The Forward Parity Puzzle

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Abstract

The forward parity condition states that the forward exchange rate is an unbiased and efficient forecast of the future spot rate. The condition is an implication of market efficiency in the absence of a time varying risk premium. There have been numerous tests of the forward parity condition in the academic and applied literature on foreign exchange markets and these tests have led to a unanimous rejection of the forward parity condition. Not only has the forward rate failed to predict the future spot rate, but it has generally pointed in the wrong direction. If the forward rate is above the current spot rate, the future spot rate is more likely to fall than rise. Since the forward rate is a reflection of short term interest rate differentials, the failure of the forward parity condition implies that high yielding currencies have tended to appreciate against low yielding currencies. This is equivalent to high dividend stocks appreciating in value against low dividend stocks. The purpose of this paper is to review and extend the early literature on the forward parity condition. The results suggest that the simple interest rate differential model should be replaced with a more sophisticated model based upon both real and nominal interest rate differentials. The extended model has continued to exhibit surprising levels of profitability through the recent period of greater monetary stability.

Introduction

The forward exchange rate plays two distinct roles in the foreign exchange market. Through the interest rate parity condition, the forward rate provides a link between the spot exchange rate and short term interest rate differentials. When offshore deposit rates are employed as representative short term interest rates, the interest rate parity condition holds very tightly. Market participants often state that forward exchange rates are mechanically priced off the interest rate parity relationship. The second role for the forward rate is as a forecast of the future spot rate. The forward parity condition states that the forward exchange rate is an efficient and unbiased forecast of the future spot rate. The immediate implication of this condition is that short term interest rate differentials are determined by expectations of the appreciation or depreciation of the exchange rate. The alternative view is that interest rates are predominantly determined by domestic economic conditions and monetary policy. If the central bank tightens monetary policy, this will cause short term interest rates to increase relative to US dollar rates. The higher interest rates will make the local currency more attractive to international investors. In their attempt to take advantage of this opportunity, the investors will bid up the local currency. In equilibrium the appreciation will be sufficient to induce expectations of a subsequent depreciation which will offset the benefit of the higher short term interest rate. This description of the market, which was originally developed by Rudiger Dornbusch (1976), neatly combined the domestic determination of interest rates with the forward parity condition.

The forward parity condition can be derived from the hypothesis that the excess rate of return from foreign exchange speculation is zero. Consider a strategy of borrowing one dollar and investing the proceeds in a foreign currency. The end of period payoff on the strategy is then:

$$(1) \qquad r_{t+1} = \frac{S_{t+1}}{S_t}(1+i_t^*) - (1+i_t)$$

where *S* represents the exchange rate, expressed as the US dollar price of a unit of the foreign currency, *i** is the foreign interest rate, and *i* is the US interest rate. The first term is the dollar payoff on the foreign currency investment and the second term is the dollar payment on the loan. An alternative way of writing the excess return is:

$$(2) \qquad r_{t+1} = \frac{S_{t+1} - S_t}{S_t}(1+i_t^*) + (i_t^* - i_t)$$

This description of the excess return emphasizes that it is comprised of two components: a capital appreciation component determined by the appreciation of the exchange rate and a dividend yield component determined by the interest rate differential. In order for the expected return to be zero, these two components must offset each other. High yielding currencies should depreciate against low yielding currencies. Another version of the relationship employs the interest rate parity condition, as defined in equation (3):

(3) $$F_t = S_t \frac{1+i_t}{1+i_t^*}$$

where *F* is the forward rate corresponding to the maturity of the interest rates. Combining (1) and (3) yields:

(4) $$r_{t+1} = \frac{S_{t+1} - F_t}{F_t}(1+i_t)$$

In this formulation, the necessary condition for the expected excess return to be zero is that the forward exchange rate be equal to the expected future spot rate, a condition that I will refer to as the forward parity condition. Empirical testing of the forward parity condition began in the late 1970's in papers by Hansen and Hodrick (1980), Bilson (1981) and Fama (1984). These studies all reported strong rejections of the forward parity condition. Subsequent studies have confirmed these results.[1] There is also an active theoretical literature which attempts to determine if the failure of the forward parity condition is due to risk aversion or market segmentation rather than market inefficiency.[2] These theological issues will not be addressed in this paper.

The simplest test of the forward parity condition involves a regression equation relating the ex-post excess return to the nominal interest rate differential.

(5) $$r_{t+1} = \alpha + \beta(i_t^* - i_t) + \varepsilon_{t+1}$$

Equation (2) demonstrated that the interest rate differential was one component of the excess return. If the currency return is unrelated to the interest rate differential, then the slope coefficient in the regression would be unity. If the forward parity condition held, one would expect both regression coefficients to be zero.

Since this equation forms the starting point for the investigation in this paper, Table 1 reports estimates of the regression for the six currencies studied. The estimates are based upon quarterly data over the period from 1976:02 to 1997:01. The starting point of the estimation period was based upon the availability of data. The end point was chosen to allow for five years of post-sample testing of the model. Quarterly observations were employed because the quarterly time horizon is the most liquid maturity in the Eurocurrency market and

[1] Lewis (1995) and Engel (1996) provide recent reviews of the literature..
[2] Fama (1984), Hodrick and Srivastava (1986) and Bekaert and Hodrick (1992).

because quarterly observations give a better signal to noise ratio than the more usual monthly approach. The model was estimated using Zellner's seemingly unrelated regression procedure with Hansen's hetroscedastic consistent estimator of the covariance matrix.[3]

Table 1: The Basic Model

	Alpha	**Beta**
Australian Dollar	-0.0068	1.2793**
	(0.0055)	(0.4698)
Canadian Dollar	-0.0055	1.5327**
	(0.0030)	(0.4924)
DeutscheMark/Euro	0.0107	1.9327**
	(0.0073)	(0.4758)
Japanese Yen	0.0307**	3.3720**
	(0.0071)	(0.5437)
Swiss Franc	0.0192*	2.0680**
	(0.0092)	(0.5453)
British Pound	-0.0069	2.0687**
	(0.0072)	(0.6647)

Notes: Asymptotic standard errors are presented in brackets beneath coefficients.
** Significantly different from zero at the 1% confidence level.
* Significantly different from zero at the 5% confidence level.

These results confirm the early studies of Hansen and Hodrick (1980), Bilson (1981), and Fama (1984) and the many subsequent studies of the forward parity condition. Specifically, all of the beta coefficients are greater than unity and all are significantly greater than zero at standard levels of statistical significance. A joint test of the hypothesis that all of the regression coefficients are zero is firmly rejected with this data. The fact that the regression coefficient is greater than unity means that the expected return from this strategy exceeds the interest rate differential. In other words, the trader not only benefits from the higher foreign interest rate – the dividend yield – but also from the appreciation of the exchange rate against the dollar – the capital gain. When the foreign interest rate is below the U.S. rate, the trader will borrow the foreign currency and invest in dollars. Because the regression coefficient is greater than unity, this strategy will also benefit from a dividend yield and the appreciation of the dollar against the foreign currency.

The failure of the forward parity condition to hold is one of the great puzzles in international finance.[4] While forward prices have often been found to be poor predictors of future spot prices in other markets – particularly equity and

[3] The estimation procedure was NLSYSTEM in RATS with the ROBUST option.
[4] Lewis (1995), Engel (1996) and Froot and Thaler (1990) discuss the forward parity puzzle.

fixed income – it is very unusual for the forward price to be a perverse forecast of the future spot price.[5] Fama (1984) attempted to explain the results in terms of a time varying risk premium but his model only works when the regression coefficient lies between zero and one. Furthermore, there is little to no evidence that the expected returns from foreign exchange speculation are related to conventional risk premia in equity markets. The more reasonable explanation is that nominal interest rates are predominantly determined by domestic economic conditions like expected inflation and the business cycle. While hedge funds and other foreign exchange speculators benefit from the interest rate differential game, they are not large enough to eliminate all major differences in interest rates.[6]

There is, however, another aspect to the results reported in Table 1. that needs to addressed. While there is a substantial commonality in the estimates of the slope coefficient, there are substantial differences in the estimates of the constant terms. In some cases, Japan and Switzerland for example, the constant terms are significantly different from zero. The constant terms are negative for the Australian Dollar, Canadian Dollar and British Pound and they are positive for the DeutscheMark/Euro, Japanese Yen and Swiss Franc. If the interest rate differentials were zero, the first three currencies would tend to appreciate against the dollar while the second three would tend to depreciate. It is noticeable that the first three currencies have tended to have interest rates that have historically been higher than U.S. rates and the second three currencies have tended to have lower rates than in the U.S. We can formalize this insight by defining the "threshold rate differential" as that interest rate differential at which the expected return on the forward exchange contract is zero.

(6) $$E(r_{t+1}) = 0 \quad \rightarrow \quad (i_t^* - i_t) = -\frac{\alpha}{\beta}$$

In Table 2., the results reported in Table 1. are used to estimate the threshold rate differentials for the basic model.

[5] Bilson and Cernauskas (2003) examine the relationship between the foreign exchange market and the eurocurrency deposit market in more detail. Their evidence suggests that eurocurrency forward prices correctly forecasts the direction of change in the spot interest rate.

[6] Alvarez, Atkeson and Kehoe (1999) develop a model of the forward parity puzzle based upon segmented asset markets.

Table 2: Threshold Rate Differentials

Currency	Threshold
Australian dollar	2.13%
Canadian dollar	1.44%
DeutscheMark/Euro	-2.21%
Japanese Yen	-3.64%
Swiss Franc	-3.71%
British Pound	1.33%

These results demonstrate that the simple axiom of borrowing the lowest yielding currency and investing in the highest yielding currency is misplaced. In the case of the yen, for example, the Japanese interest rate has to be 3.64% below the U.S. rate before it begins to be profitable to borrow Yen and invest in dollars. Similarly, Australian interest rates would have to be 2.13% above U.S. rates before it is expected to be profitable to borrow U.S. dollars and invest in Australian dollars.

Once the importance of the threshold rate differential has been recognized, it becomes necessary to provide accurate estimates of its level. On a purely econometric level, the threshold rates are estimated imprecisely because there is a strong negative covariance between constant and slope coefficients in the regressions. We can reduce the influence of this factor by imposing the constraint that the slope coefficients are the same for all of the currencies. Using the Wald test, it was not possible to reject the hypothesis that the slope coefficients were the same for all currencies over the whole sample period. Second, the data sample is split into two decade long sub-samples in order to investigate if the constant terms are varying through time. The results of this exercise are presented in Table 3.

Table 3: Estimates of the Constrained Model

	1976-1997	1976-1987	1987-1997
Australian Dollar	2.42%	3.73%	0.73%
Canadian Dollar	1.42%	1.65%	1.01%
DeutscheMark/Euro	-2.27%	-3.94%	-0.52%
Japanese Yen	-4.19%	-5.60%	-2.43%
Swiss Franc	-3.74%	-5.98%	-1.29%
British Pound	1.18%	1.39%	0.87%
Slope Coefficient	1.8007	1.9864	1.7087

The statistics presented in Table 3 include the estimated threshold rate differentials for the three periods and the common regression slope coefficient. It is clear from these results that the threshold rate differentials are not constant. In the Japanese Yen example, the differential has declined from -5.60% in 1976 to 1987 to -2.43% in 1987 to 1997. Similar changes are evident for the other countries. The main reason for the change is that the post 1987 period witnessed an important change in monetary strategy by the world's central banks away from activist Keynesian policies towards programs for monetary stability. As a consequence, the major developed countries have experienced a stabilization in interest rates and a convergence in rates across the major currencies. This is reflected in the smaller threshold rate differentials in the second period.

If this view is correct, the threshold rate differentials should reflect longer term inflationary expectations. Using the assumption that real interests rates are similar in the long run, we can use long term bond yields to represent inflationary expectations. In Figure 1, the relationship between the threshold rate differential and the 10 Year benchmark bond differential is plotted for the two sub-samples.

Figure 1: Threshold Rates

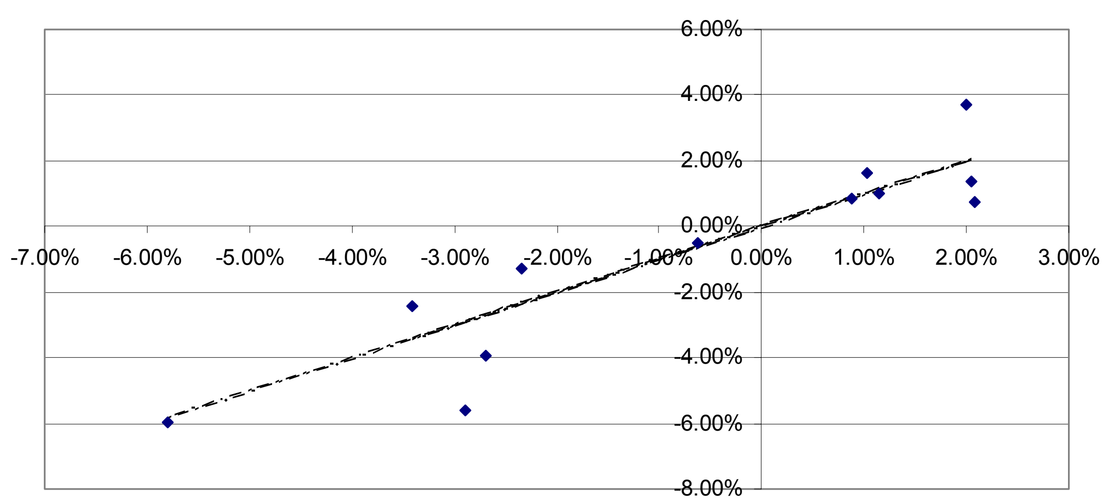


Figure 1. demonstrates that there is indeed a close relationship between the bond yield differential and the threshold rate differential. We incorporate this factor in the following extended version of the forward parity condition:

(7) $$r_{t+1} = \beta_1(i_t^* - i_t) + \beta_2(i_t^* - i_t - g_t^* + g_t) + \varepsilon_{t+1}$$

In this equation, '*g*' represents the 10 Year government bond yield. The underlying assumption is that the bond yield differential is a measure of the expected inflation rate differential. The first term in equation (7) consequently represents the nominal interest rate differential while the second term represents the real interest rate differential. There are no constant terms in equation (7) since the threshold rate differentials are assumed to be captured by the bond yield differential. Estimates of equation (4) are presented in Table 4.

Table 4: The Augmented Model

	1976-1997	1976-1987	1987-1997
Beta-1	0.6927*	0.3948	0.7898
	(0.2895)	(0.3675)	(0.4879)
Beta-2	1.0858*	1.0271	1.2412
	(0.5297)	(0.6537)	(0.8369)
Wald Test	28.56	9.40	17.27
Probability(2 d.f.)	6.26E-07	0.009	1.77E-04

While the individual coefficients are not precisely estimated in the augmented model, this is because the two indicator series are highly correlated[7]. The Wald Test in Table 4. tests the hypothesis that both coefficients are zero. The data lead to a rejection of this hypothesis at the 1% confidence level in all three data sets. Since it is difficult to distinguish the effect of the nominal and real interest rate differentials, we may as well simplify the forecasting task by assuming that both regression coefficients are unity. This hypothesis cannot be rejected at standard levels of statistical significance. The simple model is consequently:

(8) $$r_{t+1} = \left[i_t^* - i_t\right] + \left[(i_t^* - i_t) - (g_t^* - g_t)\right] = 2(i_t^* - i_t) - (g_t^* - g_t) + \varepsilon_{t+1}$$

The simple model makes a lot of sense. The return on the currency position is the sum of the yield differential and the currency appreciation. The nominal yield differential, with a coefficient of one, represents the yield component of the currency return. The real interest rate differential represents the currency appreciation factor. [8] In order to calculate the expected return on a currency forward contract, simply take twice the short term nominal interest rate differential and subtract the government bond yield differential.

While the regression models can be used to test for the statistical significance of the risk premium, they are less useful as descriptive tools for describing the size and evolution of the premium. For this reason, my early paper on this topic developed a speculative trading rule from the regression analysis.[9] By examining the expected and actual profitability of the rule, we can determine if the returns are of a sufficient size and stability to bring into question the efficiency of the market – or, more accurately, the degree of integration of the international

[7] The estimated correlation coefficient between the two parameter estimates is -.60.
[8] This approach is reminiscent of Jeffrey Frankel's synthesis of the Bilson/Frenkel monetary model and the Dornbusch overshooting model of the exchange rate. In Frankel's approach, the bond yield represented the expected inflation and the short rates represented the real cost of credit.
[9] Bilson (1981). Bilson and Hsieh (1984) provided an earlier update of the performance of the system.

financial market – or whether the effect is simply another anomaly whose influence depreciates as the market incorporates its effects into their trading strategy. It is doubtful whether any of the academics working in this area in the early eighties believed that the violations of forward parity would continue to be strong after twenty years of published research. Others feel that the convergence of nominal interest rates in the last decade has finally demonstrated that the market has caught on to the profitability of the carry trade. My own take on this issue is that the convergence in interest rates is itself the temporary phenomena because of the substantial differences between monetary and fiscal policies across the major developed countries. In the next section, the in-sample and post-sample profitability of the speculative strategy will be examined.

In-Sample Speculation

It has often been said that anyone can make money in-sample and it is true that the smallest amount of in-sample information is sufficient to make a strategy look good relative to a benchmark. The problem is that in-sample information is effectively the only information that we have to work with until the post-sample coughs up the new data. Even if the post-sample data supports the original model, there is no presumption that this will continue to work because the post-sample support will encourage new entrants to arbitrage away the profits. In this sense, demonstrating that a market is inefficient is roughly equivalent to hunting down the pot of gold at the end of the rainbow. No matter how close we get, we are always a good distance from the final objective. With this in mind, let us introduce the following exposition of the speculative strategy.

A trader has a sample of T observations on N assets. We define the in-sample utility of the trading strategy as:

(9) $$U = q'r - \frac{1}{2\lambda} q'\Sigma q$$

In this formulation, *'q'* is an NTx1 vector of positions, *'r'* is a vector of asset excess returns, λ is the coefficient of relative risk tolerance, and Σ is an NTxNT covariance matrix that is assumed to be known by the trader. For ease of exposition, we shall assume that the relative risk tolerance coefficient is unity. This corresponds to the logarithmic utility function. The trader's objective is to determine if the utility of the strategy is significantly different from zero.

Taking expectations, we have:

(10 $$E(U) = q'E(r) - \frac{1}{2} q'\Sigma q$$

The trader chooses the position vector to maximize the expected utility. The optimal position vector is given by:

(11 $$q = \Sigma^{-1}E(r)$$

Substituting this result back into (9), we obtain:

(12 $$U = E(r)'\Sigma^{-1}r - \frac{1}{2} E(r)'\Sigma^{-1}E(r)$$

As in the earlier exposition, we assume that the returns are determined by the system of regression equations:

(13) $r = X\beta + \varepsilon$

where X is an NTxK matrix of pre-determined variables and β is a conformable vector of regression coefficients. In addition, the residuals are assumed to be contemporaneously correlated but serially uncorrelated. Substituting (13)t back into (12), we express the level of in-sample utility as a function of the regression coefficients.

(14) $U = \beta' X' \Sigma^{-1} r - \frac{1}{2}\beta' X' \Sigma^{-1} X \beta$

Equation (14) expresses the in-sample utility as a function of the regression coefficients and the data. Maximizing in-sample utility through the choice of the coefficients leads to:

(15) $\hat{\beta} = (X'\Sigma^{-1}X)^{-1}(X'\Sigma^{-1}r)$

This is, of course, the GLS estimator of the regression coefficients. If the residuals are assumed to be serially uncorrelated but contemporaneously correlated, then the estimator described in equation (12) reduces to the Zellner SUR estimator.

At the optimum, the in-sample utility can be expressed as:

(16) $U = \frac{1}{2}\beta' \left[(X'\Sigma^{-1}X)^{-1}\right]^{-1} \beta$

Since $(X'\Sigma^{-1}X)^{-1}$ is the GLS estimator of the covariance matrix of the regression coefficients, the expression in equation (13) amounts to a Wald test of the hypothesis that all of the regression coefficients are zero. If we can reject this hypothesis, then we can reject the hypothesis that in-sample utility is zero.

In order to implement the test, it is necessary to calculate $X'\Sigma^{-1}X$. Under the standard assumption that the residuals are contemporaneously but not serially correlated, this quadratic form can be written as:

(17) $X'\Sigma^{-1}X = X'\,\Omega^{-1} \otimes I\ X = \sum_{i=1}^{I}\sum_{j=1}^{J}\sigma^{ij} X_i' X_j$

This matrix has dimensions K x K, where K is the number of regression coefficients, rather than NT x NT. The symbol, σ^{ij}, stands for the "i, j" element of the inverse of the covariance matrix of the residuals. A similar expression holds for $X'\Sigma^{-1}y$.

In the preceding discussion, three alternative specifications of the X matrix were explored. In the basic model, each currency had its own constant and slope coefficient. In the restricted model, the slope coefficient was assumed to be the same for all of the instruments in the portfolio. Finally, in the augmented model, the constant terms were removed and the government bond differential was introduced. In Table 5, the Wald tests for the three models are described.

Table 5: Testing for the Significance of In-Sample Utility

	Wald Test	Deg.Freedom	Probability
Model - 1	49.2730	12	1.87E-06
Model - 2	41.9487	7	5.32E-07
Model - 3	28.5666	2	6.26E-07

For each model, we can decisively reject the hypothesis that the in-sample utility of the strategy was zero. If we are prepared to assume that the process generating the returns is stationary, then these results should give some confidence that the mechanism that has worked in the past will continue to work in the future.

The test for in-sample utility proposed here is closely related to tests of market efficiency in equity markets. In their seminal contribution to this field, Gibbons, Ross and Shanken (1989) suggest the following test for the CAPM. Let the returns be generated by:

(18) $$r_{it} = \alpha_{ip} + \beta_{ip} r_{pt} + \varepsilon_{it}$$

In this formulation, r_{it} is the excess return on the i'th asset and r_{pt} is the excess return on the portfolio whose efficiency is being tested. The null hypothesis that the market is efficient corresponds to a test of the hypothesis that all of the alpha coefficients are zero. This is similar to the test proposed here. The difference is that instruments are assumed to be uncorrelated with the market portfolio, so that the beta's are assumed to be zero, and that the alpha coefficients are time varying with interest rate differentials.

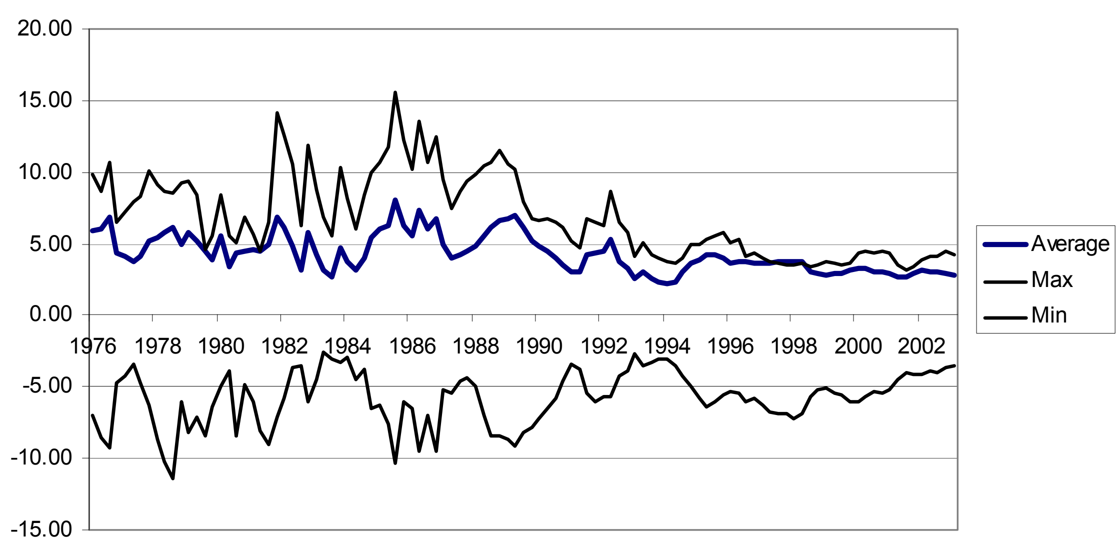
The preceding results have demonstrated that we can reject the forward parity hypothesis with a high degree of confidence in the period from 1976 to 1997. The problem with this type of result is that the regression coefficients, and indeed the trading strategy itself, could not have been known to market participants at the time since they are based upon data that was only available subsequently. As mentioned above, the first tests of the forward parity model occurred in the early 1980's and the results were well known in the subsequent period. The famous IBM-World Bank currency swap, for example, was initiated by Robert McNamara's strategy of borrowing funds in low yielding currencies. The World Bank was hitting credit limits in Switzerland and they were able to circumvent these constraints by entering into the swap contract with IBM. Since this original transaction, the global volume of currency swap transactions has grown exponentially. By 1997, the start of our post-sample analysis, any investment banker who was not aware of the historical profitability of the "carry trade" was simply not worth his bonus.

More recently, the prevailing view in the market is that the "carry trade" was profitable in the early period of high and variable interest rates but that the good times have departed with the global convergence of interest rates in the period since 1987. We can measure convergence using a simple measure of interest rate dispersion:

(19) $$D_t = \sqrt{\sum_{n=1}^{N} (i_t^n - i_t^{us})^2 / N}$$

This dispersion index measures the standard deviation of national interest rates around the U.S. rate for each period. The history of the index is described in Figure 2.

Figure 2: Yield Dispersion



.In the period from 1976 to 1990, the average interest rate differential against the dollar was around 5% and differentials on particular currencies were often sharply higher than this average value. Figure 2 demonstrates the global convergence in interest rates in the 1990's has cut the average yield spread to around 3%. Further evidence of convergence is found in the spread between the largest and smallest interest rate differentials against the dollar. In the 1980's, it was not difficult to find currencies with interest rates that were 10% above or below the U.S.[10] In the 1990's, it is difficult to find yield differentials that exceed 5% against the major trading currencies. On the other hand, it is true that there are emerging currencies with attractive yields that are beginning to be traded on a global basis. Currencies like the Mexican peso, South African rand, and Russian rouble are the new favorites of the carry trade. While these currencies are interesting, the purpose of this paper is to explore the forward parity condition as it relates to the major traded currencies.

To summarize, there are three major reasons why the post-sample performance of the strategy should be examined. First, in-sample testing is inherently suspect in markets in which learning is taking place. In the post-sample test, the regression parameters and the covariance matrix from the in-sample period are employed. Second, the post-sample test allows the use of daily data that first becomes available in 1997. The daily data is a more accurate model of the actual activities of currency traders, few of whom share Warren Buffett's view that he wouldn't care if they closed down the stock market for five years. Finally, the post-sample experiment allows us to explore the influence of globablization on the profitability of the carry trade.

Post-Sample Simulation

The data for the post-sample simulation consist of daily observations on exchange rates, three month Eurocurrency rates, and representative 10 Year government bond yields at the London (UK) close. [11] When we move from quarterly to daily data, we have to account for the influence of changes in yield on the rate of return. If a position is taken at time 't' and marked to market at time 't+k' the return on the position can be written as:

(20) $$r_{t+k} = \frac{S_{t+k}}{S_t} \frac{(1+(91/365)i_t^*)}{(1+((91-k)/365))i_{t+k}^*} - \frac{(1+(91/365)i_t}{(1+((91-k)/365)i_{t+k})}$$

where '*days*' represents the number of days until the next market close. This variable is 1 on weekdays and 3 over the weekend. The excess return on the currency is consequently:

(21)

$$r_{t+1} = \frac{S_{t+k} - F_t}{F_t} \frac{(1+(91/265)i_t}{(1+((91-k)/365))i_t^*} - \frac{(1+(91/365)i_t}{(1+(91-k)/365)i_{t+k})(1+(91-k)/365)i_{t+k}^*} \frac{91-k}{365} (i_{t+k}^* - i_{t+k})$$

[10] In my 1981 paper, I distinguished between yield differentials that were greater than or less than 10% in absolute value. The values exceeding 10% were found to be considerably more profitable that the values that were less than 10%. Bilson (1981).

[11] The data source is DataStream. Further information n the data is contained in the Data Appendix.

The first term in this expression is basically the change in the exchange rate relative to the forward rate. The second term reflects the impact of changes in interest rates on the value of the contract. Higher foreign yields reduces the value of long positions in the foreign currency. However, it is also reduces the value of the USD loan financing the position. In most instances, the effect of the interest rates on the value of the return is very minor.

As mentioned above, the post-sample simulation uses the estimated covariance matrix from the in-sample estimation. To convert the quarterly covariance to the daily, the covariance is divided by 91, the number of calendar days in a quarter. This raises the issue of what to do about the weekend. There is little or no evidence that the volatility over the weekend is larger than the volatility over a weekday. This suggests that the model should be specified in trading days rather than calendar days. The problem with this approach is that the trader still receives three days of interest over the weekend. This suggests that the return to risk tradeoff is higher over the weekend. If we were to use daily interest rates, we could explore whether the interest rate differentials tend to shrink on Friday. However, since we are using 3 Month rates, we will ignore this effect on the positions. The expected returns are consequently defined by:

(22) $$E(r_{t+1}) = \beta_1 (i_t^* - i_t)/365 + \beta_2 (i_t^* - i_t - (g_t^* - g_t))/365$$

where '*I*' and '*g*' represent the 3 Mth Eurocurrency deposit rate and the 10 Year Government bond yield respectively. Using this formulation abstracts from large over the weekend positions.

There are three variants of the expected return model that will be investigated:

Case 1: The Random Walk Model

$\beta_1 = 1, \quad \beta_2 = 0$

In Case 1, the underlying assumption is that the exchange rate follows a random walk so that the best forecast of the future spot rate is the current spot rate. The expected return on a long or short currency position is simply the nominal interest rate differential. The mean variance optimizer simply maximizes the yield on the portfolio for a given degree of risk.

Case 2: The Augmented Random Walk Model

$\beta_1 = 1, \quad \beta_2 = 1$

In Case 2, the underlying assumption is that the exchange rate appreciates at a rate equal to the real interest rate differential. The return on the currency is equal to the nominal interest rate differential – the dividend yield – and the rate of price appreciation – the capital gain. If a country is in a recession, as reflected in a positively sloped yield curve, the augmented random walk model will predict a depreciation of the exchange rate which will have to be offset by the yield differential.

Case 3: The Estimated Model

$\beta_1 = .70, \quad \beta_2 = 1.09$

In Case 3, the estimated coefficients from the regression equations are used to formulate the expected returns. In-sample, it was determined that this model is not significantly different from the augmented random walk model although it did, by definition, outperform that model in-sample. The estimated model is included in the post-sample to investigate the possibility that regression based forecasting models can improve upon rules of thumb in post-sample simulations.

Having defined the expected rates of return, we now turn to the speculative strategy. The trader is assumed to characterize her risk/return preferences in terms of a mean-variance expected utility framework.

(23) $$E(U) = q'E(r) - \frac{1}{2\lambda} q'\Omega q$$

In equation (23), *q* is an Nx1 vector of positions, *E(r)* is an Nx1 vector of expected returns, and Ω is an NxN covariance matrix. λ is the coefficient of relative risk tolerance. The degree of risk tolerance is peripheral to the issue of the statistical significance of speculative profits since it scales both the portfolio return and the standard deviation of the return. However, for illustrative purposes, we set this coefficient equal to .2 which appears to give a portfolio with risk characteristics that are similar to those of the stock market. The optimal position is found by maximizing *E(U)* with respect to *q.*

(24) $$\hat{q} = \lambda \Omega^{-1} E(r)$$

Substituting this result back into (23). we obtain:

(25) $$E(\hat{U}) = \frac{\lambda}{2} E(r)'\Omega^{-1} E(r)$$

In financial terms, the term $E(r)'\Omega^{-1}E(r)$ is clearly the square of the Sharpe ratio of the portfolio. In statistical terms, the term is a Wald test of the hypothesis that the elements of the vector of expected returns are all zero. Under the null hypothesis, the statistic follows a Chi-squared distribution with N degrees of freedom. However, since the estimates of the expected returns are based upon arbitrary coefficients, the statistical interpretation should be viewed in purely psychological terms. In other words, the trader may be 95% confident that the expected utility of the portfolio is positive, but this does not tell us too much about whether the actual utility of the strategy is significantly different from zero.

Jobson and Korkie (1981) were the first researchers to explore the statistical aspects of evaluating portfolio performance through the Sharpe ratio. Suppose that we have two vectors of portfolio returns from which we calculate two Sharpe Ratios:

(26) $$\hat{s}_1 = \frac{\hat{\mu}_1}{\hat{\sigma}_1}, \quad \hat{s}_2 = \frac{\hat{\mu}_2}{\hat{\sigma}_2}$$

We wish to examine the hypothesis $s_1 - s_2 = 0$. That is, we want to test if the two strategies have the same Sharpe Ratio. We can define the parameter set as:

(27) $$K = \left[\mu_1, \mu_2, \sigma_1^2, \sigma_2^2\right]$$

If there are T observations on each portfolio, then the following large sample result can be derived:

(28) $$\sqrt{T}(\hat{K} - K) \rightarrow N(0, \Phi)$$

with

(29) $$\Phi = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & 0 & 0 \\ \sigma_{12} & \sigma_2^2 & 0 & 0 \\ 0 & 0 & 2\sigma_1^4 & 2\sigma_{12}^2 \\ 0 & 0 & 2\sigma_{12}^2 & 2\sigma_2^4 \end{bmatrix}$$

Jobson and Korkie propose the following test statistic:

(30) $$z = \hat{\sigma}_1\hat{\sigma}_2\left(\frac{\hat{\mu}_1}{\hat{\sigma}_1} - \frac{\hat{\mu}_2}{\hat{\sigma}_2}\right) = \hat{\sigma}_2\hat{\mu}_1 - \hat{\sigma}_1\hat{\mu}_2$$

The asymptotic variance of this estimator is[12]:

(31) $$v(z) = \frac{1}{T}\left[2\sigma_1^2\sigma_2^2 - 2\sigma_1\sigma_2\sigma_{12} + \frac{1}{2}\mu_1^2\sigma_2^2 + \frac{1}{2}\mu_2^2\sigma_1^2 - \frac{\mu_1\mu_2}{\sigma_1\sigma_2}\sigma_{12}^2\right]$$

The natural starting point for the post-sample performance is with the raw returns from the three strategies. Summary statistics are provided in Table 6.

[12] This definition of the variance corrects a typographical error in the original Jobson and Korkie article. For details, see Memmel (2003).

Table 6: Comparative Analysis of Raw Excess Returns

	Model-1	Model-2	Model-3
Actual Return (Annual)	23.03%	36.15%	30.42%
Expected Return (Annual)	10.94%	22.98%	15.98%
Standard Deviation (Annual)	17.25%	24.53%	20.29%
Sharpe Ratio (Annual)	2.81	3.11	3.16
Probability	0.24%	0.09%	0.07%
JK against Model-3	-1.04	-0.71	
Probability	14.94%	23.39%	

These results strongly confirm that the carry trade has continued to be profitable in the post-sample period. The simple yield model represented in Model-1 had an average expected return on 10.94% over the 1997 to 2003 period. The actual performance was over twice the expected result. This is again due to the fact that high yield currencies have tended to appreciate against low yielding currencies over this period. The Sharpe ratio of the actual returns against a null hypothesis of zero is 2.81% so that we can reject the hypothesis that the true mean of the Model-1 returns is zero at the 1% confidence level. The Jobson-Korkie statistic against Model-3 is -1.04. This demonstrates that Model-1 is inferior to Model-3 in terms of its Sharpe Ratio but that this difference is not significant even at the 10% confidence level. This constitutes a surprising strong affirmation of the simple yield differential model.

Model-2 is the augmented random walk model that allows for the real interest rate differential to be a forecast of the change in the exchange rate. This model is the most aggressive of the three models with an average expected return of 22.98%. The high average expected return encourages the program to take more highly leveraged positions. This in turn leads to high actual excess returns averaging 36.15% per annum and a high standard deviation of 24.53% per annum. However, the Sharpe Ratio for the model is slightly less than Model-3 – 3.11 relative to 3.16. The JK statistic is -0.71. It is consequently not possible to reject the hypothesis that the two Sharpe Ratio's are the same at standard levels of statistical significance.

An alternative approach to the post-sample testing of the model is in terms of ex-post portfolio utilities. The utility of the portfolio, or the risk-adjusted excess return, is defined by:

$$(32) \quad u_t = r_t^p - \frac{1}{2\lambda}[r_t^p - E(r_t^p)]^2$$

where r_t^p is the expected portfolio excess return and $E(r_t^p)$ is the expected value of the portfolio return based upon the preceding period information set. The basic idea behind the risk-adjusted expected return is that deviations of the actual return from its expected value are penalized. These penalties are important because Model-1, the pure yield model, tends to underestimate the actual return because it neglects the interest rate induced currency appreciation while Model-2, the augmented random walk model, tends to be too aggressive in forecasting portfolio expected returns. In Table 5, we compare the three models in terms of the ex-post utility of the results.

Table 5: Ex-Post Utility Analysis

	Model-1	Model-2	Model-3
Actual Utility (Annual)	15.59%	21.11%	20.12%
Expected Utility (Annual)	10.94%	22.98%	15.98%
Standard Deviation (Annual)	17.46%	25.08%	20.65%
Sharpe Ratio (Annual)	1.88	1.77	2.05
Probability	2.97%	3.77%	1.98%
JK against Model-3	-0.52	-3.69	
Probability	30.29%	0.011%	

The ex-post utility analysis confirms the superiority of Model-3, particularly relative to Model-2. The estimated Sharpe Ratio is largest for Model-3 and the Jobson-Korkie test leads to a decisive rejection of Model-2 relative to Model-3. Somewhat surprisingly, the Jobson-Korkie test does not reject Model-1 relative to Model-3 despite the fact that Model-1 tends to underestimate the returns from the portfolio.

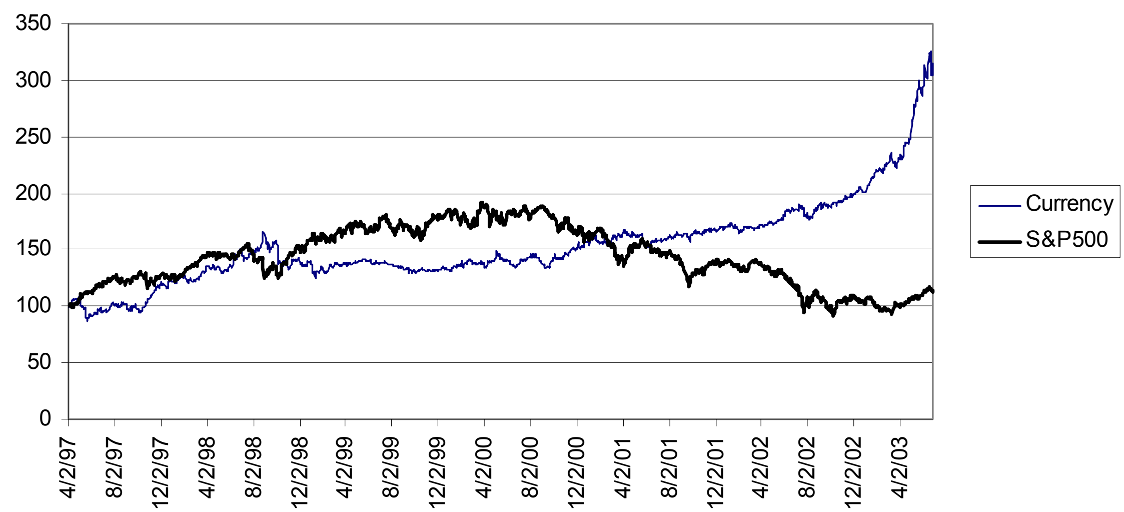
In conclusion, the results support Model-3 as the best performer in post-sample trading. Model-3's annual Sharpe Ratio is 3.16 and the probability that the true Sharpe Ratio is zero is 0.07%. It is important to note that the test of significance is based upon the distribution of the mean return. This means that the test statistic is:

(33) $$Z = \overline{X} / (\overline{s} / \sqrt{T})$$

where $\overline{X}$ is the average daily excess return and $\overline{s}$ is the estimated daily standard deviation. T is the total number of observations in the period running from April 1, 1997 to June 26, 2003 or 1628 daily observations. An alternative view of the results is to calculate the Sharpe Ratio for a single year. With an excess return of approximately 30% and a standard deviation of 20%, this single year Sharpe Ratio is around 1.5. If the expected return is used rather than the actual return on the grounds that the actual returns are influenced by uncertain events, the single year Sharpe Ratio is still around .8. By way of comparison, the Sharpe Ratio for traditional investment classes like common stocks, small firm stocks, corporate bonds and long term government bonds are typically around

.5.[13] In Figure 3., we compare the performance of the currency strategy with the performance of the S&P500 over the sample period. This comparison is biased against the currency model because the performance is based upon cumulative excess returns over the U.S. risk free rate while the S&P500 is based upon gross returns including dividends.[14]

Figure 3: Relative Performance



This chart demonstrates that the high rate of return on the currency portfolio is not simply a highly leveraged bet on the U.S. equity market. In fact, to the extent that a correlation exists, it appears to be negative rather than positive. This appearance is confirmed by the following regression equation, which relates the quarterly return on the currency portfolio to the return on the S&P500 index.

Table 6: The CAPM Regression

	Constant	Slope
Estimated Coefficient	.0499	.0337
Standard Error	(.0187)	(.1797)
T-State	2.66	0.18
S.E.E.	.0917	

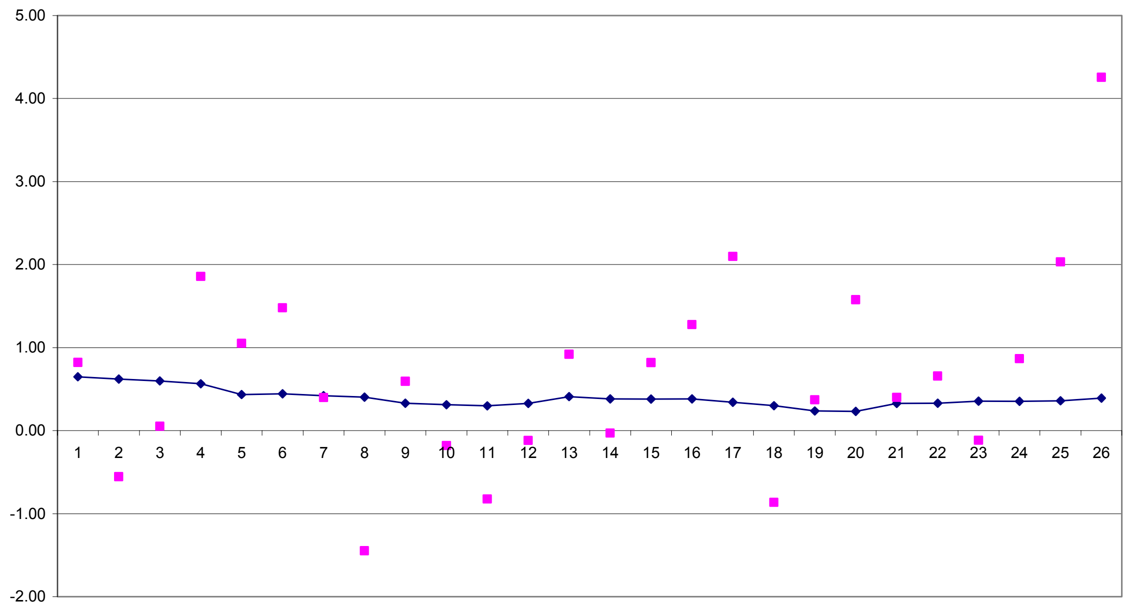
In the style of Gibbons, Ross and Shanken (1989) we can interpret this regression as a test of the efficiency of the S&P500 as a market portfolio. If the constant term is not significantly different from zero, then the statistical evidence

[13] This statement is based upon the SBBI return series from Ibbotson Associates over the time period from 1926 to 1999. See Francis and Ibbotson (2001).
[14] S&P 500 total returns were obtained from Yahoo Finance.

supports the efficiency of the index portfolio. In this case, the constant term is approximately 5% per quarter and the estimate is significantly different from zero at standard levels of statistical significance. The results consequently support the rejection of the null hypothesis. Basically, the currency portfolio has a zero beta against the S&P500 and an estimated alpha of around 5% per quarter.

In Figure 4, the post-sample Sharpe ratio is plotted along with the standardized residuals.

Figure 4: Post-Sample Sharpe Ratios



Over this period, the prior Sharpe ratio has declined from a value of approximately .65 per quarter to .40 per quarter. If we assume that the annual Sharpe ratio in the equity market is .5 (for example, a 10% expected excess return divided by a standard deviation of 20%), then the quarterly Sharpe would be .25. This suggests that the risk/return tradeoff in the currency market has continued to be favorable relative to traditional investments despite the global convergence of short term interest rates. In order to understand why this has happened, we need to be able to decompose the Sharpe Ratio into its component parts.

The portfolio Sharpe Ratio can be written as:

(34) $$R = \sqrt{E(r)'\Omega^{-1}E(r)}$$

Take the derivative of R with respect to the vector of expected returns:

(35) $$\frac{dR}{dE(r)} = \frac{\Omega^{-1}E(r)}{\sqrt{E(r)'\Omega^{-1}E(r)}}$$

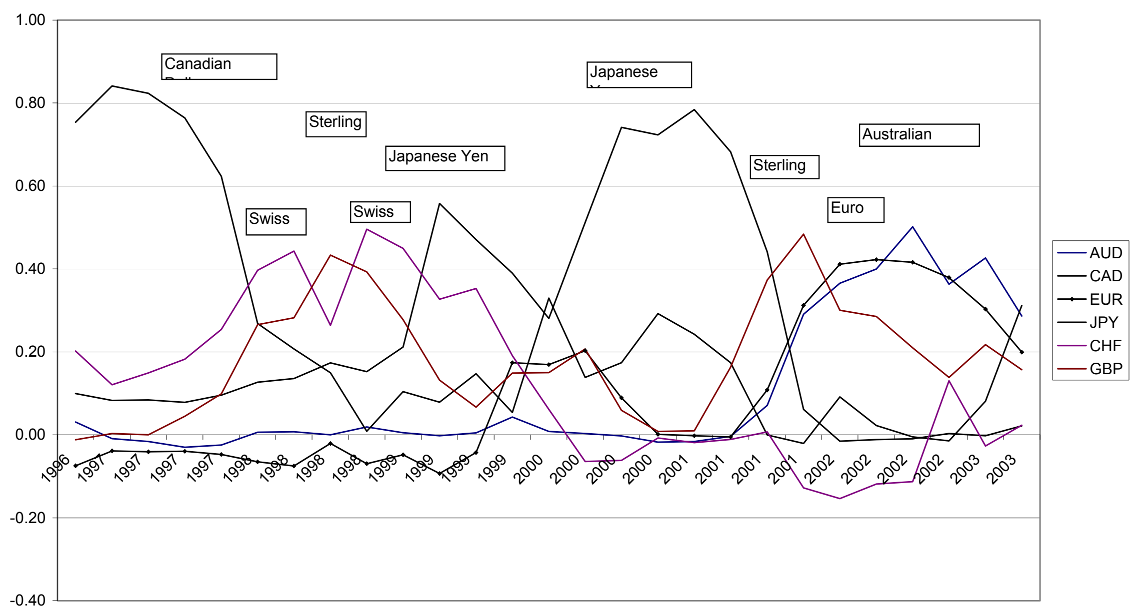
The component Sharpe of the n'th asset in the portfolio is defined as the product of the expected return on the asset and the derivative:

(36) $$R_n = E(r_n)\frac{dR}{dE(r_n)}$$

These components have the desirable property that they sum to the total Sharpe ratio.[15]

(37) $$R = E(r)'\frac{dR}{dE(r)}$$

Figure 5: Sharpe Components



In Figure 5, the estimated Sharpe components are expressed as percentages of the total exposure. It is clear from the chart that the strategy involves heavy weightings – up to 80% of the total – to particular currencies at certain times. At the start of the post-sample simulation, most of the exposure was to the Canadian dollar. In the 1998-1999 period, the position shifted towards the European currencies, particularly the Swiss franc and the pound Sterling. In 2000-2001, short positions in the Japanese Yen were the dominant source of

[15] This methodology is based upon Mark Garman's approach to component value at risk. Garman (2000).

exposure. Finally, in 2002-2003, long positions in the Euro and the Australian dollar were dominant. Negative Sharpe components indicate hedge positions. In 2001-2002, the program was taking long positions in Sterling and Euro's and partially hedging these positions with short positions in Swiss francs. During this period, the Swiss franc was making a negative contribution to the Sharpe ratio. For the most part, however, negative components are small and unimportant. This indicates that spread trading is not of particular importance for the model.

Conclusion

The forward parity puzzle was one of many anomalies discovered in the 1970's as academic researchers began to confront financial theory with empirical evidence. In some cases, the anomalies have remained anomalous while in other cases the anomalies have either disappeared or become imbedded in other theories. In the equity market, the size and value effects have survived while the January effect appears to have lost its predictive power. However, none of these anomalies is comparable to the forward parity puzzle both in terms of the size of the risk premium, its predictability, and its lack of correlation with other risk premia. The original purpose of this study was to demonstrate that the currency risk premium had declined due to the convergence of global interest rates and the adoption of programs emphasizing monetary stability by the world's central banks. However, despite the best efforts of the author, the currency risk premium was found to be large, predictable, and uncorrelated with other risk premia. In fact, the last few years have witnessed one of the best periods for the premium in its entire history. While the U.S. and world equity markets declined, the profitability of the carry trade in currencies rose as U.S. rates fell relative to European, Canadian, and Australian rates. While there is some evidence that central banks are paying attention to the effect of their policies on the exchange rate, they remain primarily concerned with inflation and domestic financial market considerations. This appears to give rise to an opportunity for hedge funds and large banks to profit from differences in yields. The advent of electronic currency trading platforms is making this game accessible to retail investors with small amounts of capital. It may be that the solution to the puzzle must wait until these participants have a sizeable influence on the market. In some future time, we may observe households financing their homes in Japanese Yen and holding their bank accounts in New Zealand dollars.

Data Appendix

Exchange rates and eurodeposit rates from 1976 to 1988 are taken from the Harris Bank data set maintained by Richard Levich at New York University. The Harris Bank data is a weekly data set. Quarterly series were based upon the last day closest to the end of the month.

The Harris Bank data set does not include Australian dollar deposit rates for the period 1976 to 1988. For this period, three and six months deposit rates are taken from the Reserve Bank of Australia and represent the yields on domestic Bankers Acceptances.

Exchange rates and eurodeposit rates from 1989 to 2003 are taken from the Economagic web sit at www.economagic.com. The original source is the British Bankers Association. Quarterly series were developed from the daily data by taking the last observation for the month.

Yields on 10 Year Government Bonds from 1980 to 2003 were taken from Datastream. Early data on bond yields was taken from the International Financial Statistics CD-ROM published by the International Monetary Fund.

Data on the total return on the S&P500 was taken from Yahoo Finance at http://finance.yahoo.com.

A complete set of the data employed in this study is available from the author at j.bilson@mbs.edu.

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