Quantitative methods for economics, finance and management

2019/2020

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LAB 3

07.10.2019

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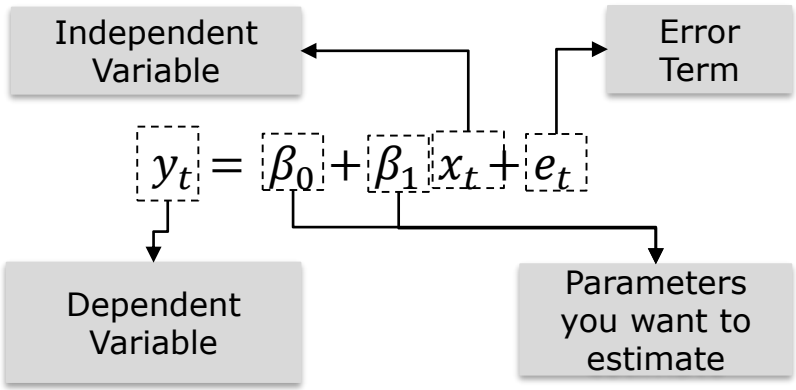
Lab-3: Simple Linear Regression Model (SLRM)

- In this lecture you are introduced to the simple linear regression model, which is estimated using the principle of least squares.
- Given a generic Bivariate Linear Regression Model:



- The main assumptions of the SLRM are:
  - ✓ Assumption 1: *The regression model is linear in parameters and it is correctly specified*
  - ✓ Assumption 2: *The expected value of error terms is zero*
  - ✓ Assumption 3: *Homoscedasticity of error terms*
  - ✓ Assumption 4: *No autocorrelation of error terms*
  - ✓ Assumption 5: *The X variables are not stochastic*
  - ✓ Assumption *6: Gaussian-Distribution of residuals*

Assumptions 2-3-4 are also called "Classical Assumptions Of Gauss-Markov"

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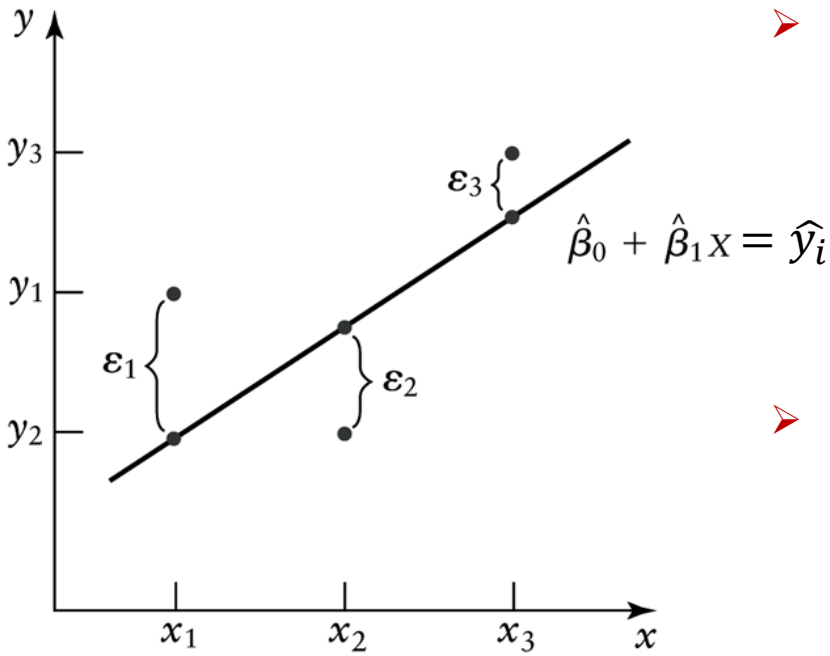

- To find analytically the parameters $\widehat{\beta_0}$ and $\widehat{\beta_1}$ you need to solve the following optimization problem:

$$\min_{\widehat{\beta_0}\,\widehat{\beta_1}} \sum_{i=1}^{N} \left(y_i - \widehat{\beta_0} - \widehat{\beta_1} x_i\right)^2$$

- In other words we are looking for those values ($\widehat{\beta_0}$ and $\widehat{\beta_1}$) such that minimize the square of the residuals (or the distance between y_i and $\hat{y}_i$) for each observation.

- The solutions of the minimization problem are:

$$\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\,\bar{x}$$

$$\widehat{\beta_1} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i}^{N}(x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}} = \frac{Cov(x,y)}{Var\,(x)}$$

- If the "classical assumptions" hold, Ordinary Least Square (OLS) is the Best Linear Unbiased Estimator (BLUE) – Gauss Markov Theorem

Lab-3: Simple Linear Regression Model (SLRM)

- THE COEFFICIENT (β_1) OF AN EQUATION USING OLS REGRESSION ANALYSIS PROVIDES AN ESTIMATE OF THE SLOPE OF A STRAIGHT LINE THAT IS ASSUMED TO BE THE RELATIONSHIP BETWEEN THE DEPENDENT VARIABLE (CONSUMPTION) AND THE INDIPENDENT " " (INCOME).

- THE SLOPE OF THE LINE IS THE FIRST DERIVATIVE AND TELL US THE MAGNITUDE OF THE IMPACT OF A ONE UNIT CHANGE IN THE X VARIABLE

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UPON THE VALUE OF THE y VARIABLE, MEASURED IN THE UNITS OF THE y (VARIABLE)

- THIS INTERPRETATION HOLDS WHEN USING THE LEVEL-LEVEL SPECIFICATION
- IN EVIEWS WE WANT TO OBTAIN THE ESTIMATES OF β_0, THAT IS $\hat{\beta}_0$ AND
  " " " β_1, " " $\hat{\beta}_1$

  BY USING THE FORMULA.

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Lab-3: Consumption dataset - SLRM

Equation: EQ01 Workfile: QMEFM_LAB3_1_081018::Castdataclass_v...

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: CONSUMPTION
Method: Least Squares
Date: 10/08/18 Time: 19:31
Sample: 1 40
Included observations: 40

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.383218	4.008356	1.841956	0.0733
INCOME	0.232253	0.055293	4.200378	0.0002

R-squared	0.317077	Mean dependent var	23.59450
Adjusted R-squared	0.299105	S.D. dependent var	8.176025
S.E. of regression	6.844922	Akaike info criterion	6.733598
Sum squared resid	1780.413	Schwarz criterion	6.818042
Log likelihood	-132.6720	Hannan-Quinn criter.	6.764130
F-statistic	17.64318	Durbin-Watson stat	2.370272
Prob(F-statistic)	0.000155		

IF INCOME INCREASES BY 1 $ THEN CONSUMPTION RISES BY 0,23 $

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$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{N} (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$$

WHERE $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$ AND $\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$

- FIRST, FIND $\hat{\beta}_1$; THEN FIND $\hat{\beta}_2$ (IN EVIEWS) WITHOUT USING THE COMMAND LS (LEAST SQUARES)

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Lab-3: Simple Linear Regression Model (SLRM)

Program: QMEFM_LAB3_071019 - (c:\users\asseg1\desktop\didattic

Run | Print | Save | SaveAs | Cut | Copy | Paste | InsertTxt | Find | Replace | Wrap+/- | Li

```
'Consumption-income model'
'Run the SLRM by using the "ls" (least squares) command'
ls consumption c income
'generate a  new vector x1 equals 1 (constant)'
genr x1 = 1
'generate a new vector x2 equals income (explanatory variable)'
genr x2 = income
'generate a new vector y equals consumption (dependent variable)'
genr y = consumption
'compute the sample average of consumption'
scalar avgy = @mean(y)
'compute the sample average of  income'
scalar avgx2 = @mean(x2)
'compute consumption in deviation from the mean '
genr ydevavgy = y - avgy
'compute income in deviation from the mean '
genr x2devavgx2 = x2 - avgx2
'compute the square of income in deviation from the mean '
genr x2devavgx2sq = x2devavgx2^2
'define new variable equals the number of observations'
scalar nobs = @obs(y)
'define k=2=number of explanatory variables (constant+ income)'
scalar k = 2
genr x2devydev= @sum(x2devavgx2*ydevavgy)
scalar x2dev_ydev = x2devydev(1)
```


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```
'define the sample covariance between x and y'
scalar covx2y =  x2dev_ydev/(nobs-1)
genr x2devAvgx_sq= @sum(x2devavgx2^2)
'define the sample variance of x '
scalar varx2 =  x2devAvgx_sq(1) /(nobs-1)
'compute the estimated coefficients'
scalar beta2hat = covx2y/varx2
scalar beta1hat = avgy- beta2hat*avgx2
show y
'compute the fitted values'
genr yhat = beta1hat*x1 + beta2hat*x2
'compute the residuals'
genr res = y-yhat
show res
'actual values  = fitted values +residuals'
genr y1 = yhat+res
```