours

The Penny Fab (WIP=1)



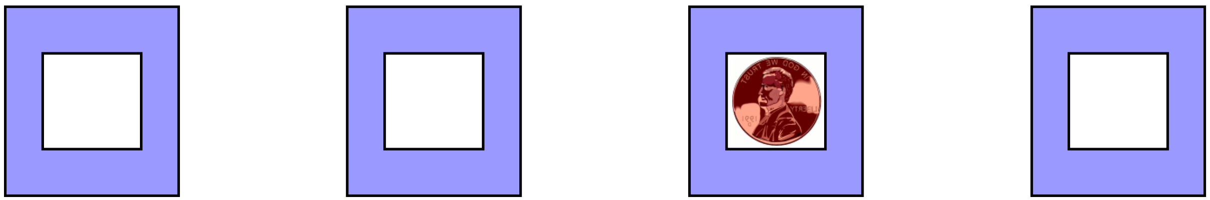
Time = 8 hours

The Penny Fab (WIP=1)



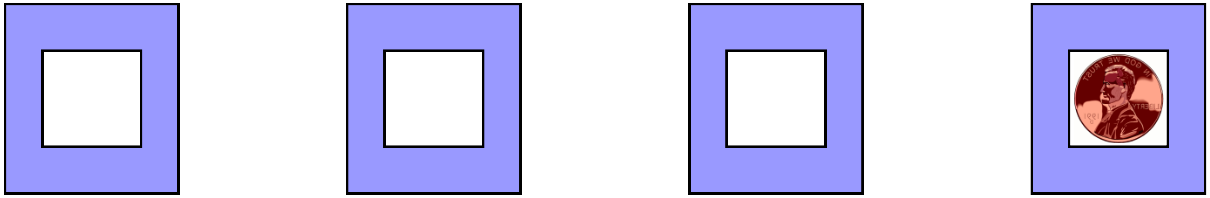
Time = 10 hours

The Penny Fab (WIP=1)

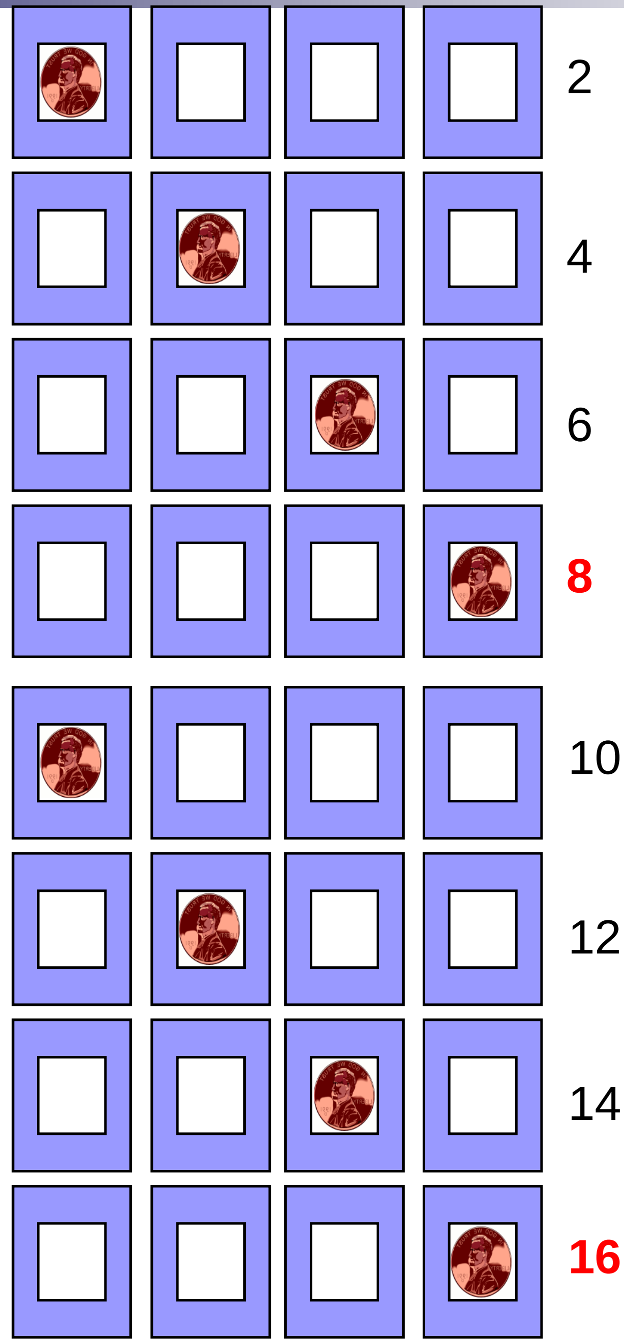


Time = 12 hours

The Penny Fab (WIP=1)



Time = 14 hours

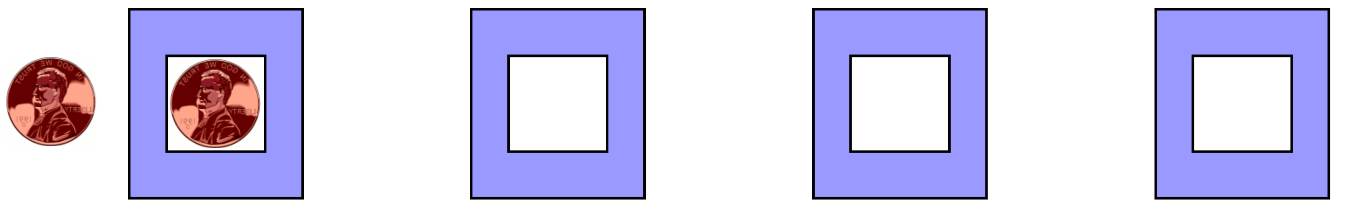


2 MONETE IN 16 MINUTI = 2/16 = 0,125

Performance with WIP = 1

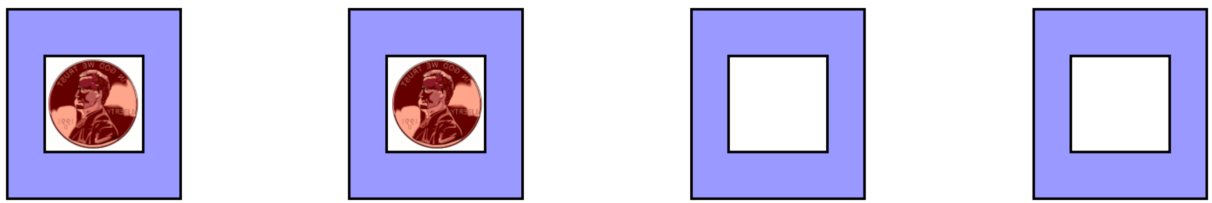
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2			
3			
4			
5			
6			

The Penny Fab (WIP=2)



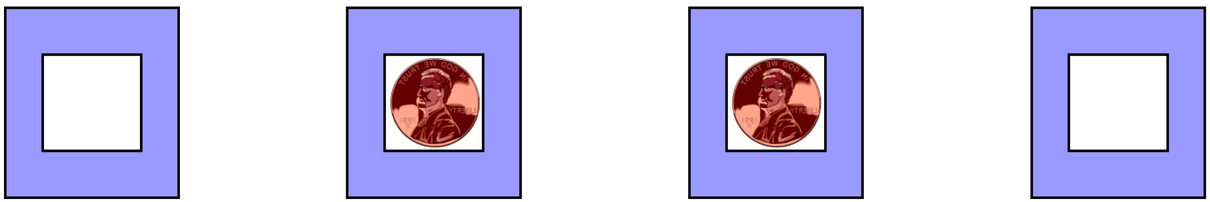
Time = 0 hours

The Penny Fab (WIP=2)



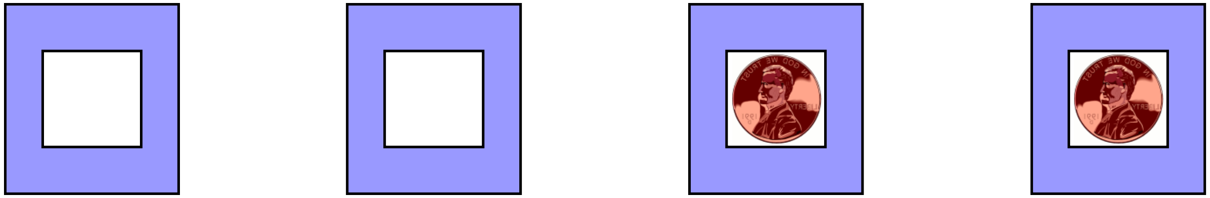
Time = 2 hours

The Penny Fab (WIP=2)



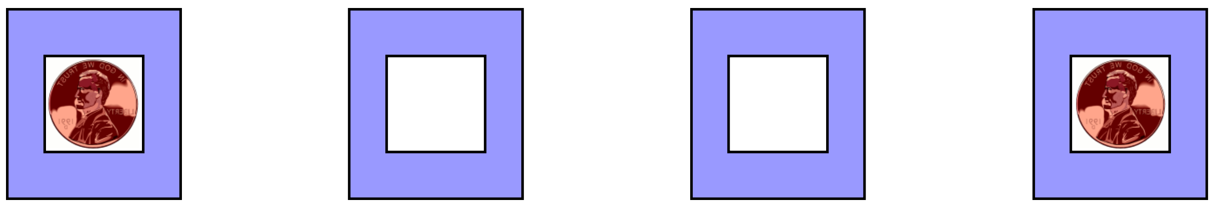
Time = 4 hours

The Penny Fab (WIP=2)



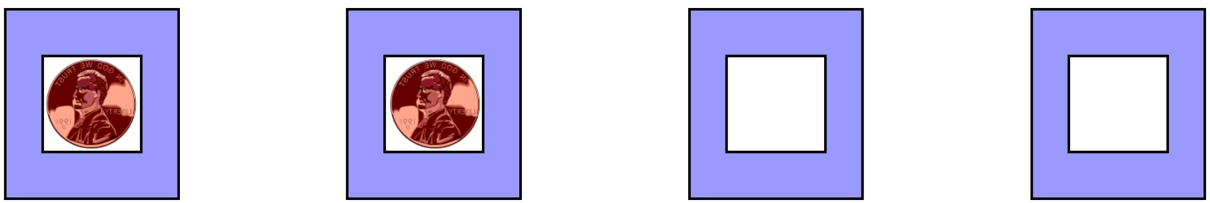
Time = 6 hours

The Penny Fab (WIP=2)



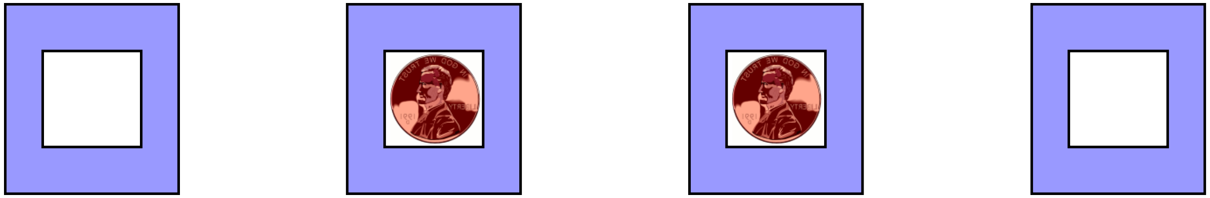
Time = 8 hours

The Penny Fab (WIP=2)



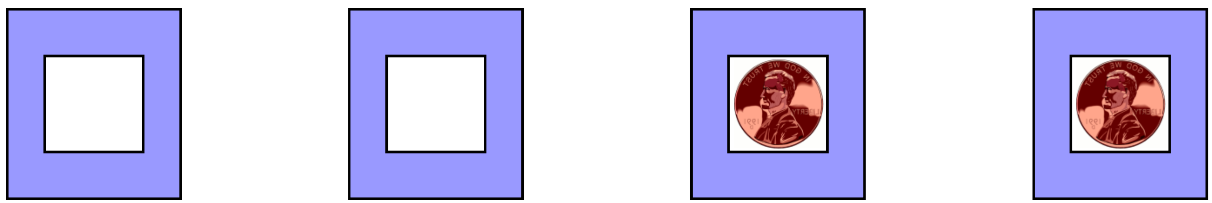
Time = 10 hours

The Penny Fab (WIP=2)

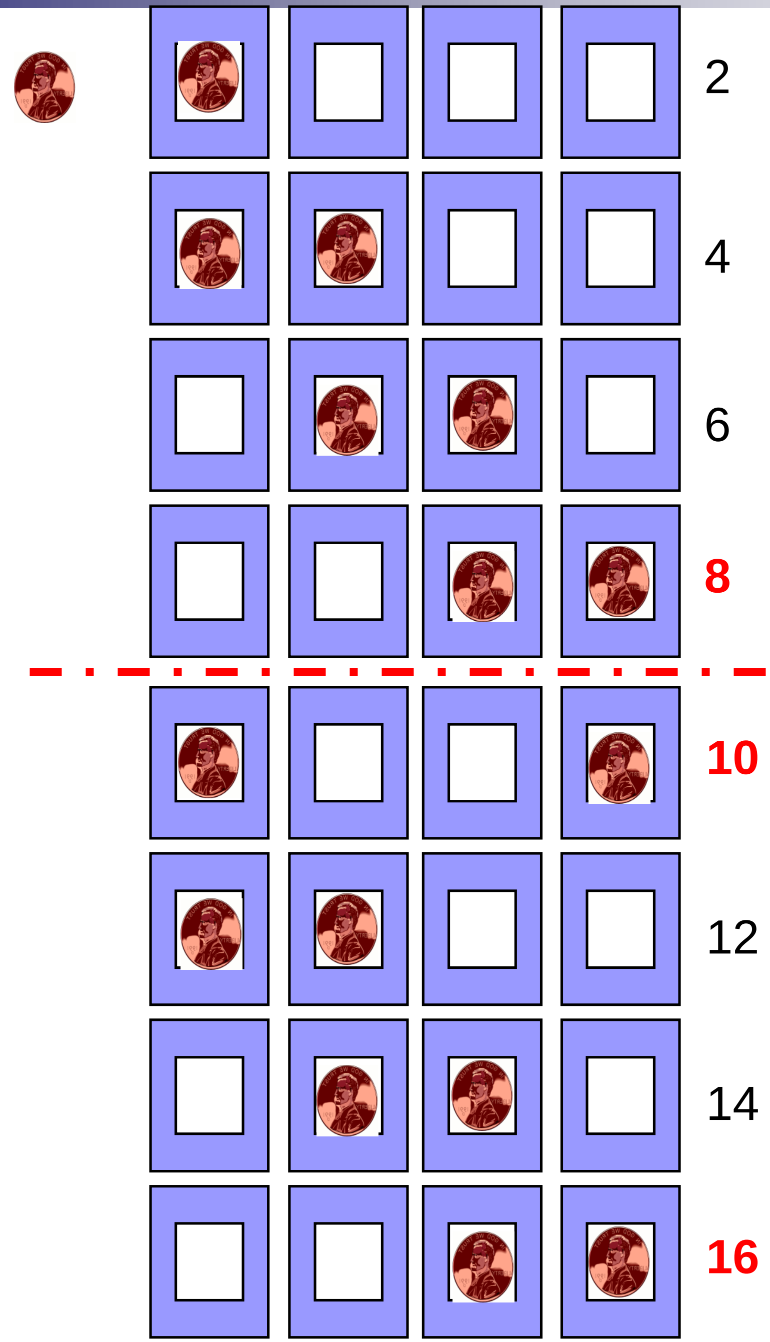


Time = 12 hours

The Penny Fab (WIP=2)



Time = 14 hours

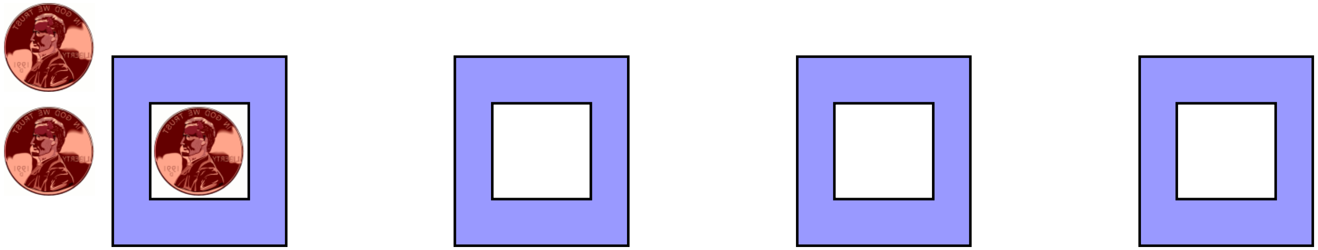


Dopo la fase di transitorio (riempimento del sistema)
2 MONETE IN 8 MINUTI
= 2/8 = 0,250

Performance with WIP = 2

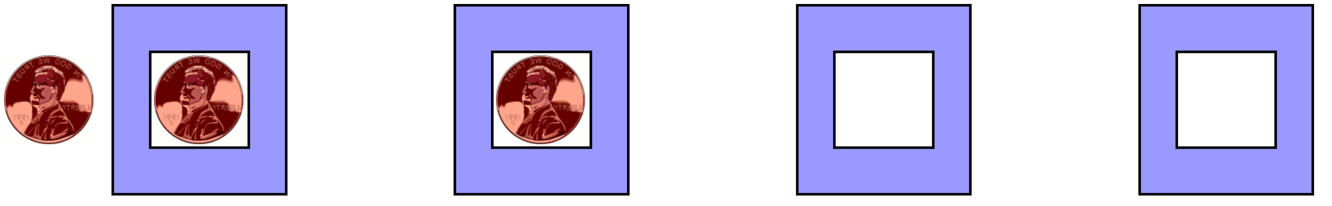
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3			
4			
5			
6			

The Penny Fab (WIP=3)



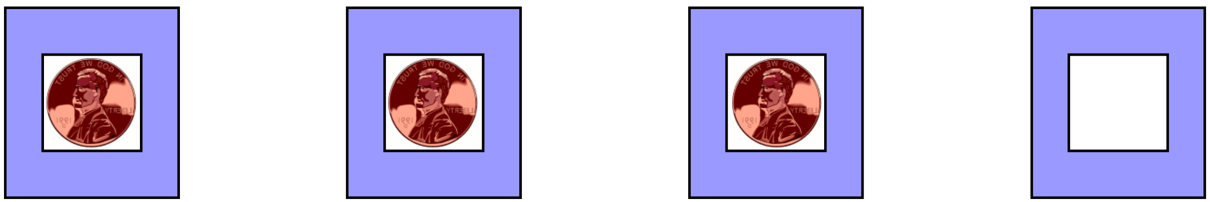
Time = 0 hours

The Penny Fab (WIP=3)



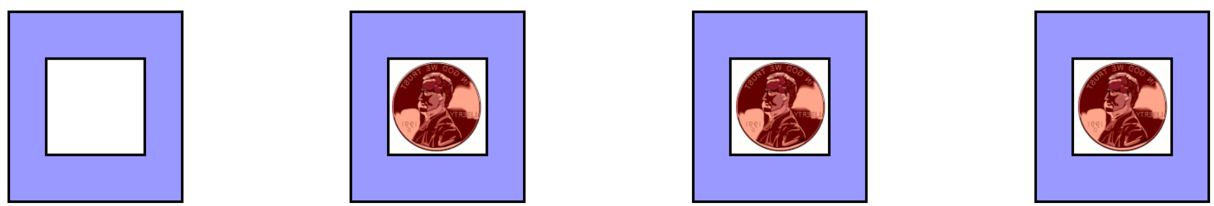
Time = 2 hours

The Penny Fab (WIP=3)



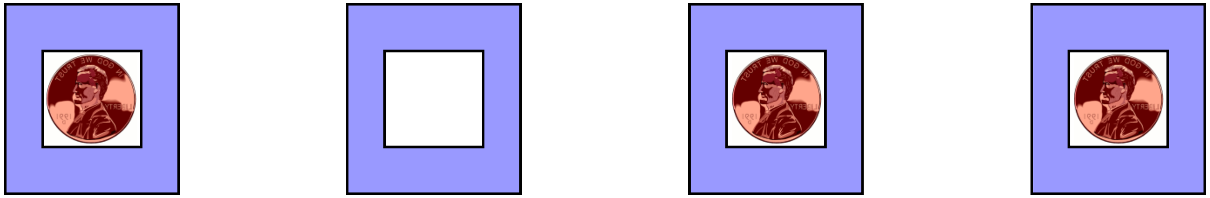
Time = 4 hours

The Penny Fab (WIP=3)



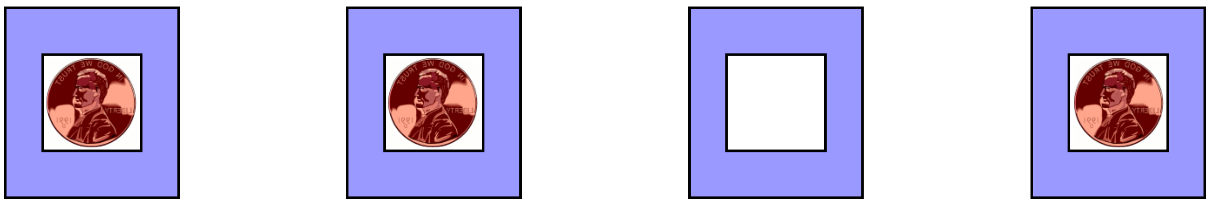
Time = 6 hours

The Penny Fab (WIP=3)



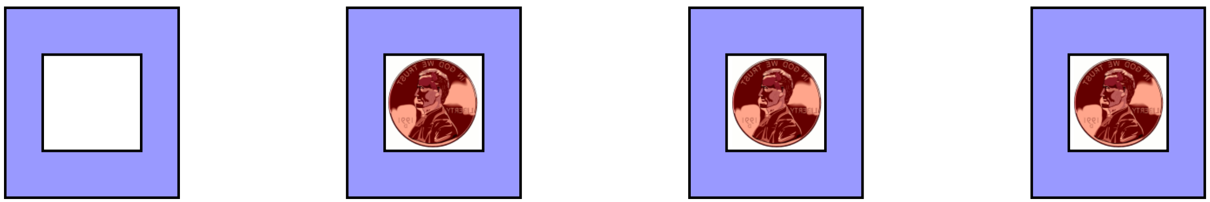
Time = 8 hours

The Penny Fab (WIP=3)

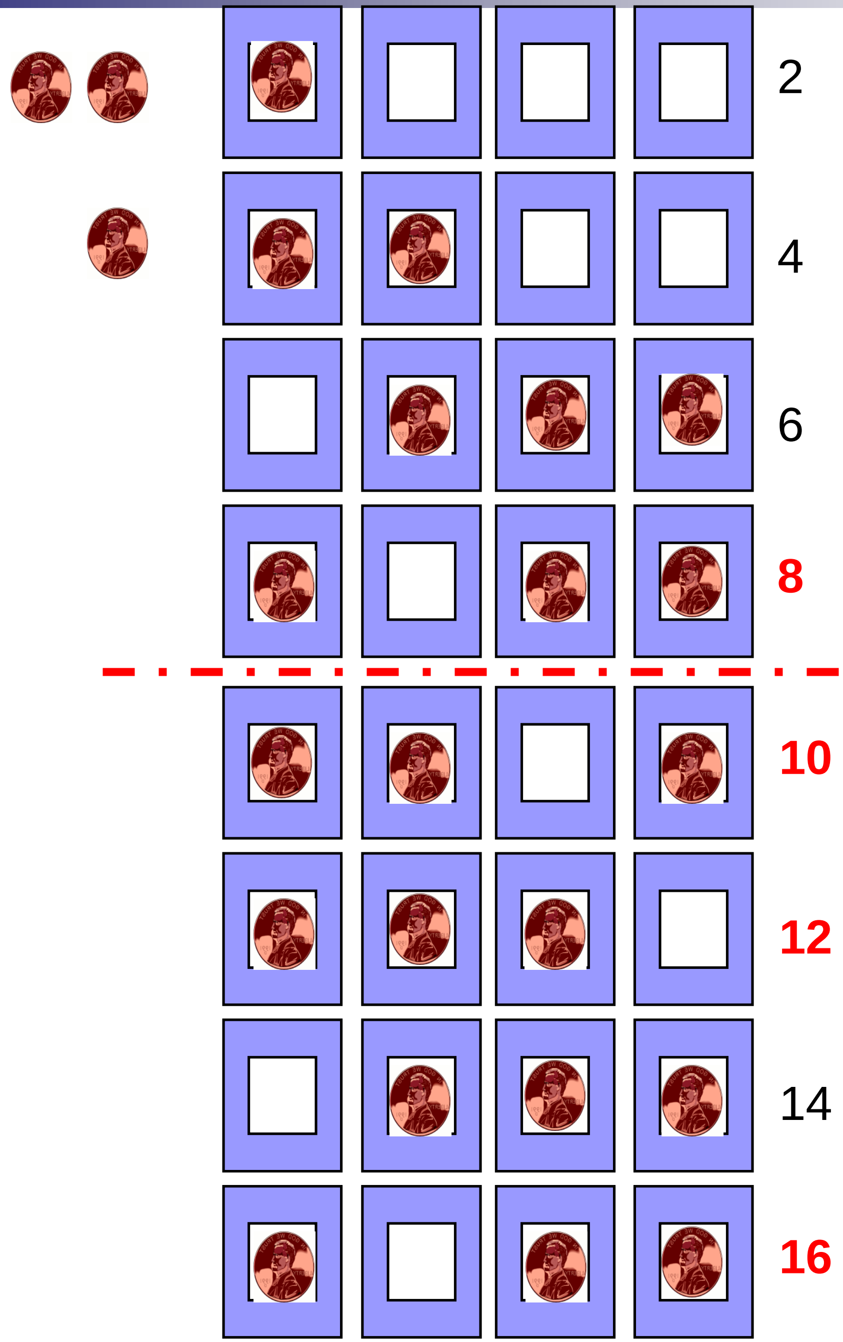


Time = 10 hours

The Penny Fab (WIP=3)



Time = 12 hours

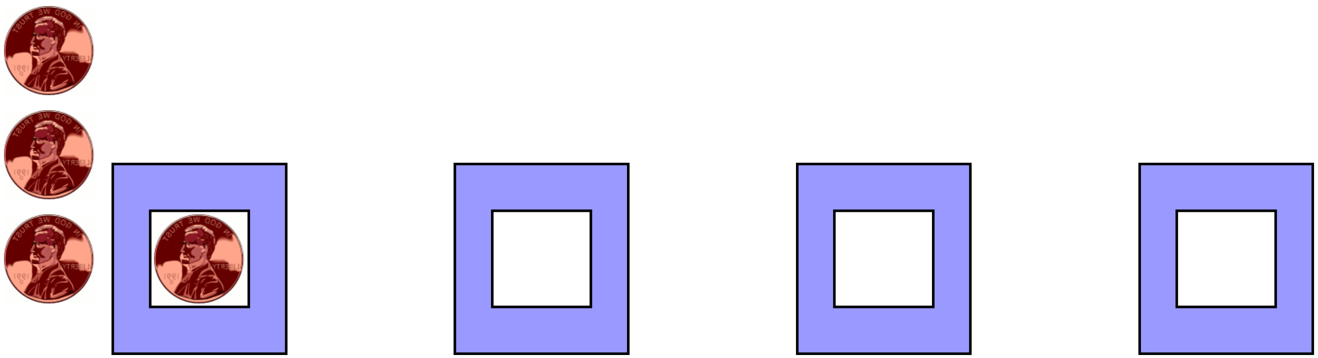


Dopo la fase di transitorio (riempimento del sistema)
3 MONETE IN 8 MINUTI
= 3/8 = 0,375

Performance with WIP = 3

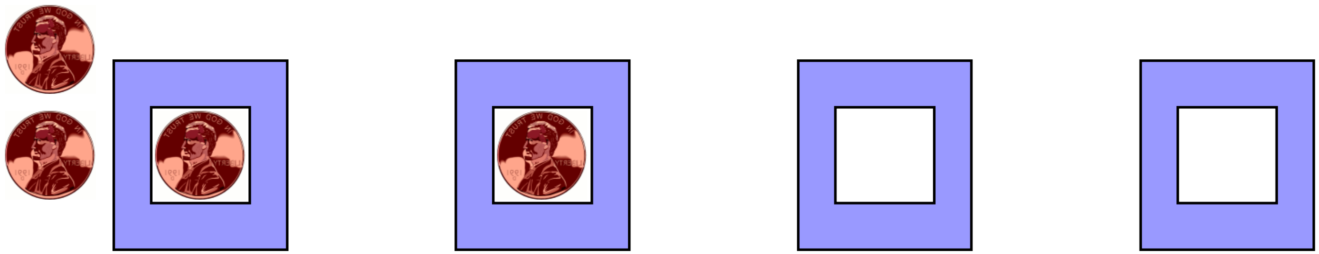
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4			
5			
6			

The Penny Fab (WIP=4)



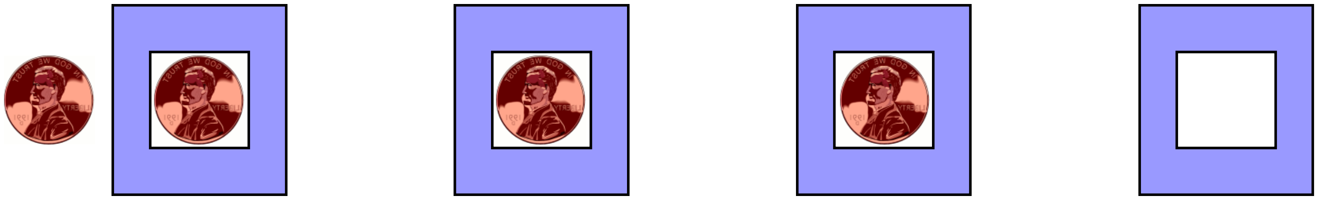
Time = 0 hours

The Penny Fab (WIP=4)



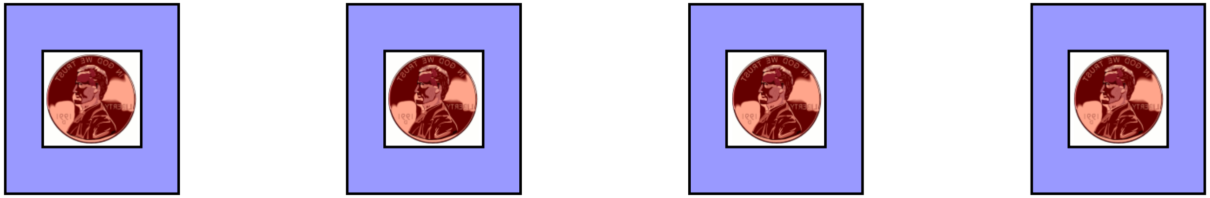
Time = 2 hours

The Penny Fab (WIP=4)



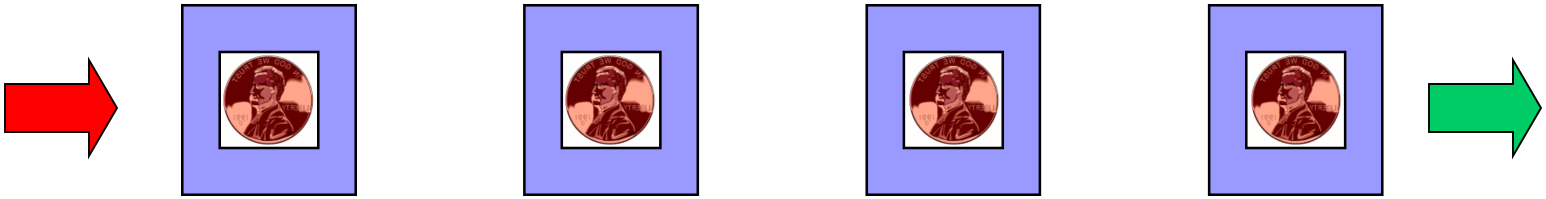
Time = 4 hours

The Penny Fab (WIP=4)



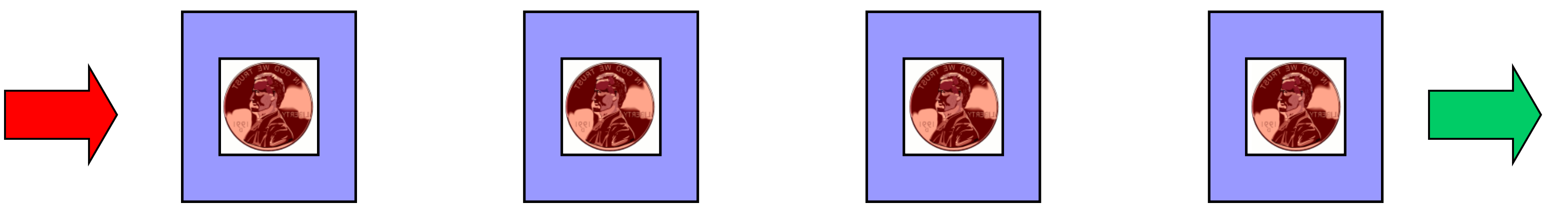
Time = 6 hours

The Penny Fab (WIP=4)



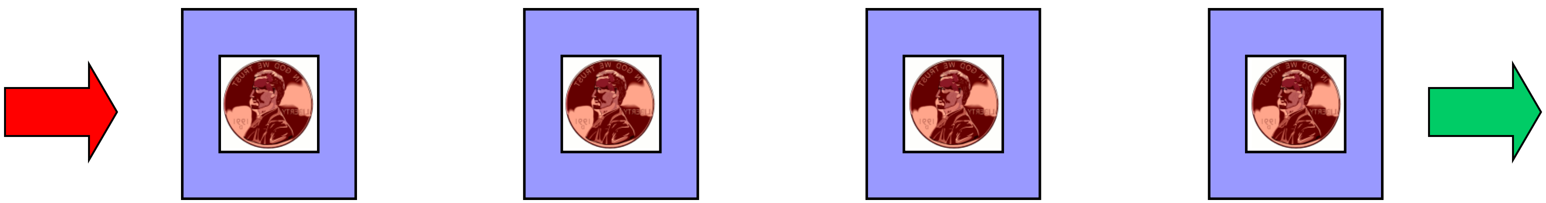
Time = 8 hours

The Penny Fab (WIP=4)



Time = 10 hours

The Penny Fab (WIP=4)

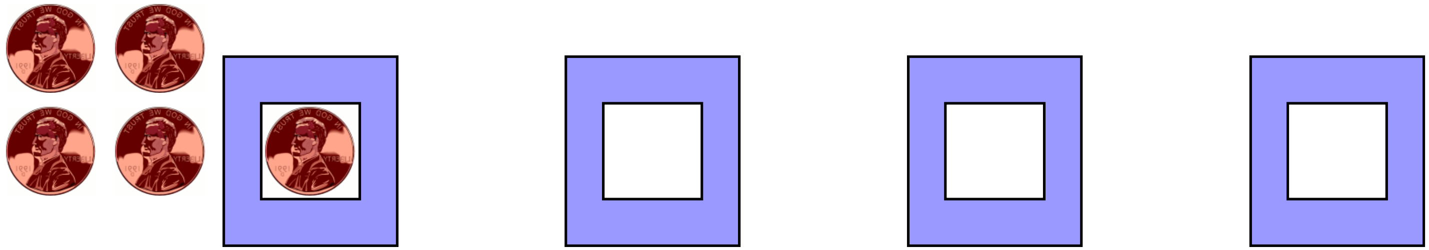


Time = 12 hours

Performance with WIP = 4

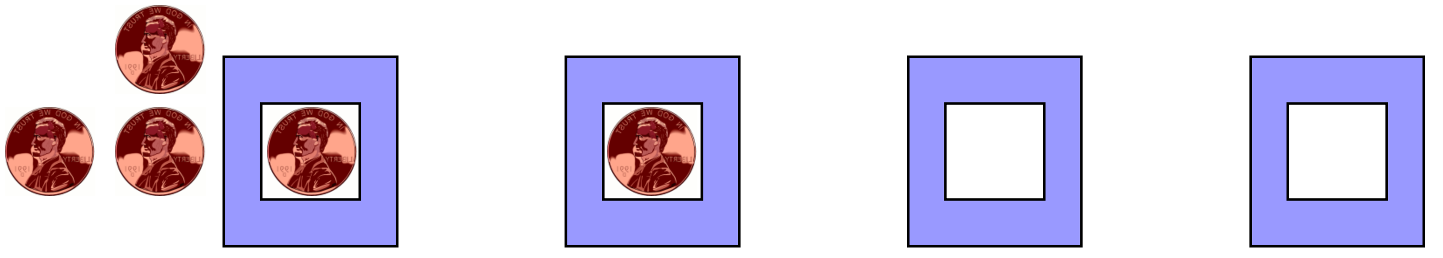
WIP [pz]	TH [pz/h]	LT [h]	THxLT [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5			
6			

The Penny Fab (WIP=5)



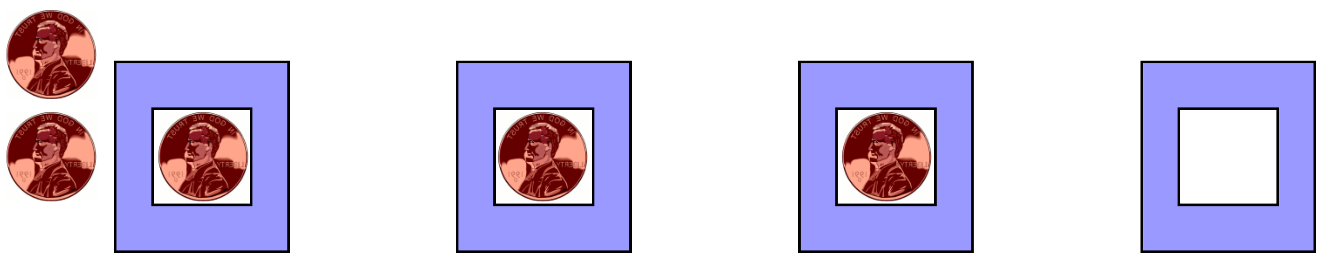
Time = 0 hours

The Penny Fab (WIP=5)



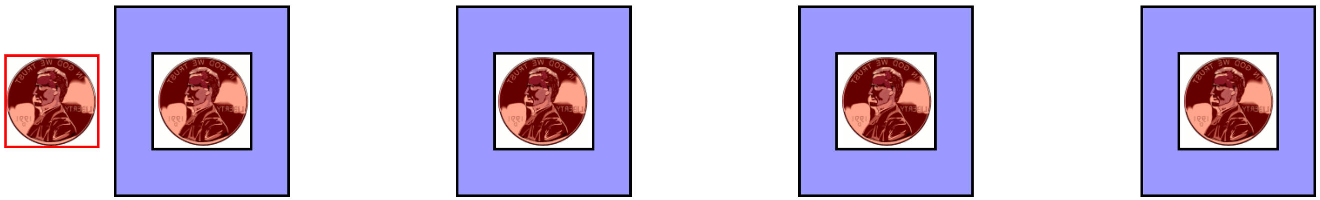
Time = 2 hours

The Penny Fab (WIP=5)



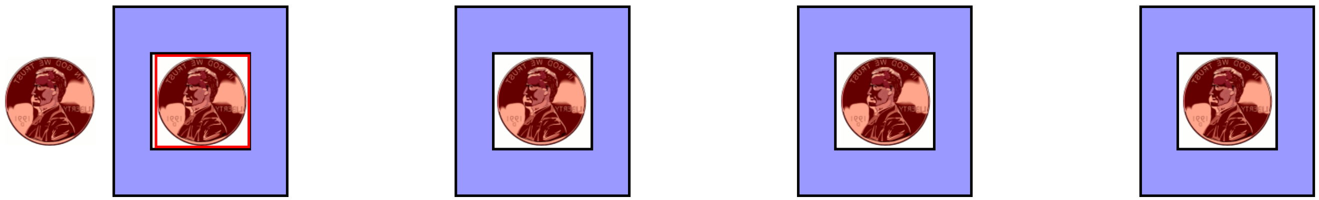
Time = 4 hours

The Penny Fab (WIP=5)



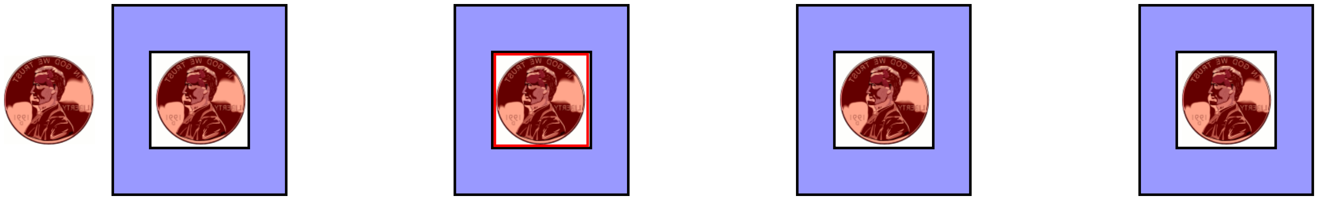
Time = 6 hours

The Penny Fab (WIP=5)



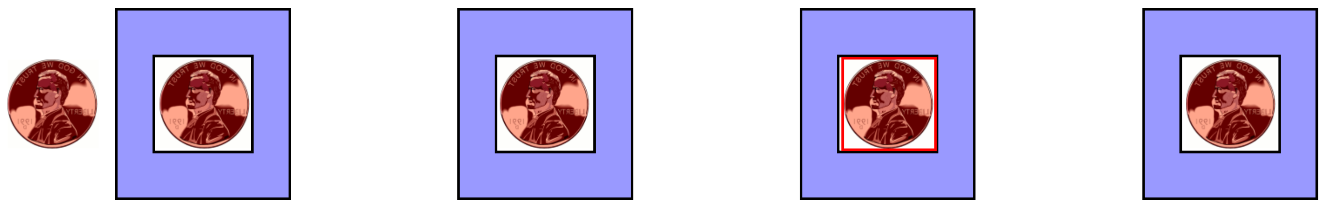
Time = 8 hours

The Penny Fab (WIP=5)



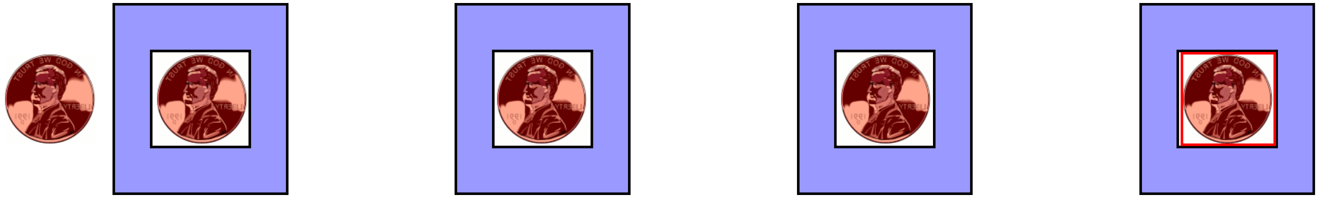
Time = 10 hours

The Penny Fab (WIP=5)



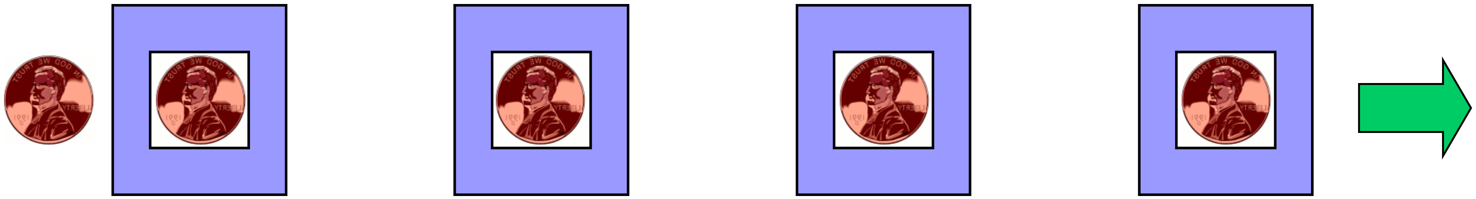
Time = 12 hours

The Penny Fab (WIP=5)



Time = 14 hours

The Penny Fab (WIP=5)

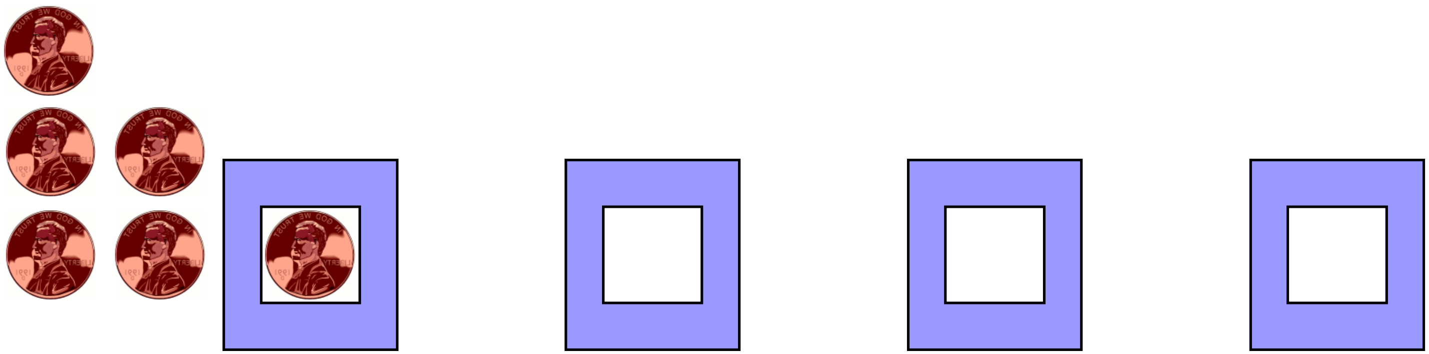


Time = 16 hours

Performance with WIP = 5

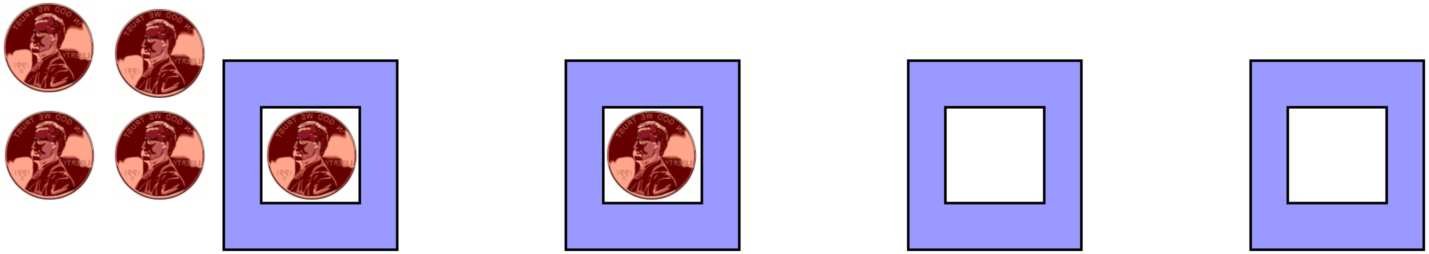
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5	0,500	10	5
6			

The Penny Fab (WIP=6)



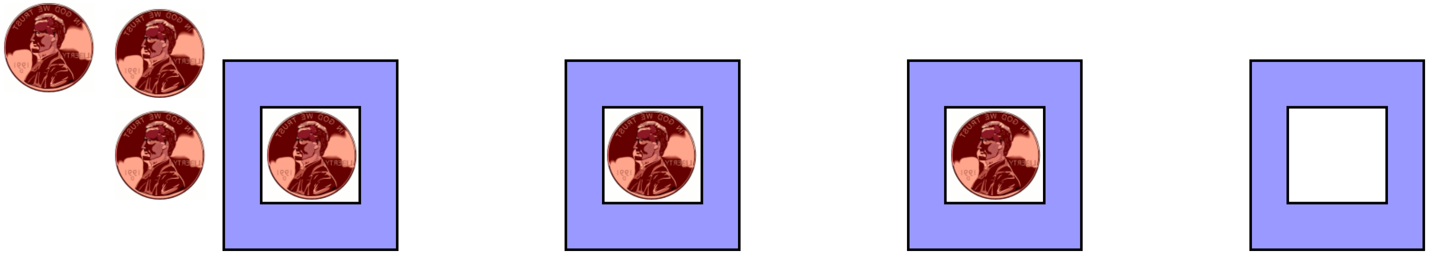
Time = 0 hours

The Penny Fab (WIP=6)



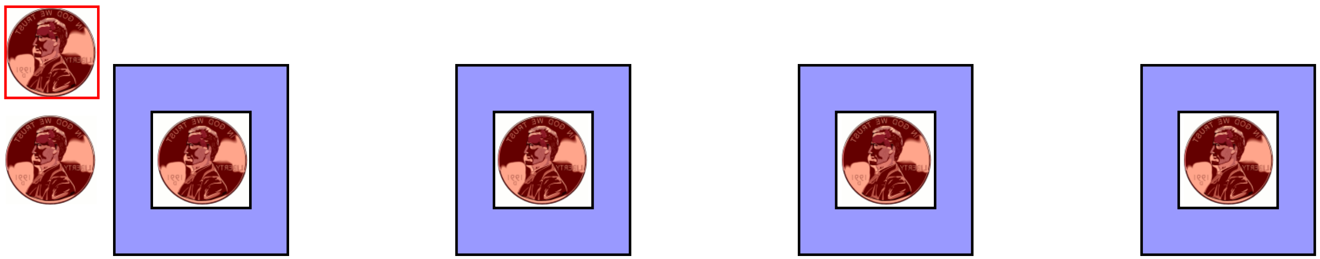
Time = 2 hours

The Penny Fab (WIP=6)



Time = 4 hours

The Penny Fab (WIP=6)



Time = 6 hours

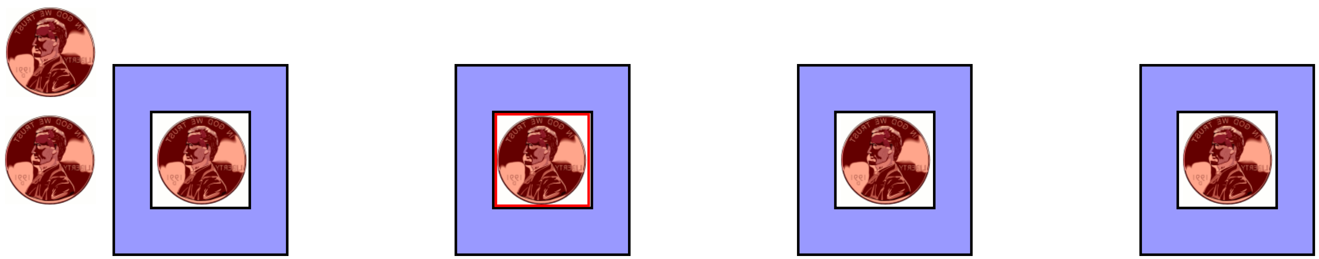
The Penny Fab (WIP=6)

Time = 8 hours

The Penny Fab (WIP=6)

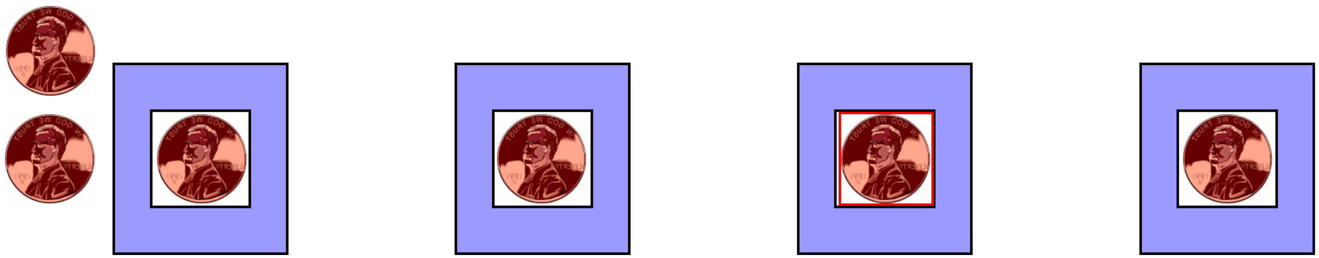
Time = 10 hours

The Penny Fab (WIP=6)



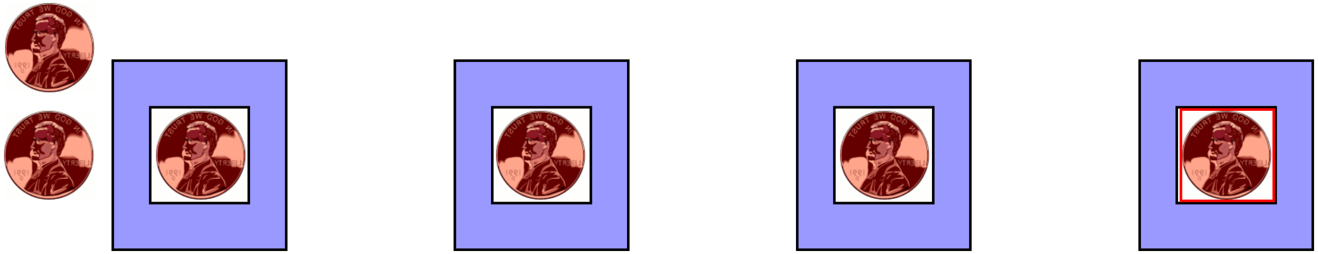
Time = 12 hours

The Penny Fab (WIP=6)



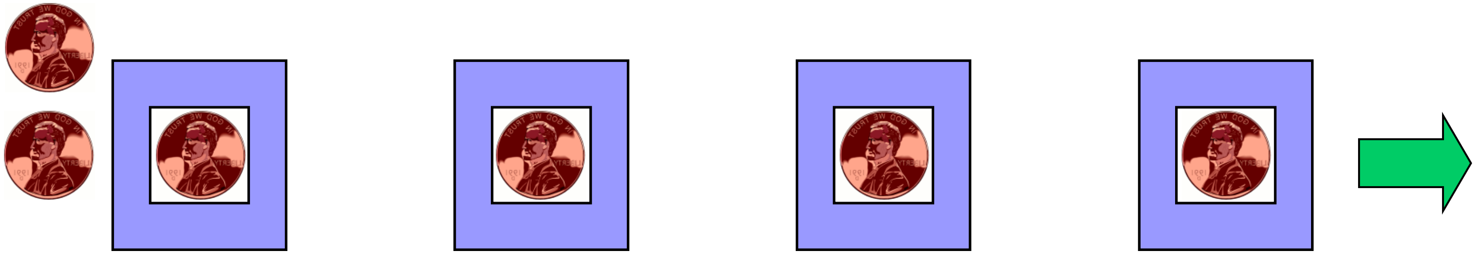
Time = 14 hours

The Penny Fab (WIP=6)



Time = 16 hours

The Penny Fab (WIP=6)

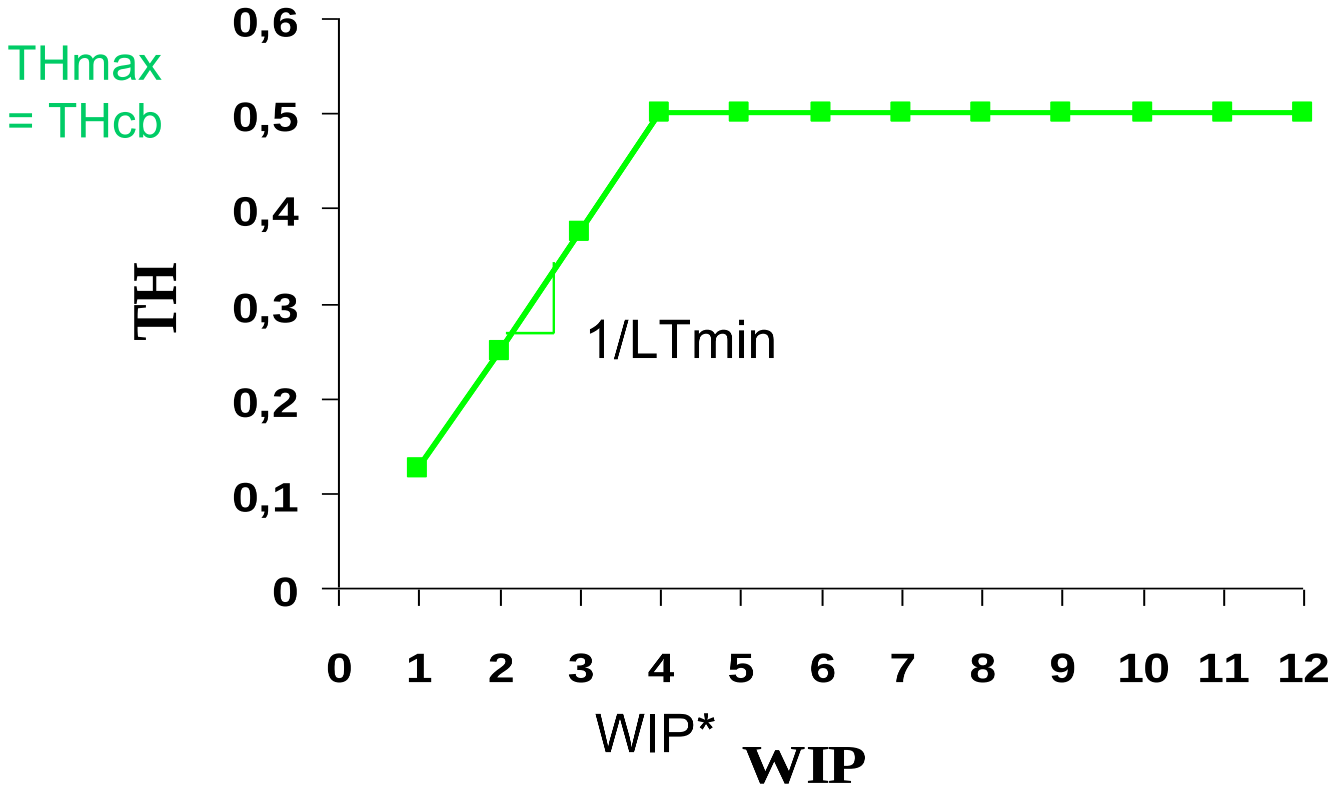


Time = 18 hours

Performance with WIP = 6

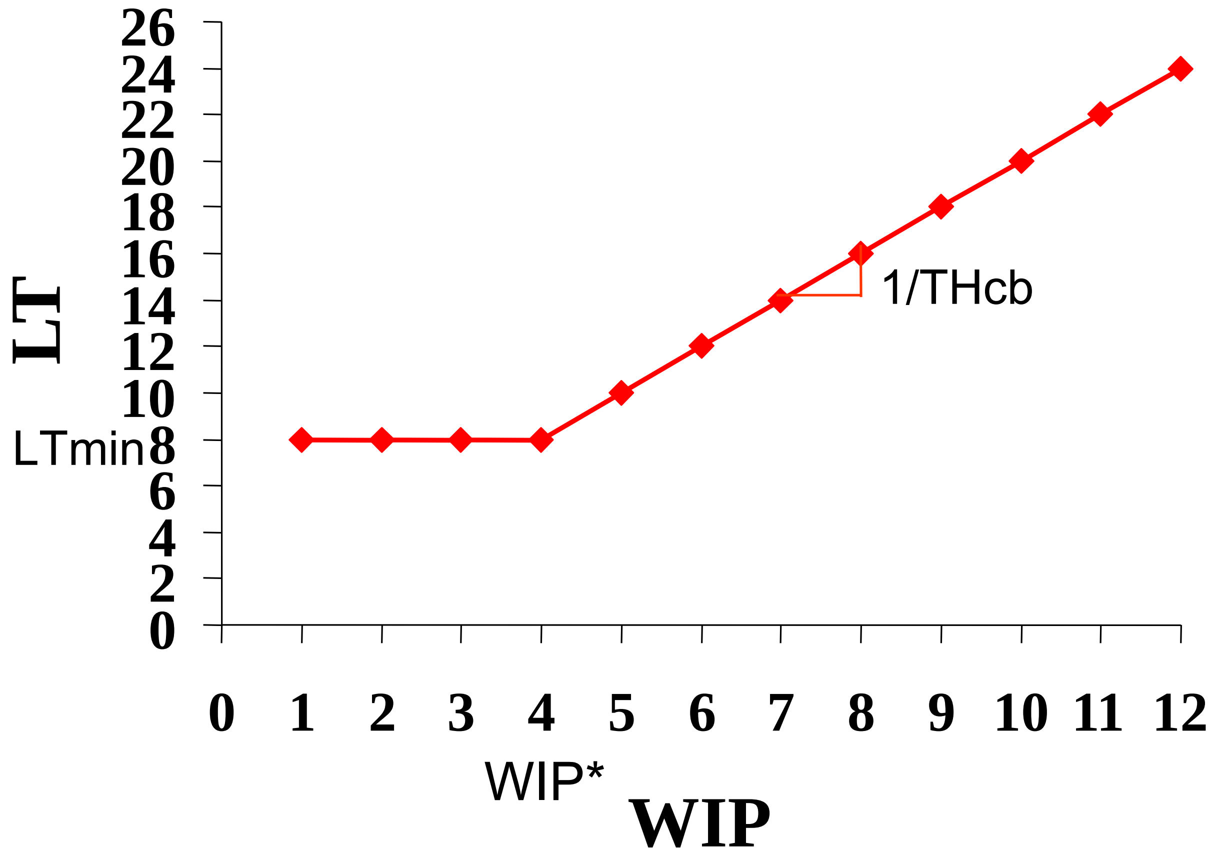
WIP [pz]	TH [pz/h]	LT [h]	THxLT [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5	0,500	10	5
6	0,500	12	6

TH vs. WIP



THmax
= THcb
0,6
0,5
0,4
0,3
0,2
0,1
0
TH
1/LTmin
0 1 2 3 4 5 6 7 8 9 10 11 12
WIP*
WIP

LT vs. WIP



LT
26
24
22
20
18
16
14
12
10
LTmin
8
6
4
2
0
1/THcb
0 1 2 3 4 5 6 7 8 9 10 11 12
WIP*
WIP

THmax and LTmin

LTmin is the sum of effort (time) for each machine in order to realize a single penny

- Minimum LT (LTmin) given the WIP level is

$$LT_{\min} = \begin{cases} \sum Tprocess, & \text{if } wip \le WIP^* \\ wip / TH_{\max}, & \text{otherwise} \end{cases}$$

- Maximum throughput (THmax) given the level WIP is

$$TH_{\max} = \begin{cases} wip / LT_{\min}, & \text{if } wip \le WIP^* \\ TH_{bn}, & \text{otherwise} \end{cases}$$

THmax is the TH of the system bottleneck (the slowest resource)

THmax and LTmin

- Penny Fab, THcb = 0.5 and LTmin = 8
  - WIP* = 0.5 × 8 = 4,

$$\mathrm{LT}_{\min} = \begin{cases} 8, & \text{if } wip \leq 4 \\ 2wip, & \text{otherwise} \end{cases}$$

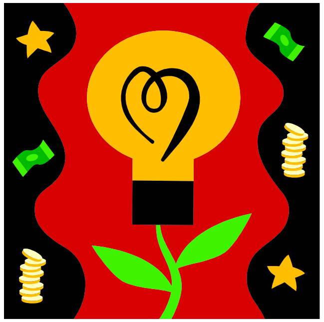
$$\mathrm{TH}_{\max} = \begin{cases} wip/8, & \text{if } wip \leq 4 \\ 0.5, & \text{otherwise} \end{cases}$$

Little's Law

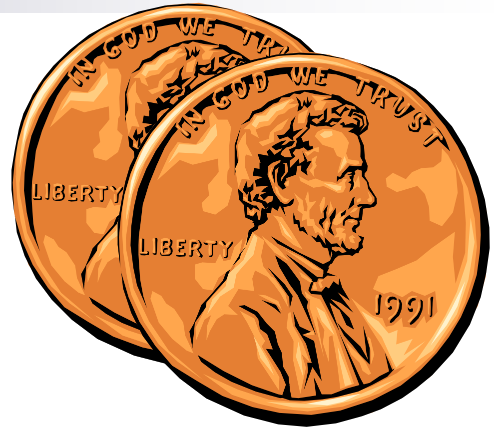
- The main relationship among WIP, LT and Th is

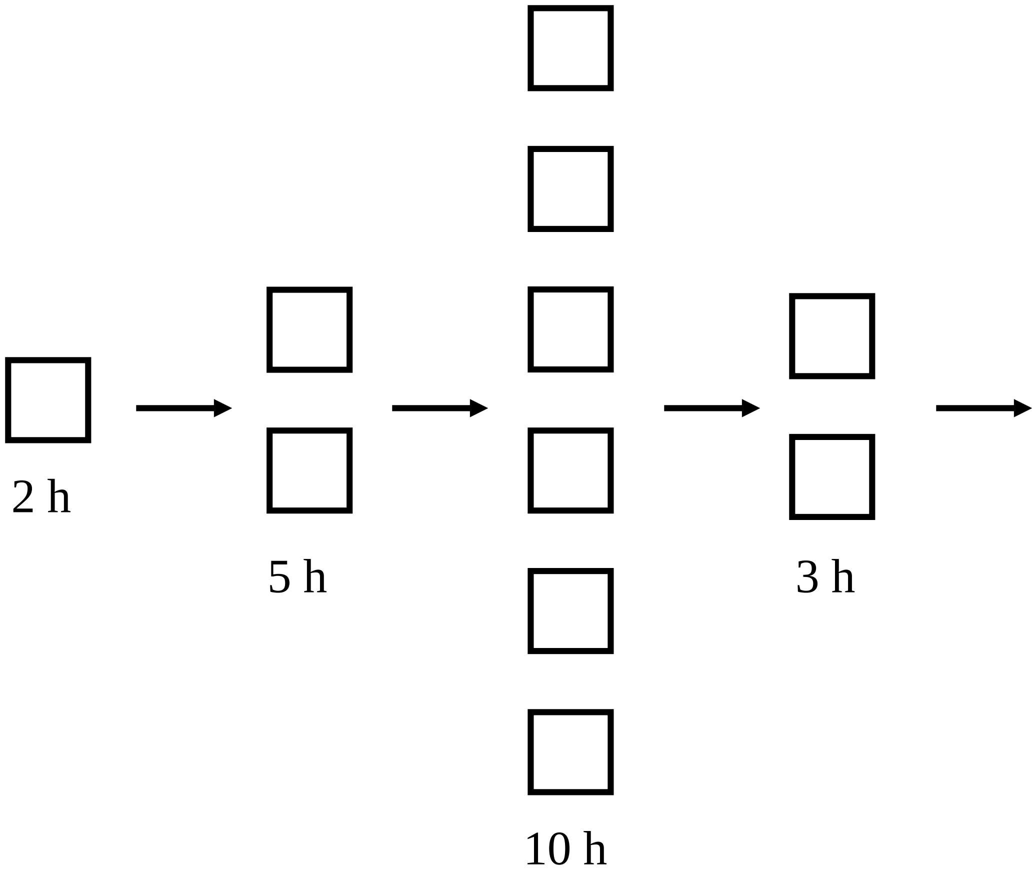
$$\mathrm{WIP} = \mathrm{TH} \times \mathrm{LT} \qquad pcs = \frac{pcs}{h} \times h$$

- LT = WIP/TH



Penny Fab Two



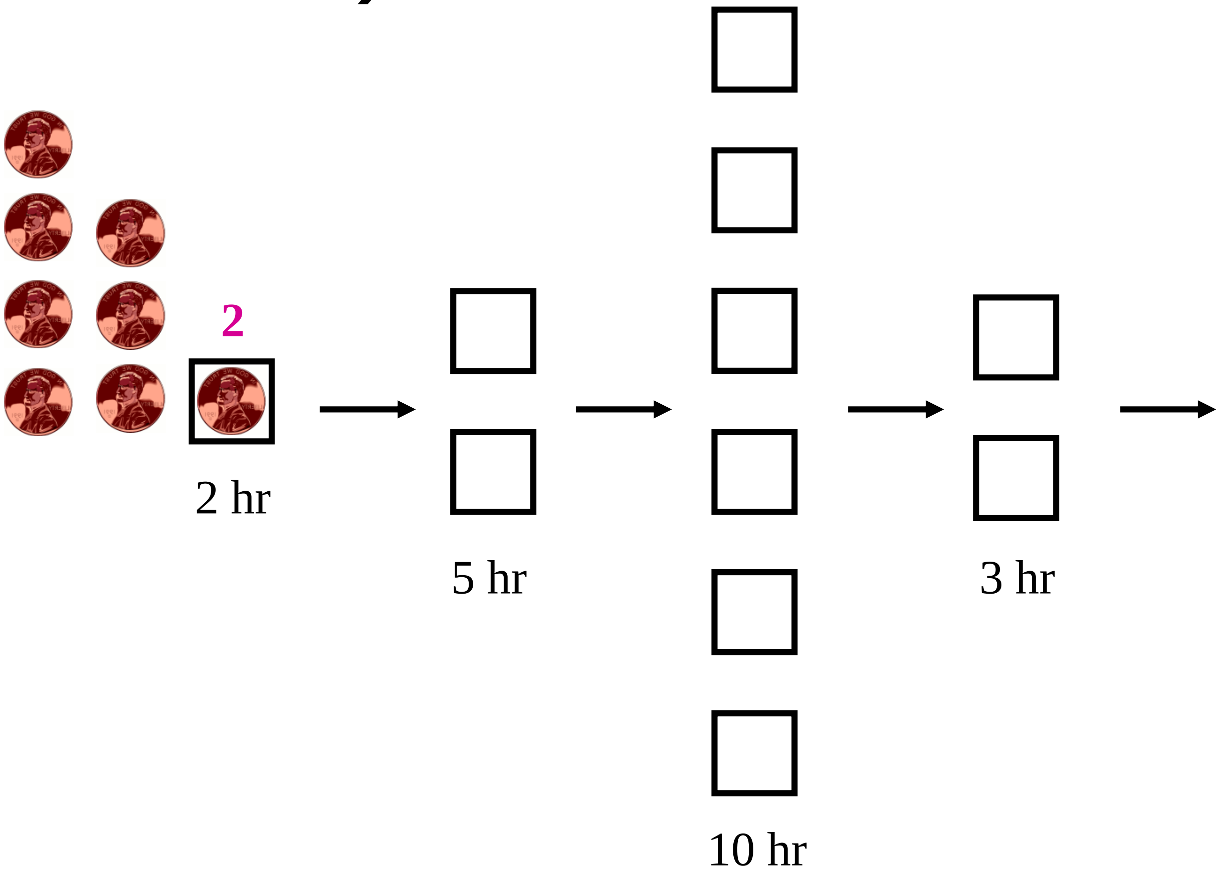


Penny Fab Two

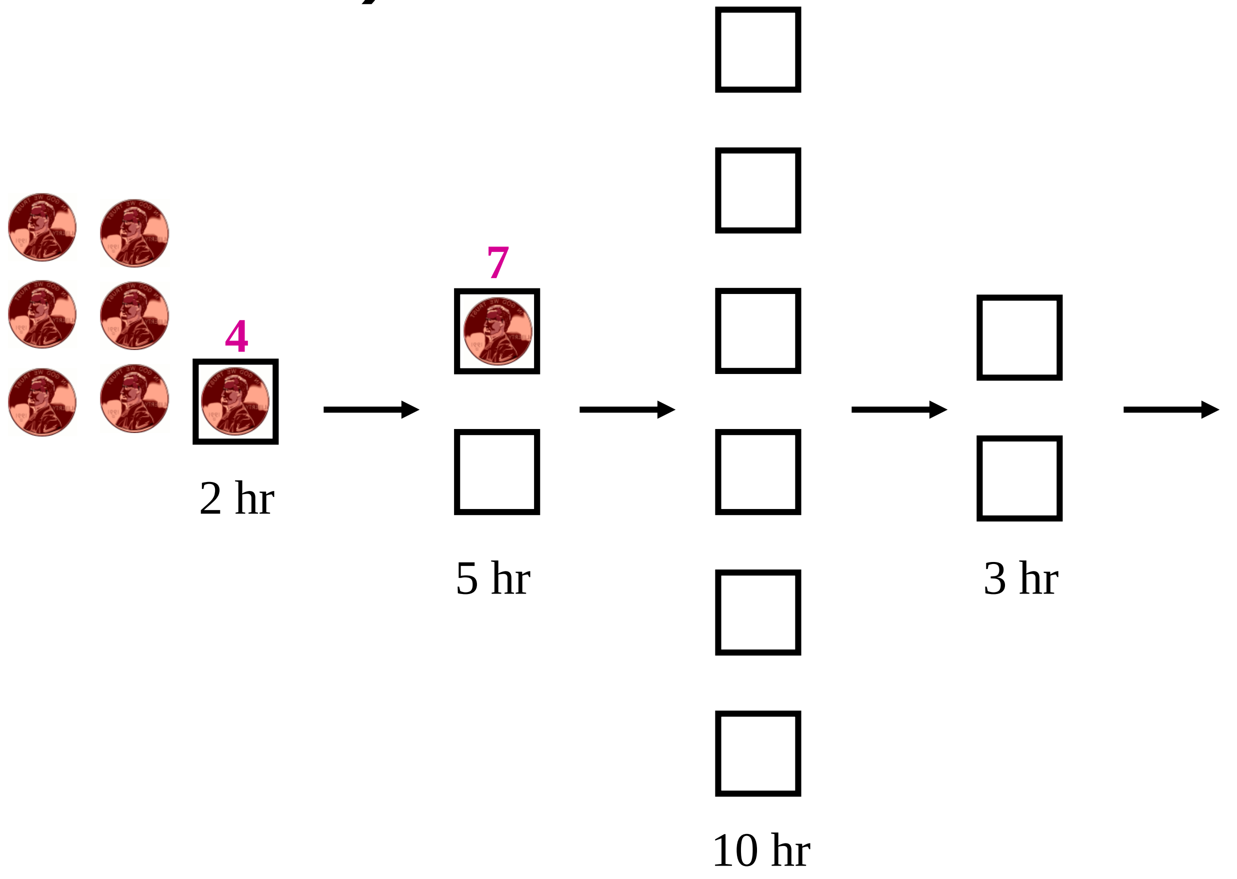
Department	**Number ot machines**	**Work time** [h]	**Department TH** [job/h]
1	1	2	0,5
2	2	5	0,4
3	6	10	0,6
4	2	3	0,67

THcb = **0.4 p/h** *LTmin* = **20 h** *WIP** = **8 penny**

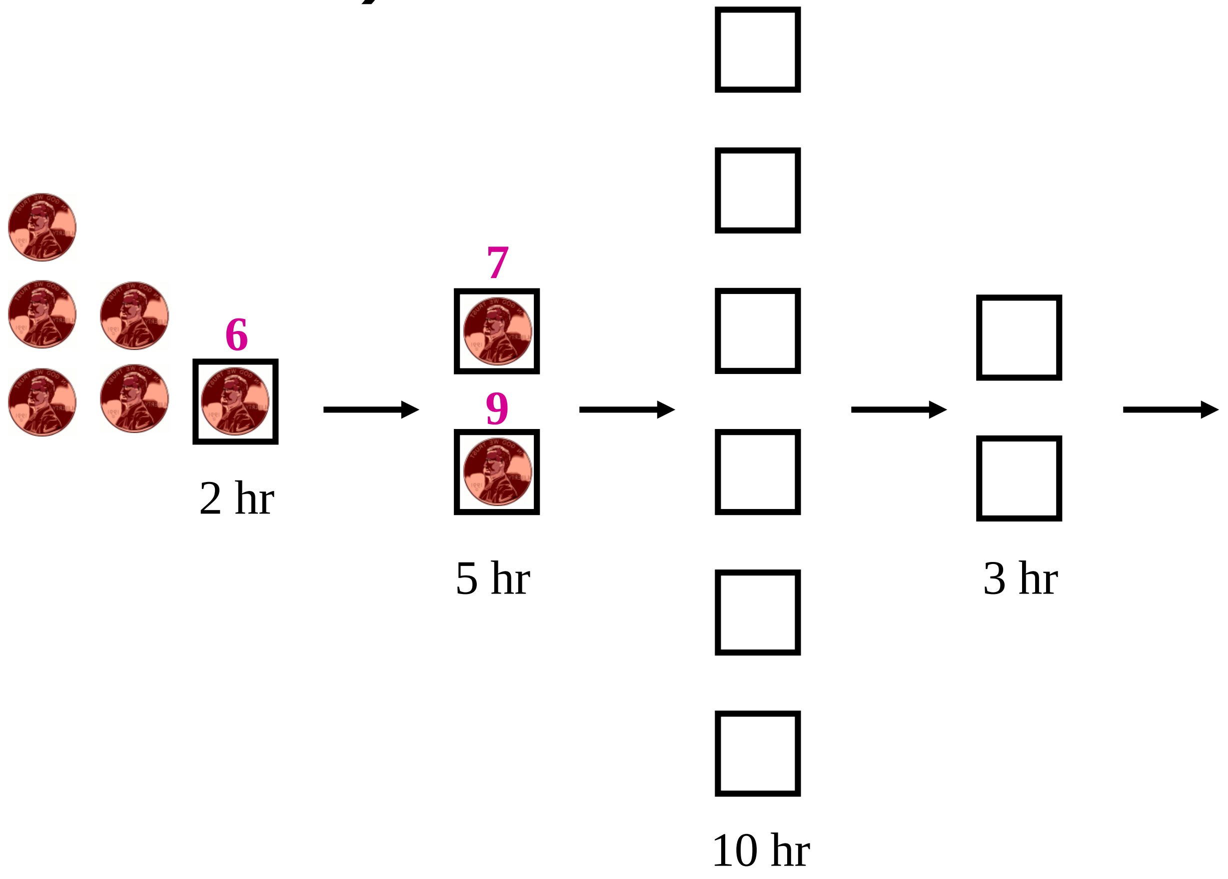
Penny Fab Two Simulation (Time=0)



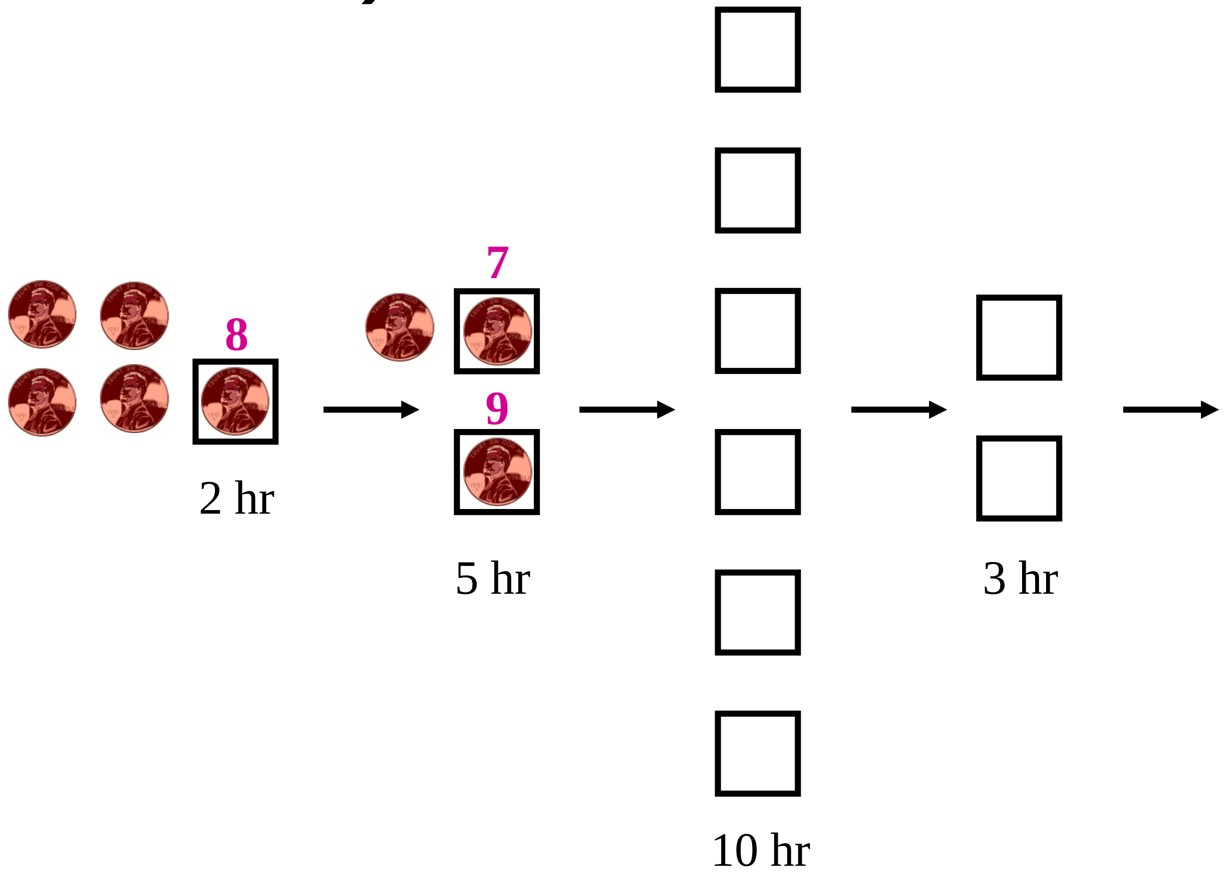
Penny Fab Two Simulation (Time=2)



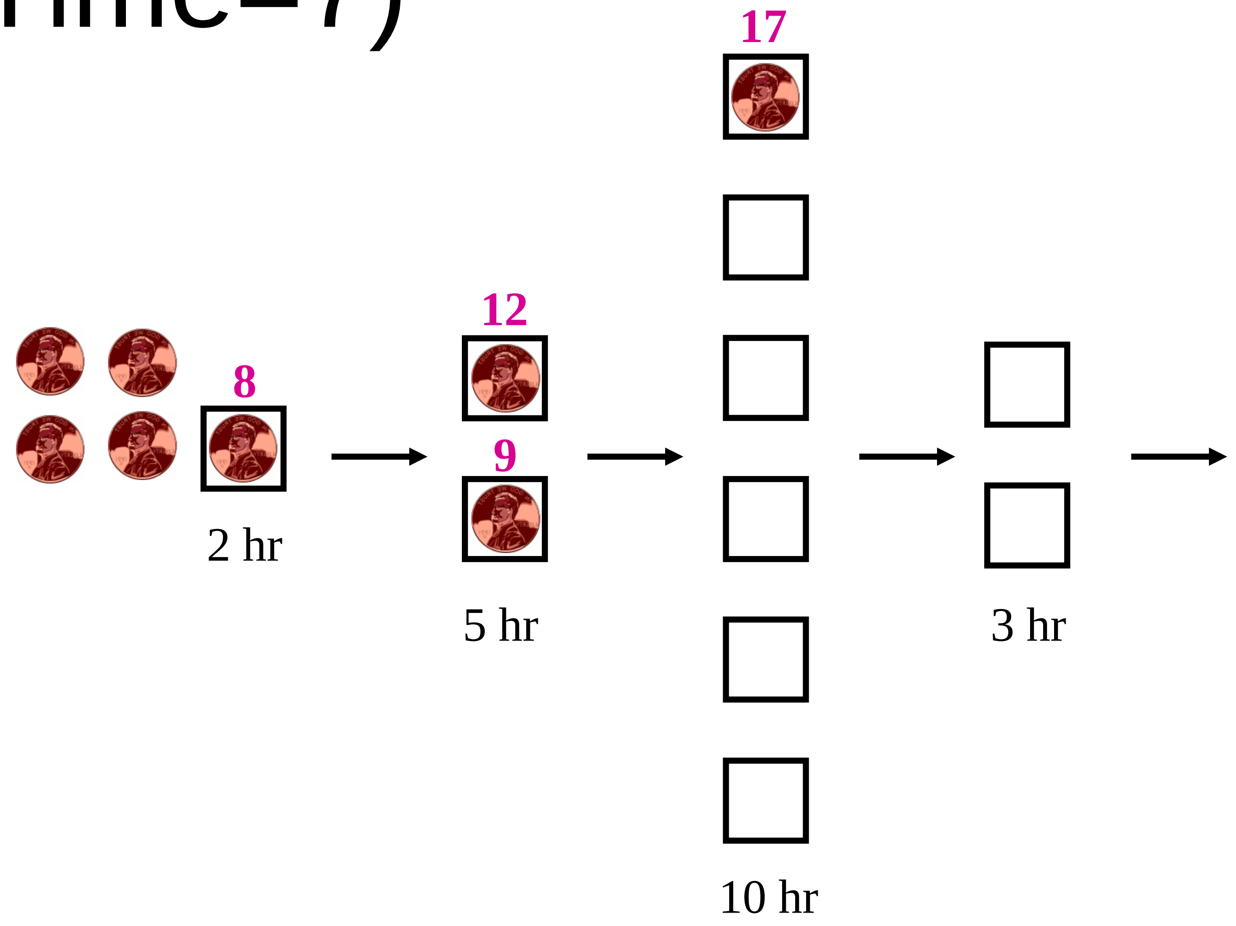
Penny Fab Two Simulation (Time=4)



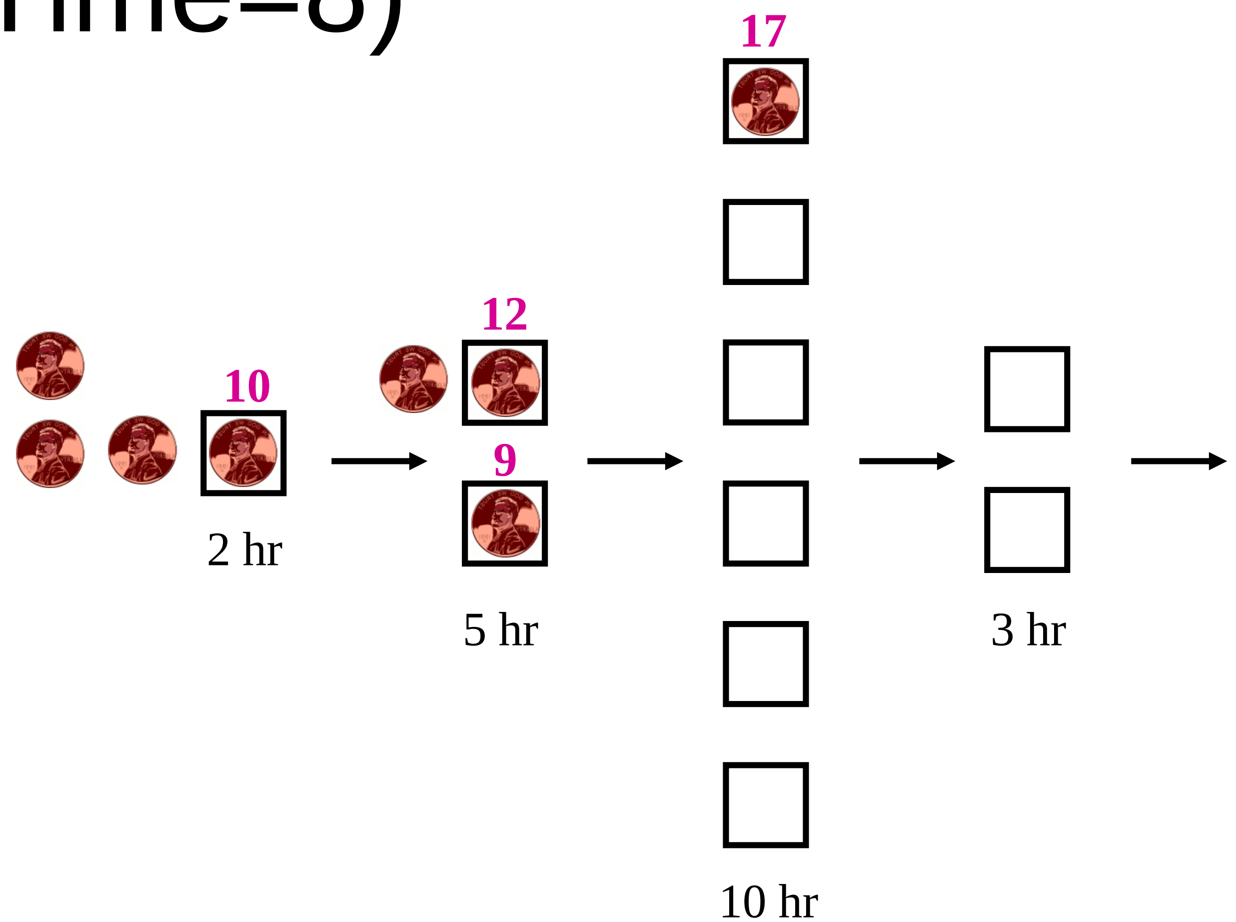
Penny Fab Two Simulation (Time=6)



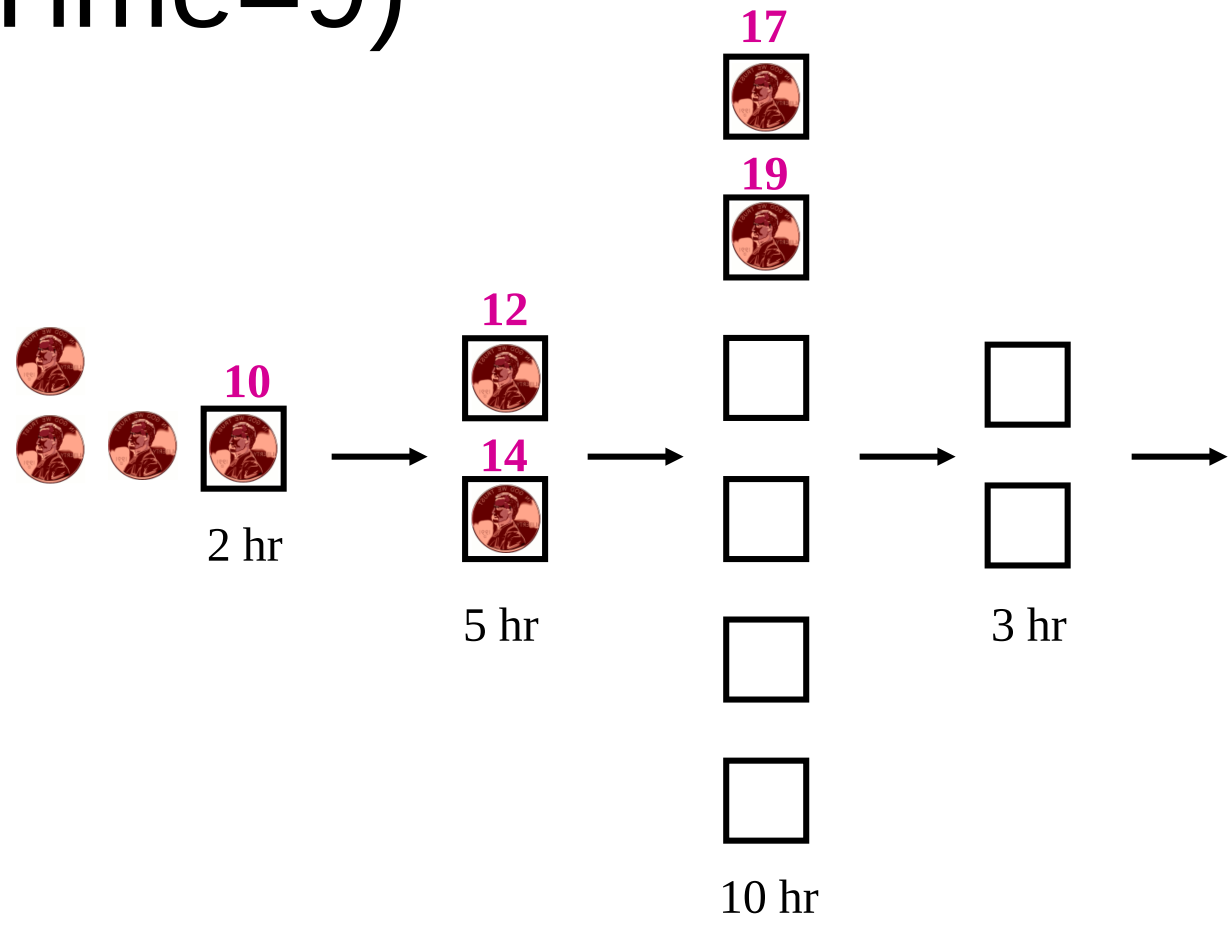
Penny Fab Two Simulation (Time=7)



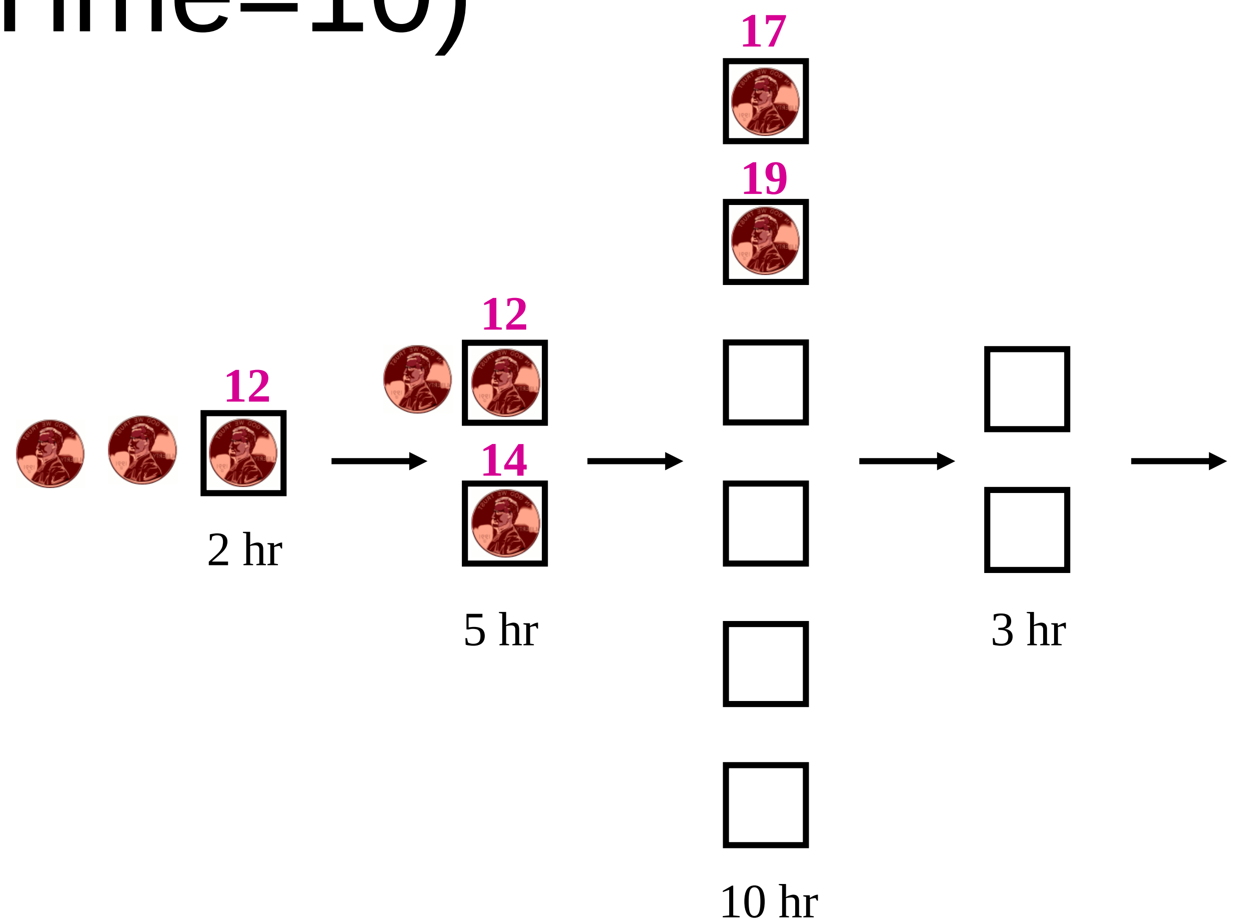
Penny Fab Two Simulation (Time=8)



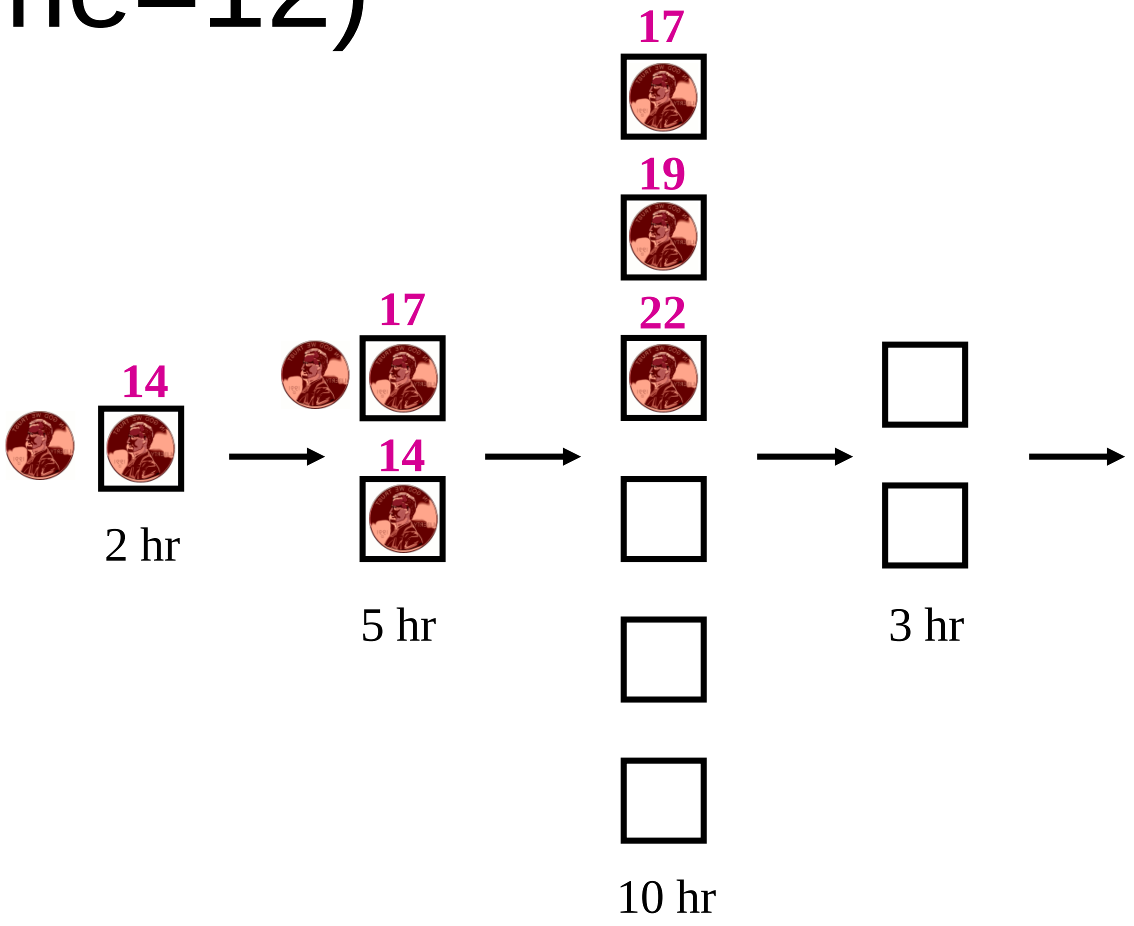
Penny Fab Two Simulation (Time=9)



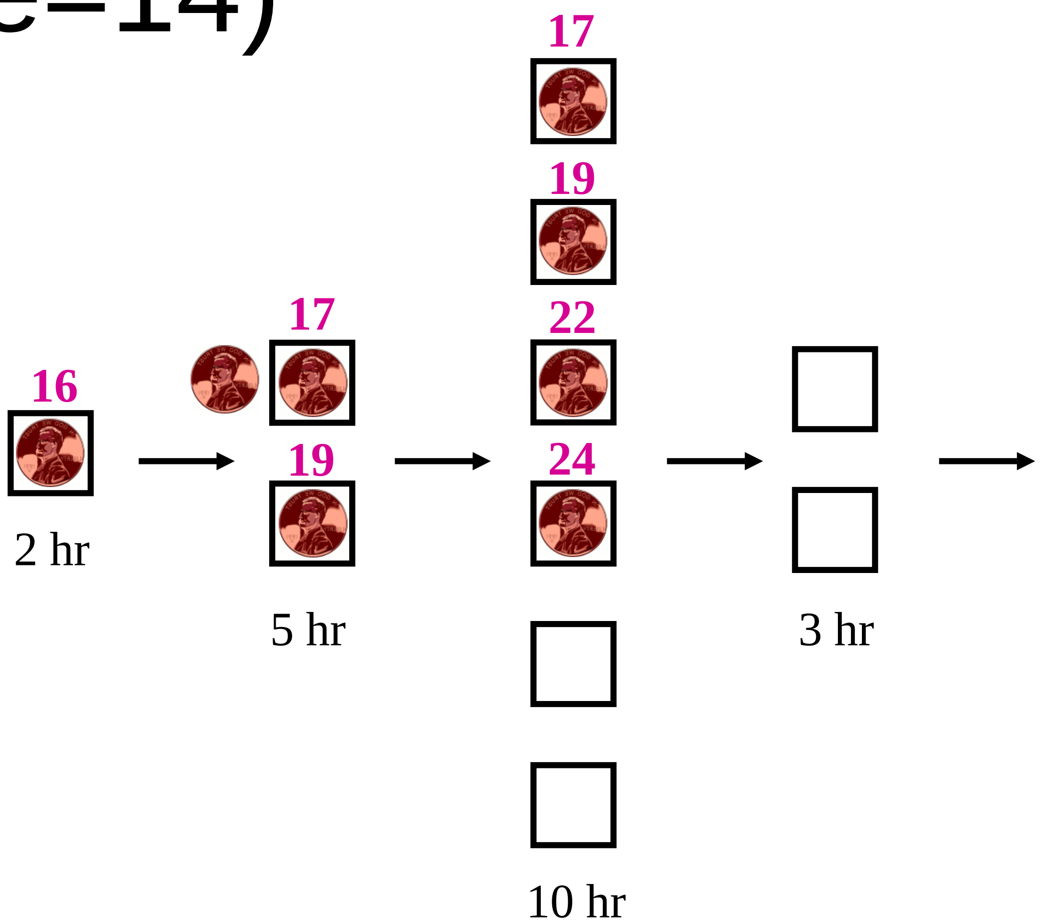
Penny Fab Two Simulation (Time=10)



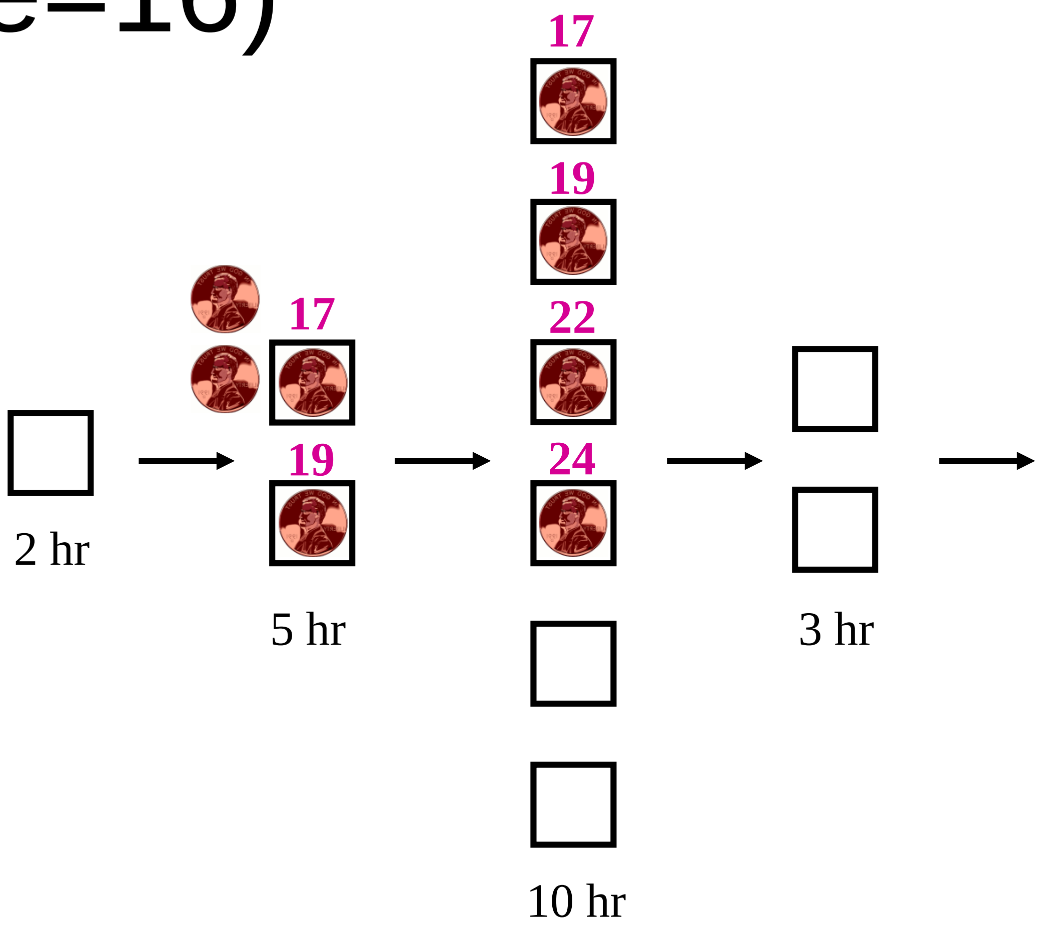
Penny Fab Two Simulation (Time=12)



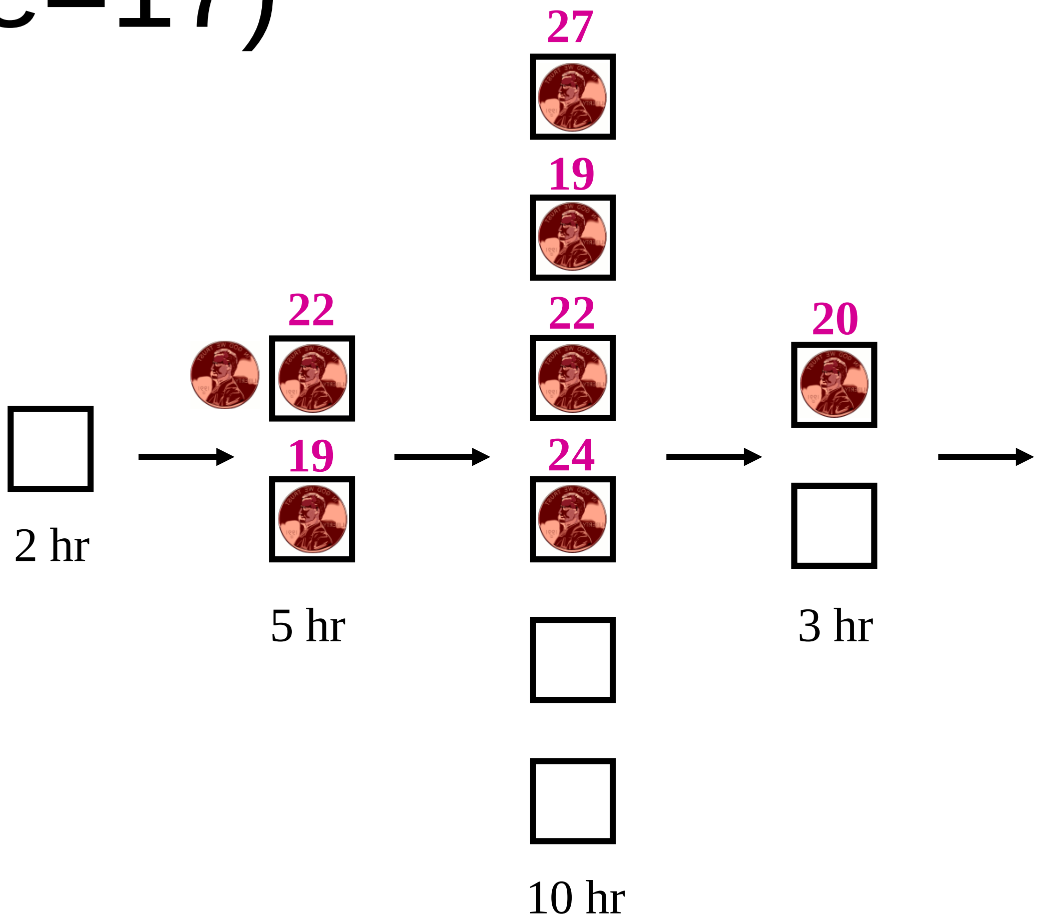
Penny Fab Two Simulation (Time=14)



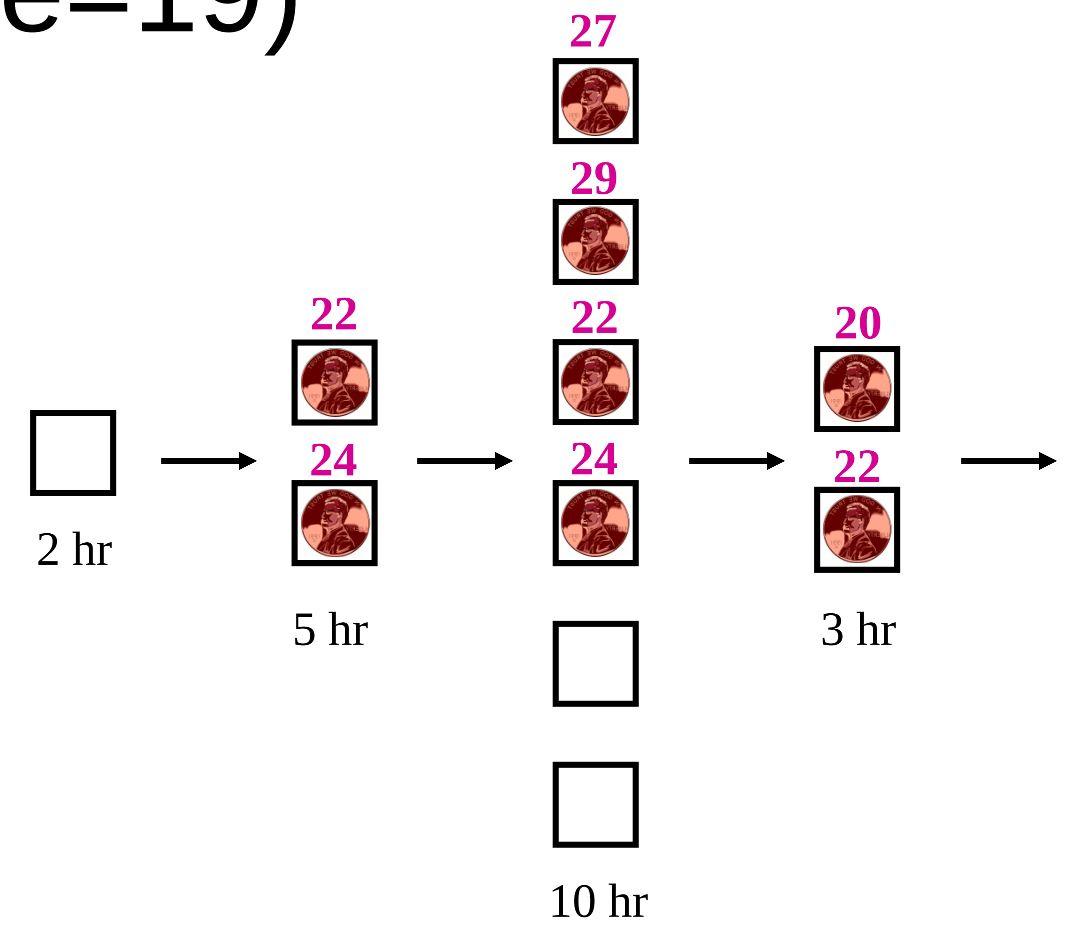
Penny Fab Two Simulation (Time=16)



Penny Fab Two Simulation (Time=17)

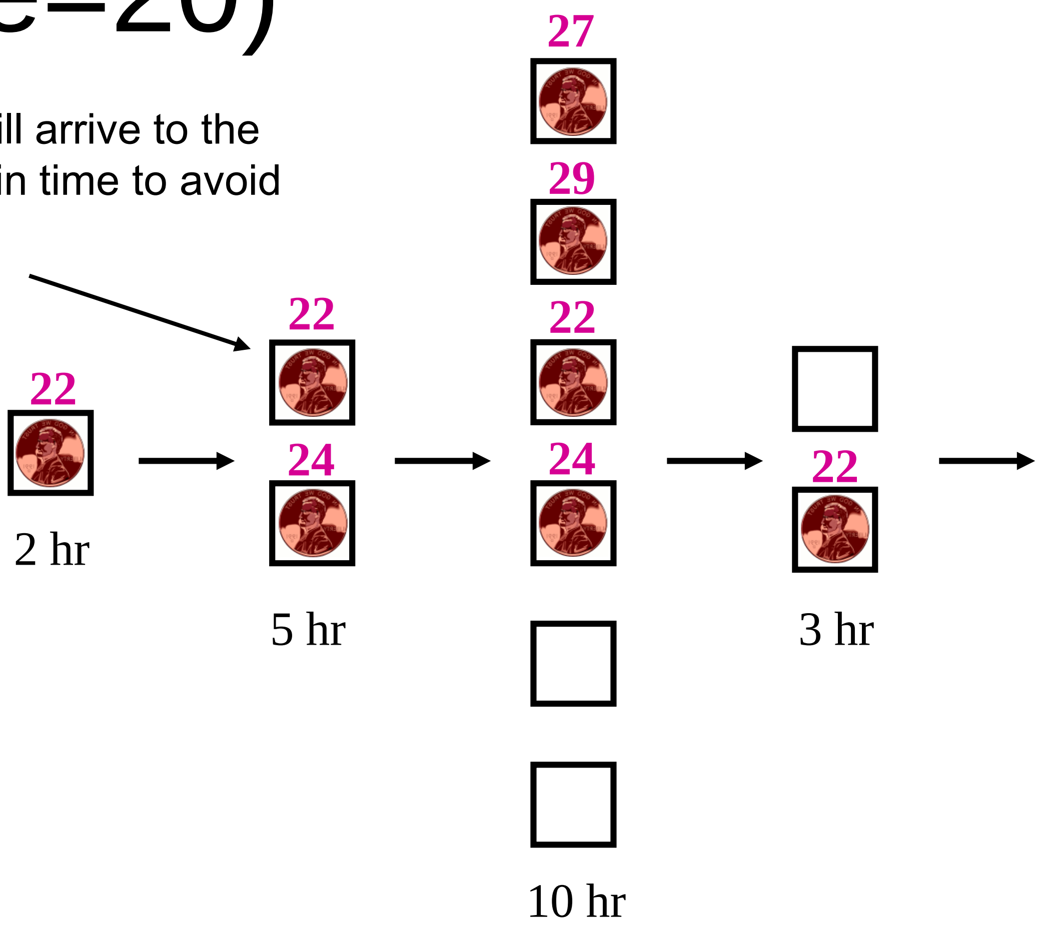


Penny Fab Two Simulation (Time=19)

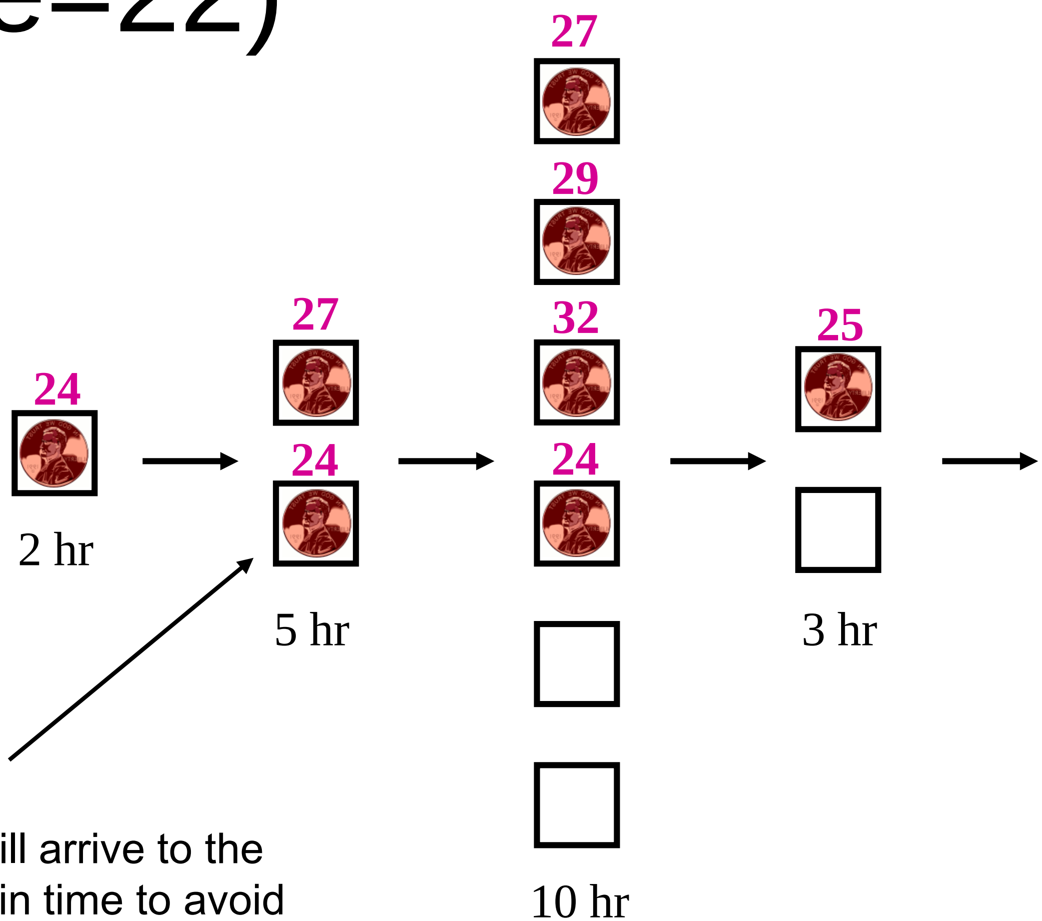


Penny Fab Two Simulation (Time=20)

Note: the job will arrive to the bottleneck just in time to avoid starvation

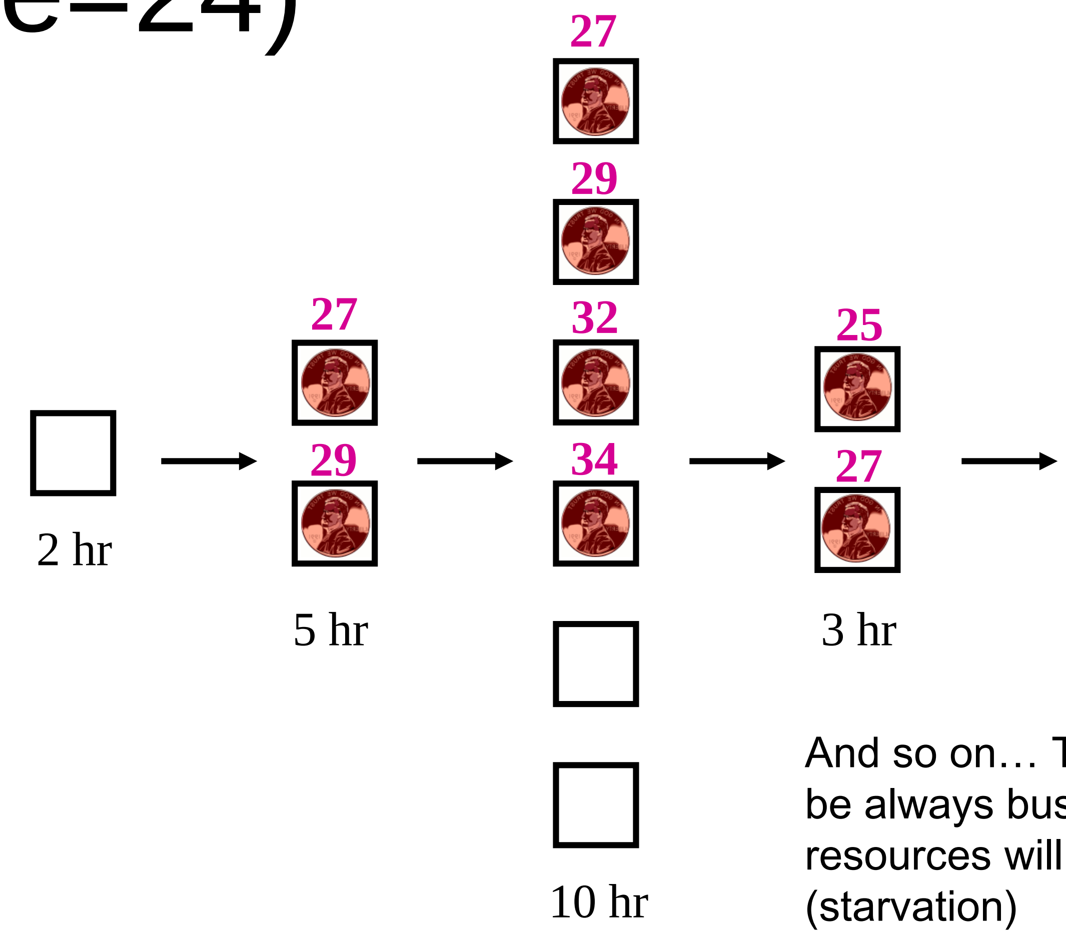


Penny Fab Two Simulation (Time=22)



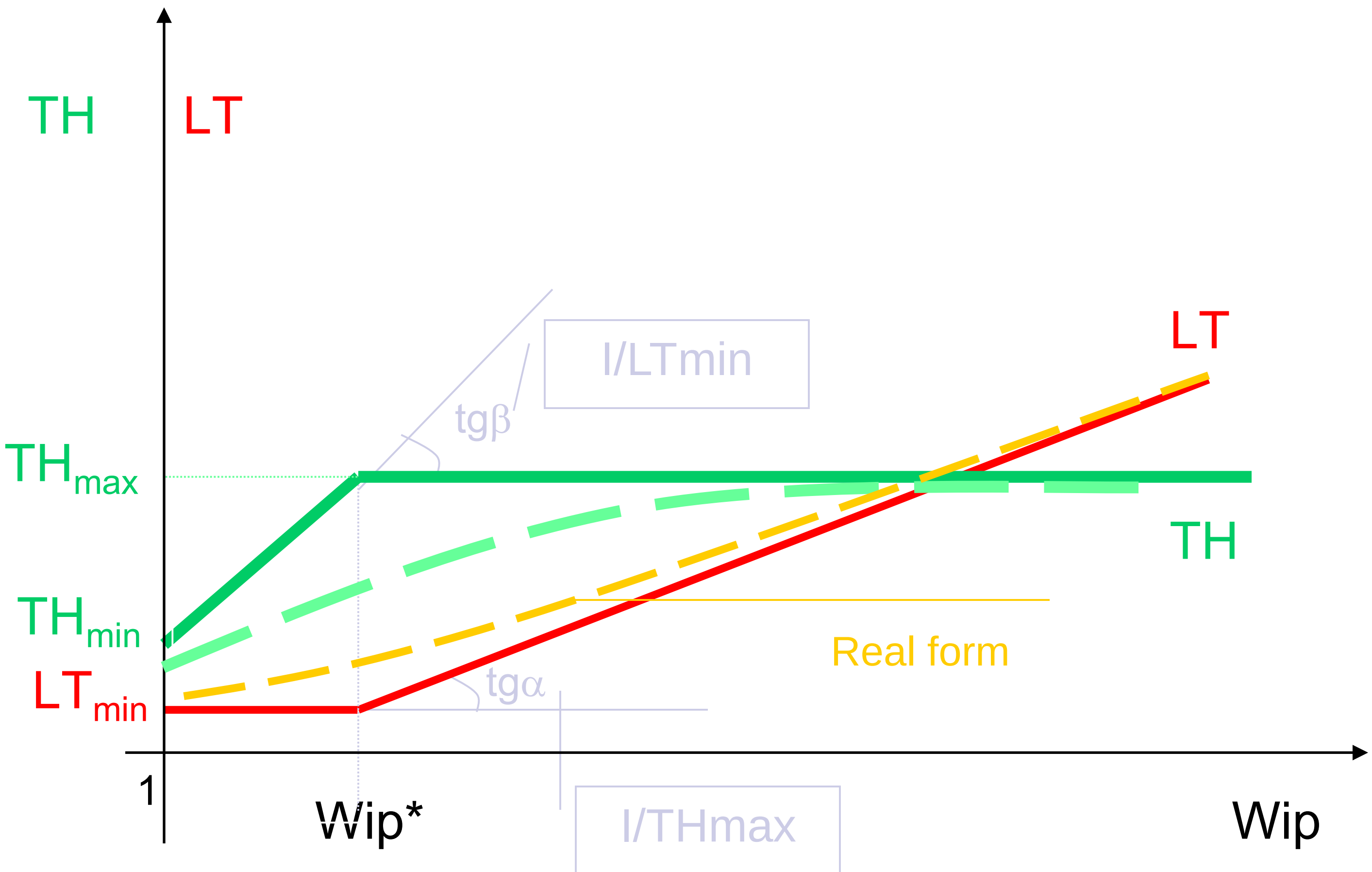
Note: the job will arrive to the bottleneck just in time to avoid starvation

Penny Fab Two Simulation (Time=24)



And so on… The bottleneck will be always busy, while the other resources will have idle periods (starvation)

Little's diagram



Conclusions

- In real cases, also for low level of WIP, the different pieces concurr in order to obtain system resources (not only machines, but also operators, transporters, tools) and, for this reason, the real form of the LT and TH diagrams is not exactly the one described by Little's Law.
- Penny Fab is only an ideal case with fixed time and balanced line
  - Wip* is equal to stations number and TH grows to the maximum value with LT = LTmin; from that point on LT grows without advantages.
  - In a non-ideal case (not balanced line) Wip* is less than stations number (e.g. Penny Fab Two).
- Also for balaced line, in case of non-fixed times, a wating time can occurr
- In all these cases, the specific form of diagrams should be evaluated case by case, for example with simulation
- On the basis of Little's Law, in order to reduce the LT of a system the Wip value should be reduced in order to mantain a constant TH
  - For this reason, if a system is characterized by the presence of queues, it should be possible to reduce LT and WIP.