

QMETM - LECTURE 10 Bi's

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Elon Autoregression

Model of interest:

$$y_t = \rho_1 + \sum_{j=2}^k \rho_j x_{t-j} + u_t \quad t = 1 - T$$

Model for u_t

$$\left\{ \begin{array}{l} u_t = \phi_1 u_{t-1} + \varepsilon_t \quad \text{AR(1)} \\ \text{Autocor} \quad u_t = u_t + \alpha_1 \varepsilon_{t-1} \quad \text{MA(1)} \end{array} \right.$$

CONCENTRATION OR AND(1)

TESTS FOR ENTER AUTOMATION

MOTIVATION: WE NEED TO CHECK IF THE
ERRORS ARE AUTOMATICALLY OR NOT - (IF THEY)
ARE, WE NEED TO TAKE INTO ACCOUNT THIS PROPERTY
SINCE ALSO ANE NO HAVING "BEST" -

$$A_n(z) := \boxed{y_t = \phi_1 y_{t-1} + \epsilon_t}$$

$$H_0 : \phi_1 = 0$$

(No Error Autocorrelation.)

$$H_1 : \phi_1 \neq 0$$

vs

In practice, $H_0 : \phi_1 = 0$ can be tested
 $\frac{\text{S.E.}(A)}{\text{S.E.}(A)} - t\text{-STAT} = \frac{\hat{\phi}_1 - 0}{\text{S.E.}(\hat{\phi}_1)} \sim \mathcal{T}_{(T-k)}$

Problems :

- 1) Since U_t and U_{t-1} are not observed,
we cannot use model $U_t = \phi_1 U_{t-1} + \epsilon_t$
To estimate ϕ_1

Solution: Replace U_t and U_{t-1} with
residuals \tilde{U}_t and \tilde{U}_{t-1} of

THE MODEL OF WINTERFJ

$$Ar(z): \hat{v}_t = \phi_1 \hat{v}_{t-1} + \epsilon_t$$

observed
observed

variable
variable

$$ols: \hat{y}_t = \frac{\sum_t \hat{v}_t \hat{v}_{t-1}}{\sum_t \hat{v}_{t-1}^2}$$

(
standard
error)

2) IT IS POSSIBLE TO SHOW THAT

$$t\text{-STRAT} = \frac{\text{SET}(\varphi_1)}{\text{SET}(\varphi_1) \cap \{T\}}$$

↳

IT DOES NOT HAVE A STUDENT'S
DICTIONARY SINCE φ_1 IS ROUNDED.

↙

CONCLUSION: WE CANNOT USE $t\text{-STRAT}$ TO TEST
 $H_0: \varphi_1 = 0$ (NO CONTRADICTION.)

$$U_t = \phi_1 U_{t-1} + \epsilon_t$$

$H_0: \phi_1 = 0$ (no error autocorrelation)

DURBAN - WATSON TEST

$$DW_{TEST} = \frac{\sum_{t=2}^T (O_t - \hat{O}_{t-1})^2}{\sum_{t=1}^T \hat{U}_t^2} \sim DW \text{ distribution}$$

WHERE O_t residual from the model
of interest

$$PV_{TEST}$$

$$= \frac{-T}{\sum_{t=2}^T v_t + (\sum_{t=2}^T v_{t-1}) - 2 \sum_{t=2}^T v_t v_{t-1}} \left(\sum_{t=1}^T v_t \right)$$

If T is large enough, then:

$$\sum_{t=1}^T v_t \approx \sum_{t=2}^T v_t = c_{t=2} v_{t-1}$$

$$\begin{aligned}
 D\chi_{\text{rest}} &= \frac{\sum_{t=2}^T v_{t-1}^2 + \sum_{t=2}^T \hat{v}_{t-1}^2 - 2 \sum_{t=2}^T v_t \hat{v}_{t-1}}{\sum_{t=2}^T \hat{v}_{t-1}^2} \\
 &= \frac{2 \sum_{t=2}^T \hat{v}_{t-1}^2 - 2 \sum_{t=2}^T v_t \hat{v}_{t-1}}{\sum_{t=2}^T \hat{v}_{t-1}^2} \\
 &= 2 \left(1 - \frac{\sum_{t=2}^T v_t \hat{v}_{t-1}}{\sum_{t=2}^T \hat{v}_{t-1}^2} \right)
 \end{aligned}$$

IF T IS LARGE ENOUGH

$$D\chi_{\text{test}} \approx 2(1 - \phi_1)$$

$$|\phi_1| < 1$$

DRAZIN-INVERSE DISTINCTION

CASE 1: $\boxed{\phi_1 = 0}$ (NO AUTONOMY)
SINCE $\hat{\phi}_1$ IS CONSISTENT FOR ϕ_1 (PROOF ON PAGE),
THEN $\hat{\phi}_1 \rightarrow 0 \Rightarrow \boxed{D\chi_{\text{test}} \rightarrow 2}$

CASE 2 :

$$\hat{\phi}_1 \rightarrow +1 \Rightarrow \text{DVR} \rightarrow 0$$

(positive photon)

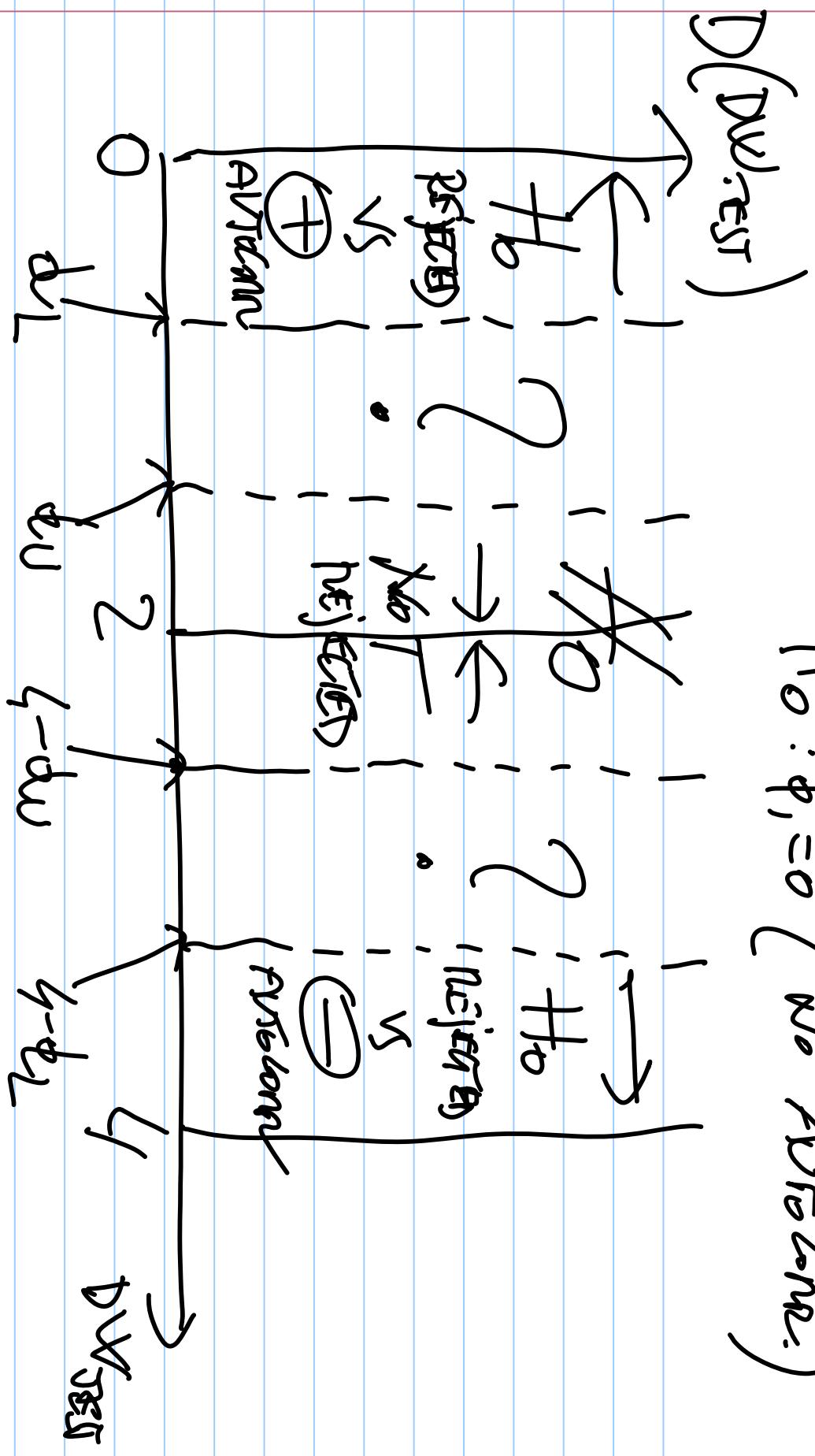
CASE 2 :

$$\hat{\phi}_1 > 0 \quad (\hat{\phi}_1 \rightarrow +1)$$

(negative photon)

$$\hat{\phi}_1 \rightarrow -1 \Rightarrow \text{DVR} \rightarrow 4$$

$\text{Ho} : \phi_1 = 0$ (no autocorr.)



χ^2_{ν} , T-k

ONE PAIR OF CRITICAL

VALUES

USEFUL TO IDENTIFY THE REJECTION

REGIONS (OR Z_α) OF THE DISTR-X TESTS

DISTRIBUTION

LIMITATIONS OF DUT TEST

- 1) ? REGIONS = INDEPENDENT REGIONS
- 2) TEST FOR AN(S) ONLY -

DEGENERATE

BREUSCH - GODFREY TEST

PETENDE MODEL FOR ALTERNATIVEN IS :

$$AR(p) \quad U_t = \phi_1 U_{t-1} + \phi_2 U_{t-2} + \dots + \phi_p U_{t-p} + \epsilon_t$$

/ $\phi \geq 1$

Auxiliary Test Equation

$$U_t = \beta_0 + \sum_{j=1}^k \beta_j X_{t-j} + \phi_1 U_{t-1} + \phi_2 U_{t-2} + \dots + \phi_p U_{t-p} + \varepsilon_t$$

Random Variables

Model of
INTEREST

RECALL FROM THE MODEL OF INTEREST
IT IS POSSIBLE TO SHOW THAT:

Asymptotically

$$R^2_{\text{TEST}} = T \cdot R^2_A \underset{H_0}{\approx} \chi^2_n$$

R^2 from the

Auxiliary estimator

where

$$H_0: \left[\phi_1 = \phi_2 = \cdots = \phi_p = 0 \right] \quad \text{(No linear relationship)}$$

Testing

SINCE $\phi_1, \phi_2, \dots, \phi_p$ ARE NOT KNOWN
IN ORDER TO CONSTRUCT R_{22} WE NEED TO
ESTIMATE $\phi_1, \phi_2, \dots, \phi_p$ JUMPING WITH $\beta_1,$
 β_2, \dots, β_p IN THE AUXILIARY EQUATION -
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$ ME CONSIDERED ESTIMATION
OF THE UNKNOWN PARAMETERS $\phi_1, \phi_2, \dots, \phi_p -$

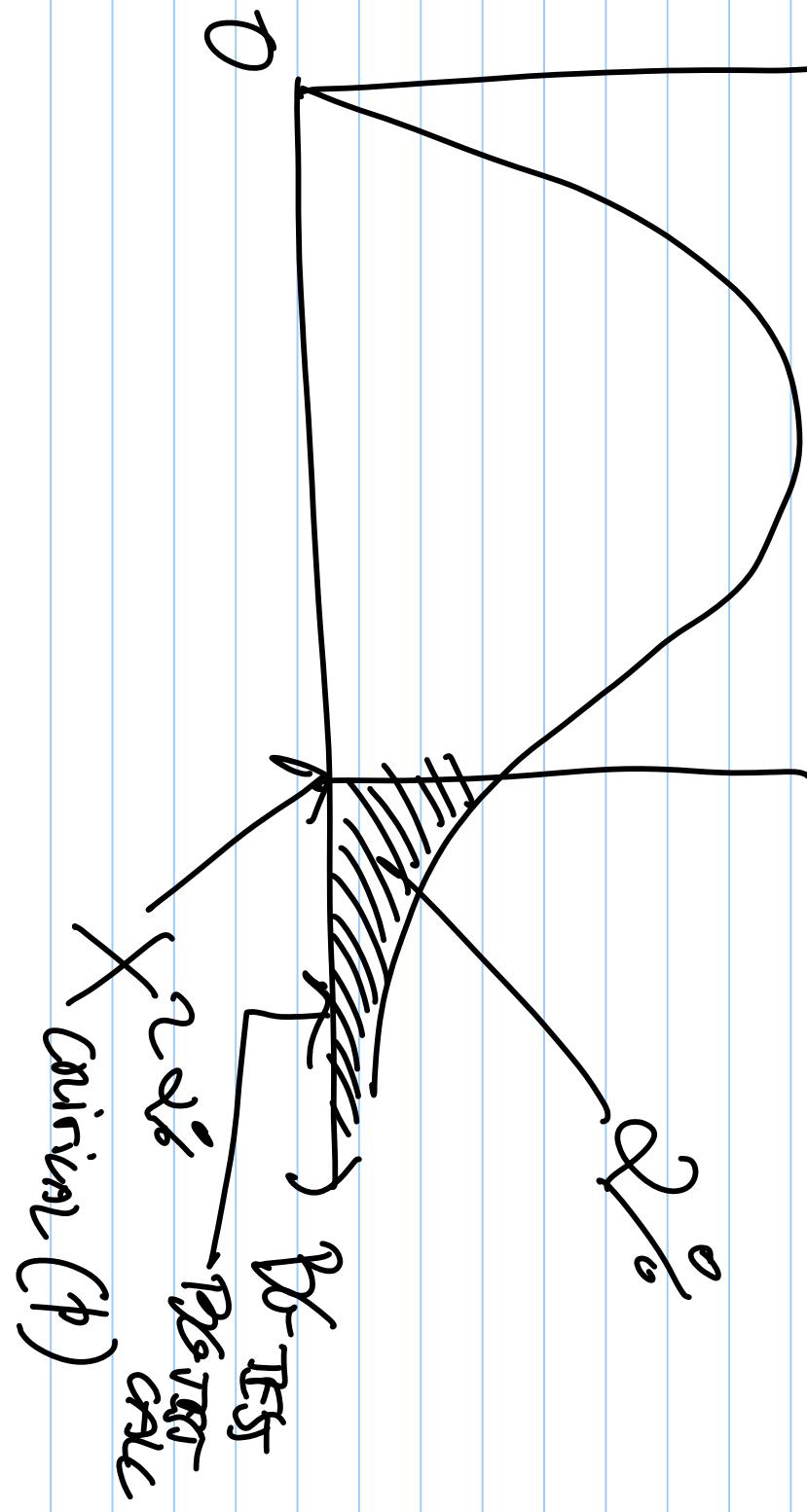
Consistency is an asymptotic property of
an estimator (i.e. it holds if \underline{T} is large

Enough)

When T is large enough,

$$\begin{bmatrix} Y \\ \bar{Y} \end{bmatrix} \xrightarrow{\text{large } T} N(\mu, \Sigma)$$

D (BC-TST)



If $B_{\text{TEST}} \text{ calc} = T \cdot n^2$ is lower
than $\chi^2_{\nu, p}$, then H_0 is not accepted.

is rejected at $\alpha \%$

Comment

W/ WHEN TO REJECT H₀: No error banevolon'

THE VALUE OF THE COMPUTED P-VALUE HAS TO

BE LOW

$$Bl_{\text{TEST}} = T \cdot n_A^2$$

T =

μ_0 observations
(fixed)

SINCE T IS FIXED, THE LENTER IS n_A^2

THE LENTER IS Bl TEST CALL

A CARIE \Rightarrow means that only the

VALUED PREDICATE ONE INHERIT EXPRESSION

Or I SOLVE THE X'S ONE BY ONE

To \cap — This means WILL THAT

IF P_2 IS VALUE WE CAN CONNECT TAKE

A DEFINITION AGAINST \Rightarrow : NO FORM CONNECTION!

ADVANTAGES OF THE BG TEST OVER THE DW TEST

- 1) BG TEST ALLOWS TO TEST FOR HOMOGENEITY OF VARIANCE ($p \geq 1$)
- 2) BG TEST HAS A ~~F~~ DISTRIBUTION, WHICH IS CONTINUOUS (NO "HOLES") -

WE HAVE USED BRE TEST / DCF TEST TO TEST

$p=1$ / i.e. $H_0: \phi_1 = 0$ vs $H_1: \phi_1 \neq 0$ INTO

THE $\rho_n(2)$ MODEL: $U_t = \phi_1 U_{t-1} + \varepsilon_t -$

BOTH TESTS HAVE REJECTED H_0

WHAT TO DO?

$$y_t = \beta_1 + \sum_{j=1}^k \beta_j \cdot x_{tj} + u_t$$

(1) (2)

$$u_t = \phi_1 v_{t-1} + \epsilon_t$$

CASE 1 : ϕ_1 known (Theoretical case)

WE NEED TO TEST FOR (1) v_{t-1} AND (2) ϕ_1

ORDEN TO APPLY DURBIN-WATSON TESTS

ON THE TRANSFORMED MODEL WITH DEJ
WE HAVE AUTOCORRELATED ERROR TERMS

$$(2) \quad U_t = \varphi U_{t-1} + \varepsilon_t$$

DIFFERENT FROM MODEL (1)

$$U_t = y_t - \rho_1 - \sum_{j=2}^k \beta_j \cdot x_{t-j}$$

$$y_{t-1} = y_{t-1} - \rho_1 - \sum_j \rho_j \cdot x_{t-1,j}$$

y_t

y_{t-1}

$$(y_t - \beta_1 - \sum_i \beta_i x_{ti}) = \phi_1 (y_{t-1} - \beta_1 - \sum_i \beta_i x_{t-1i})$$

TRANSFORMED MODEL

$$y_t - \phi_1 y_{t-1} = \underbrace{\left[\beta_1 (1 - \phi_1) + \beta_2 (x_{t2} - \phi_1 x_{t-12}) + \beta_3 (x_{t3} - \phi_1 x_{t-13}) + \dots + \beta_k (x_{tk} - \phi_1 x_{t-1k}) \right]}_{\hat{x}_{t*}} + \varepsilon_t$$

z_t

$$z_t^* = z_t - \varphi_1 z_{t-1}$$

(QUASI-DIFFERENCE
TRANSFORMATION)

TRANSFORMED MODEL :

$$y_t^* = \beta_1^* + \beta_2 x_{t2}^* + \beta_3 x_{t3}^* + \dots + \beta_k x_{tk}^* + \varepsilon_t$$

IS IT POSSIBLE TO REMOVE $\{x_t\}$ TO THIS TRANSFORMED

MODEL ADMINISTERED ? $E(x_t) = 0 \quad \forall t$

$$\text{Var}(x_t) = \sigma_x^2, \quad \forall t$$
$$\text{cov}(x_t, \varepsilon_t) = 0, \quad \forall t \neq s$$

SINK ϕ_1 IS known,
 $x_{t_2}^*, x_{t_3}^*, \dots, x_t^*$ ARE
NOT known

GENERATED
VIEWS
SERVES (LHS)

AS ON THE TRANSFORMED
MODEL

EXPLANATION IS BLUE -

CASE 2 : ϕ_1 not known (Empirical)

$$A_t(t) = \hat{C}_t = \phi_1 C_{t-1} + \varepsilon_t$$

↳ residuals from the model
of interest

$$\hat{\phi}_1 = \frac{\sum t C_t C_{t-1}}{\sum t^2 C_{t-1}^2}$$



) TRANSFORMED MODEL :

$$y_t^{**} = \beta_1^{**} + \beta_2 x_{t2}^{**} + \beta_3 x_{t3}^{**} + \dots + \beta_k x_{tk}^{**} + \varepsilon_t$$

WHERE

$$y_t^{**} = y_t - \phi_1 y_{t-1}$$

Estimation of ϕ_1

$$x_{t2}^{**} = x_{t2} - \phi_1 x_{t-12}$$

Previous variable

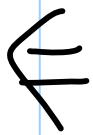
$$x_{tk}^{**} = x_{tk} - \phi_1 x_{t-k}$$

IS THE OLS ESTIMATOR APPROPRIATE IN THIS HETEROGENEOUS MODEL STILL BLUE?

$$\begin{aligned}E(\hat{\epsilon}_t) &= 0, \quad \forall t \\Var(\hat{\epsilon}_t) &= \sigma^2, \quad \forall t\end{aligned}$$

$$Cov(\hat{\epsilon}_t, \hat{\epsilon}_s) = 0, \quad \forall t \neq s$$

$$x_{t1}, x_{t2}, x_{t3}, \dots, x_{tk} \text{ are random variables}$$



WLS APPLIED TO THE TRANSFORMED
DATA (** VARIANCE) IS BIJED
IT IS POSSIBLE TO STUDY IT IS
CONSISTENT

The OLS Estimator applied to the
TRANSFORMED model (**^{*} variable) is
CONSISTENT, thus it is also known
as THE NOME TEST

FEASIBLE

Error

"PME, HETEROGENEITY
CHARGE DENSITY"
 $i = 1 \dots N$

Covalent-
non-covalent

(2) Model of interest

$$y_i = \beta_0 + \sum_{j=2}^k \beta_j x_{ij} + v_i$$

(1) $y_i = (\mu_i)_{\text{with}} \quad (2)$

Variables

Model for heteroscedasticity

(white noise)

$$y_i \sim (\alpha_0 + \alpha_1 x_{i1}^2 + \alpha_2 x_{i2}^2 + \dots + \alpha_k x_{ik}^2 + \gamma_1 x_{i1} + \gamma_2 x_{i2} + \dots + \gamma_k x_{ik}) + \epsilon_i$$

$$\text{Homo: } \alpha_2 = \alpha_3 = \dots = \alpha_k = \gamma_2 = \dots \quad (\text{Homoskedastic})$$

Point - - - $\gamma_k = 0$

$$\sigma_i^2 = \alpha_1$$

TEST FOR HOMOSKEDASTICITY (WHITE TEST)

AUXILIARY / TEST EXPRESSION

$$U_i^2 = \alpha_1 + \alpha_2 X_{i1}^2 + \dots + \alpha_k X_{ik}^2 + \beta_1 X_{i1} + \dots + \beta_k X_{ik} + \eta_i$$

↓
variance
residuals of the model of observation

constant
error term

Since σ_i^2 is not observed, we can
 measure $\underline{\sigma_i^2}$ by a proxy, i.e. $\hat{\sigma}_i^2$
 $\hat{\sigma}_i^2 = \sum_{i=1}^n \hat{U}_i^2$ is
 $\hat{\sigma}_i^2 = N - K$

} $\hat{\sigma}_i^2$ can be considered a proxy of σ_i^2

A proxy is a
represented variable
with measures
An unobserved variable with error

$$\hat{U}_i = \hat{\sigma}_i + \eta_i$$

white noise
Auxiliary equation

Auxiliary TEST expression

$$D_i^2 = \left[\alpha_1 + \alpha_2 x_i^2 + \dots + \alpha_k x_{ik}^2 + \beta_1 v_i + \dots + \beta_k v_{ik} + m_i \right]$$

$$D_i^2$$

$$\hookrightarrow H_0 : \alpha_2 = -\alpha_k = \beta_1 = \dots = \beta_k = 0 \\ (\text{Hypothesis})$$

$\text{Whitney} = N \cdot \pi^2 \tilde{\chi}^2_{(2(k-1))}$

If $\lambda_j \approx "large"$ then is if
the x_{ij} and the y_{ij} variables
 $(j=1 \dots k)$ are truly important to explain
the y_{ij} .

A SNE WITHIN TEST - REJECTS THE NULL -

WE NEED TO TAKE HETERO SKED AND

ALLOW) I.F. WE NEED TO TRANSFORM

THE MODEL OR STATEMENT IN SUCH A WAY THAT

OLS CAN BE APPLIED PROBABLY TO THE TRANSFORMING

MODEL WHICH HAS NOT HETERO SKED AND

CASE 1 :

$\alpha_1, \alpha_2, \dots, \alpha_k, \beta_1, \dots, \beta_k$

known

$$(1) \quad Y_i = \mu + \sum_{j=2}^k \beta_j X_{ij} + V_i$$

$$(2) \quad \hat{\alpha}_i = \alpha_1 + \alpha_2 X_{ii} + \dots + \alpha_k X_{ik} + \beta_1 X_{i1} + \dots + \beta_k X_{ik}$$

TRANSFORMED MODEL

TRANSFORM (1) BY TRANSFORM (2) IS ALLOWED

$$\rightarrow \text{Var}(U_i) = \sigma_i^2 \quad (2)$$

\rightarrow original error

$$U_i^* = \frac{U_i}{\sigma_i} \quad \text{TRANSFORMATION}$$

\rightarrow transformed error

$$\text{Var}(U_V^*) = \text{Var}\left(\frac{U_i}{\sigma_i}\right) =$$
$$= \frac{1}{\sigma_i^2} \text{Var}(U_i) = \frac{\sigma_i^2}{\sigma_i^2} = 1$$

$\text{Var}(U_i^*)$ is constant

Transformation of model (1) to take

into account model (2): Divide all
elements of model (1) by σ_i

$$\frac{y_i}{\sigma_i} = \beta_1 \frac{x_{i1}}{\sigma_i} + \beta_2 \frac{x_{i2}}{\sigma_i} + \dots + \beta_k \frac{x_{ik}}{\sigma_i} + \epsilon_i$$

y_i^* constant x_{i1}^* x_{i2}^* x_{ik}^*

↓ can we apply it to the transformed model (\neq uniques)?

$$E(V_i^*) = E\left(\frac{U_i}{\sigma_i}\right) = \frac{1}{\sigma_i} E(U_i) = 0$$

$$\text{Var}(V_i^*) = 1 \quad (\text{Homoscedasticity})$$

$$\text{Cov}(U_i^*, U_j^*) = 0 \quad (\text{No Error Correlation})$$

↓ $X_{t2}^*, X_{t3}^*, \dots, X_t^*$ not random

obj on the Transformation Model (* note)

is BLUE

GUS

CASE 2 : $\alpha_1, \alpha_2, \dots, \alpha_k, \beta_2, \dots, \beta_k$

Unknown

↓
WE CAN ESTIMATE THESE PARAMETERS

FROM THE ANALYSIS EQUATION:

$$\left. \begin{array}{l} \hat{\alpha}_1, \hat{\alpha}_2, \dots, \hat{\alpha}_k, \hat{\beta}_2, \dots, \hat{\beta}_k \\ \rightarrow \hat{\alpha}_1, \hat{\alpha}_2, \dots, \hat{\alpha}_k, \hat{\beta}_2, \dots, \hat{\beta}_k \end{array} \right\}$$

$$\boxed{\hat{\alpha}_i^2 = \hat{\alpha}_1 + \hat{\alpha}_2 x_{i2}^2 + \dots + \hat{\alpha}_k x_{ik}^2 + \hat{\beta}_2 x_{i2}}$$

TRANSFORMED MODEL WITH σ_i

$$y_i = \beta_0 + \beta_1 \frac{x_{i1}}{\hat{\sigma}_i} + \beta_2 \frac{x_{i2}}{\hat{\sigma}_i} + \dots + \beta_k \frac{x_{ik}}{\hat{\sigma}_i} + \epsilon_i$$

IS THE OLS ESTIMATOR ON THIS MODEL TRANSFORMED BLUE? NO

SINCE $x_{i2}^{**}, x_{i3}^{**}, \dots, x_{ic}^{**}$ ARE
PASSED REFLECTIONS

IT IS POSSIBLE TO HAVE THAT ONLY ONE

TRANSFORMED MODEL IS CONSIDERED

TG L

(THE END)